Fluidodinamica Numerica

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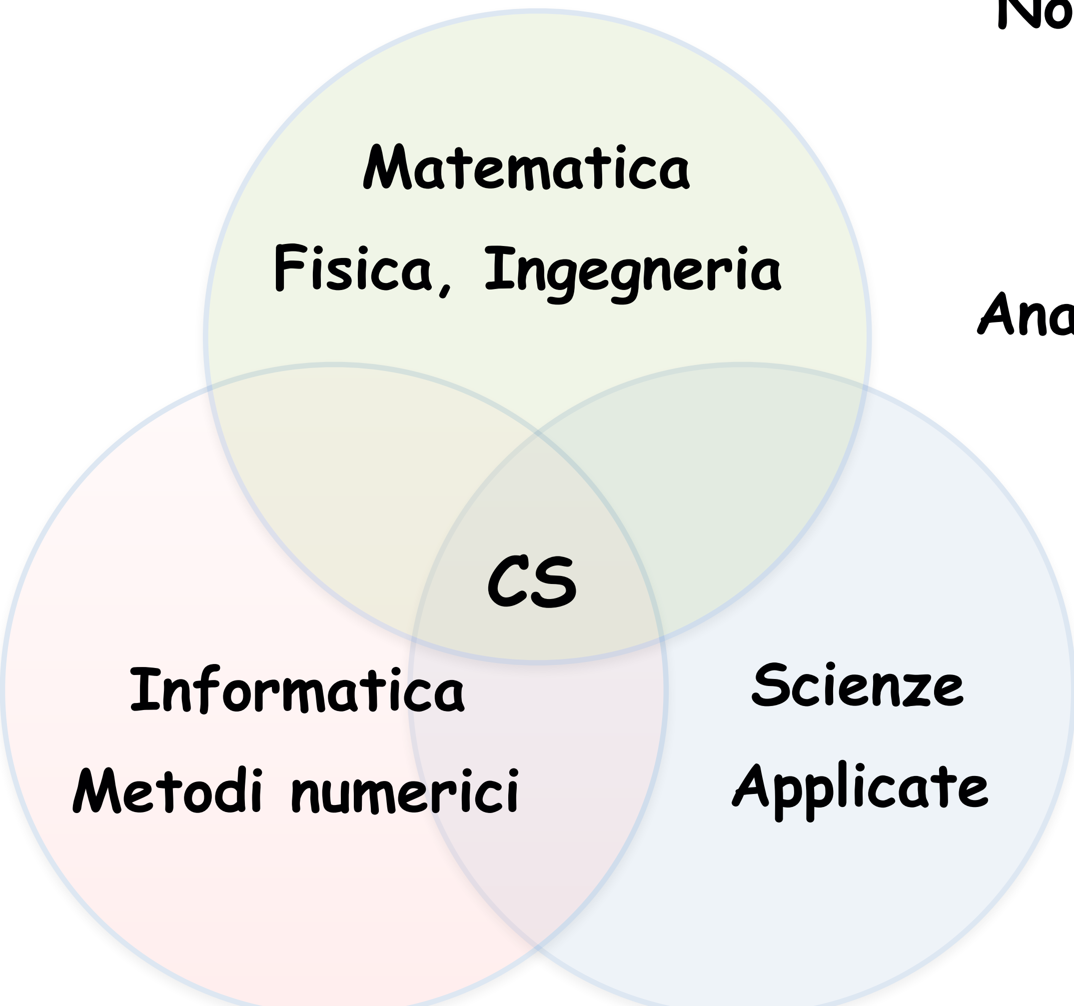
Computational Engineering

- Un insieme di discipline complementari utilizzate per costruire **modelli predittivi**



