Mini-course on filtering and mobile robot localization

Abstract. We deal with discrete time stochastic systems of the type

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}, w_{k-1}) \\ y_k &= h(x_k, v_k), \end{aligned}$$

where x_k is the state of the system at time k, y_k the measurement, u_k the control input and w_k and v_k are noises. The problem is to provide an estimate of the state x_k given all available measurements $y^k = \{y_1, y_2, \ldots, y_k\}$ and past controls $u^{k-1} = \{u_0, u_1, \ldots, u_{k-1}\}$.

Let $p(x_k|u^{k-1}, y^k)$ denote the probability density function (pdf) characterizing the state x_k given y^k and u^{k-1}. If the model (i.e. f and h) is linear and the noises are Gaussian and satisfy an independence condition, $p(x_k|u^{k-1}, y^k)$ is a Gaussian pdf. In this case, the Kalman Filter (KF) will provide much more than the optimal Least Squares (LSQ) and the Maximum A Posteriori (MAP) estimation of x_k: since a Gaussian pdf is completely determined by its mean and covariance, the KF will actually furnish in this case a *complete description* of $p(x_k|u^{k-1}, y^k)$, and can be considered for this reason an *exact* filter.

In the non linear case this strong property is lost: $p(x_k|u^{k-1}, y^k)$ is a generic pdf and non parametric approximations of it, like the Histogram Filter (HF) or the Particle Filter (PF), are often adopted. It is also possible in the non linear case to derive the equations of the optimal *linear* LSQ estimator. Two approximate practical implementations of it are: the Extended Kalman Filter (EKF), based on a linearization of the nonlinearities, and the Unscented Kalman Filter (UKF), based on the Unscented Transformation. Both of them are not optimal estimators (in any sense, being only an approximation of an estimator which is optimal in the LSQ sense only among linear estimators) and can also be seen as a Gaussian (hence unimodal) approximation of $p(x_k|u^{k-1}, y^k)$.

The description of all these methods (and properties) and their application to the localization of a mobile robot equipped with internal sensors (like encoders) and external sensors (like lasers) is the objective of this mini-course (which is perhaps more focused on filtering than on mobile robotics...).