LAPLACE 110302

Sia $\delta_{-1}(t)$ il gradino unitario centrato in $t=0$.

a) Calcolare la trasformata di Laplace delle seguenti funzioni:

$$y_1(t) = \left(\frac{t^5}{5} + \frac{t^2}{2}\right)\delta_{-1}(t)$$

$$y_2(t) = \left(e^{-t} + 2t\,e^{-2t}\right)\delta_{-1}(t)$$

$$y_3(t) = \left(e^{t} + t^2 e^{t}\right)\delta_{-1}(t)$$

$$y_4(t) = (t-2)^3\,\delta_{-1}(t)$$

$$y_5(t) = \left(\operatorname{sen}(t) + \cos(t)\right)\delta_{-1}(t)$$

$$y_6(t) = \operatorname{sen}(10t+5)\,\delta_{-1}(t)$$

$$y_7(t) = t\cos(2\pi t)\,\delta_{-1}(t)$$

$$y_8(t) = t\,e^{-t}\operatorname{sen}(3t)\,\delta_{-1}(t)$$

$$y_9(t) = e^{\alpha(t-t_0)}\,\delta_{-1}(t)$$

$$y_{10}(t) = e^{3t}\,\delta_{-1}(t-1)$$

LAPLACE 140302

b) Calcolare la trasformata inversa di Laplace delle seguenti funzioni

$$F_1(s) = \frac{1}{s+2}$$

$$F_2(s) = \frac{s+1}{4s^2+\pi^2}$$

$$F_3(s) = \frac{s^2+3}{(s+2)(s+1)}$$

$$F_4(s) = \frac{s^2+3}{(s+2)(s+1)^2}$$

$$F_5(s) = \frac{7s^2-8s+5}{s^3+2s^2+5s}$$

Soluzioni

a) $$Y_1(s) = \frac{24}{s^6} + \frac{1}{s^3}, \quad Y_2(s) = \frac{1}{s+1} + \frac{2}{(s+2)^2},$$

$$Y_3(s) = \frac{s^2-2s+3}{(s-1)^3}, \quad Y_4(s) = \frac{-8s^3+12s^2-12s+6}{s^4},$$

$$Y_5(s) = \frac{s+1}{s^2+1}, \quad Y_6(s) = \frac{\operatorname{sen}(5)\,s + 10\cos(5)}{s^2+100},$$

$$Y_7(s) = \frac{s^2-4\pi^2}{(s^2+4\pi^2)^2}, \quad Y_8(s) = \frac{6(s+1)}{(s^2+2s+10)^2},$$

$$Y_9(s) = \frac{e^{-\alpha t_0}}{s-\alpha}, \quad Y_{10}(s) = \frac{e^{3-s}}{s-3}.$$

b) $$f_1(t) = \frac{1}{\sqrt{2}}\operatorname{sen}(\sqrt{2}\,t)\,\delta_{-1}(t),$$

$$f_2(t) = \left(\frac{1}{4}\cos\left(\frac{\pi}{2}t\right) + \frac{1}{2\pi}\operatorname{sen}\left(\frac{\pi}{2}t\right)\right)\delta_{-1}(t),$$

$$f_3(t) = \delta_0(t) + \left(4e^{-t} - 7e^{-2t}\right)\delta_{-1}(t), \quad (*)$$

$$f_4(t) = \left(7e^{-2t} - 6e^{-t} + 4t\,e^{-t}\right)\delta_{-1}(t),$$

$$f_5(t) = \left(1 + 10\,e^{-t}\operatorname{sen}(2t+\varphi)\right)\delta_{-1}(t),$$

con $\varphi = \angle\left(-\frac{8}{3} + 2j\right) = \operatorname{arctg}\left(-\frac{3}{4}\right) + \pi$.

(*) $\delta_0(t)$ rappresenta l'impulso unitario centrato in $t=0$.