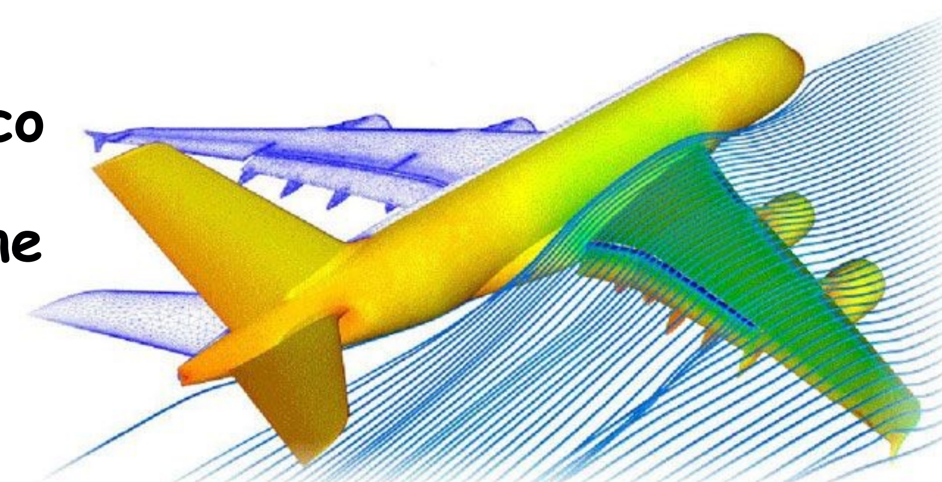
Non serve modello Matematico

Analisi dimensionale

Modello matematico

Metodo di soluzione

Analisi dati



La "**computational engineering**" è molto sviluppata in alcuni campi (aerospace, automotive) e si basa sull'analisi numerica

The Navier-Stokes equations

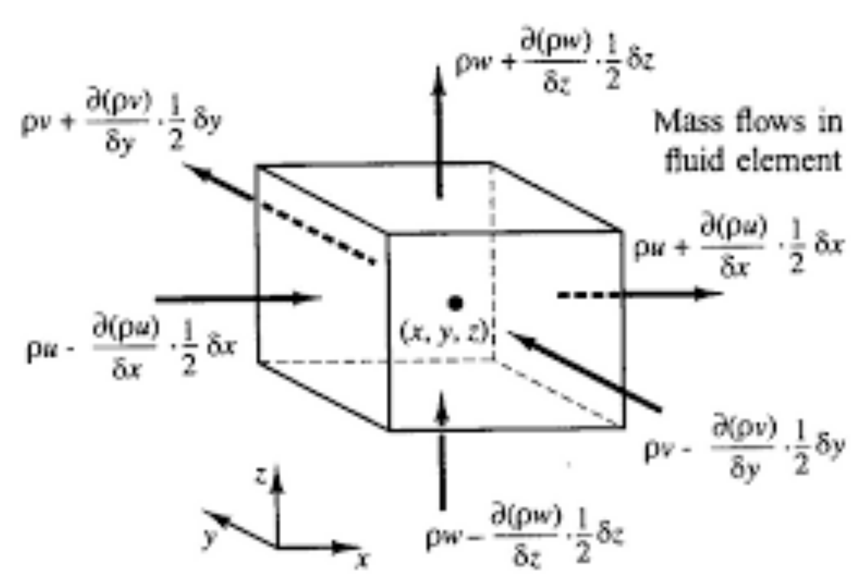
The Navier-Stokes equations govern the motion of fluids and can be seen as Newton's second law of motion for fluids. In the case of a compressible Newtonian fluid, this yields

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

$\mathbf{u}$ **velocity**

p **pressure**



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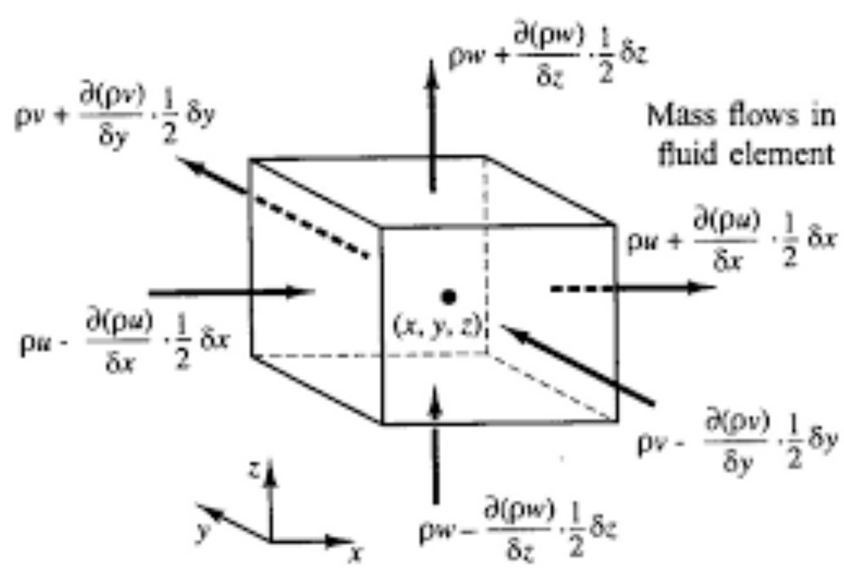
F $\qquad$ ma

$\mathbf{u}$ velocity

p pressure

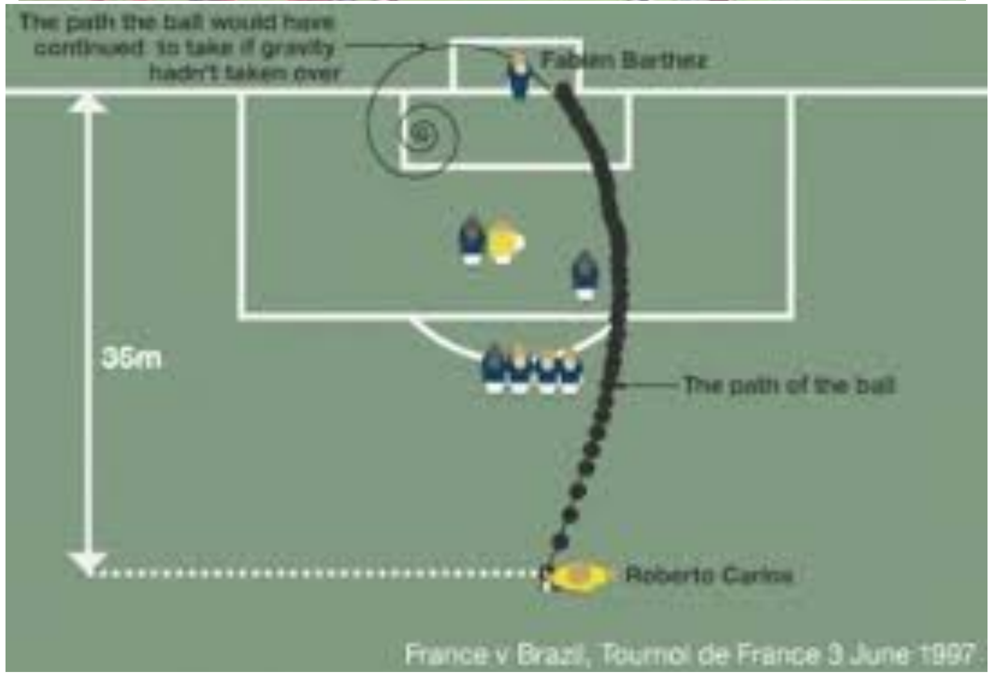


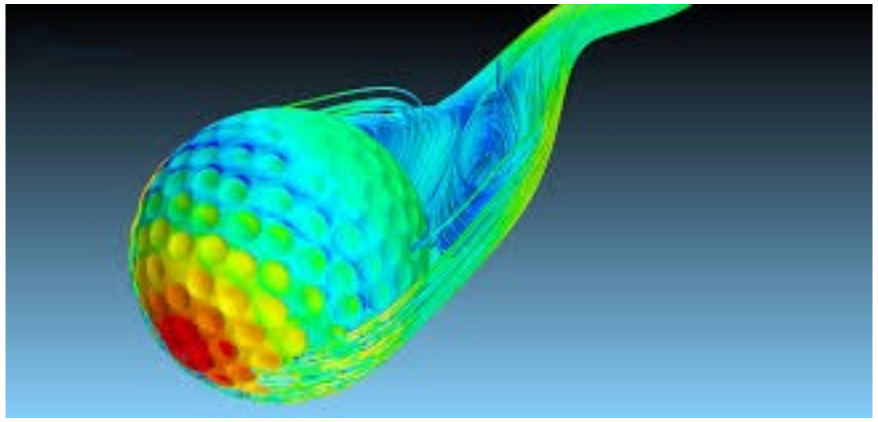
The Navier-Stokes equations in *Aeronautics*

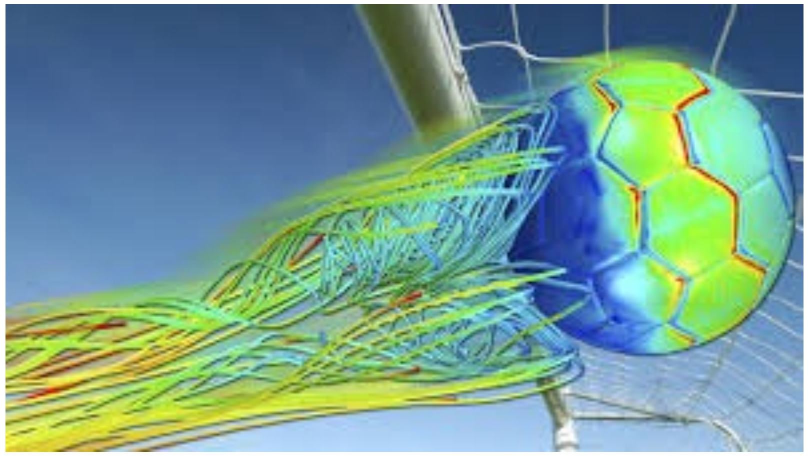
The Navier-Stokes equations in *Energy Harvesting*

The Navier-Stokes equations in *Car Aerodynamics*

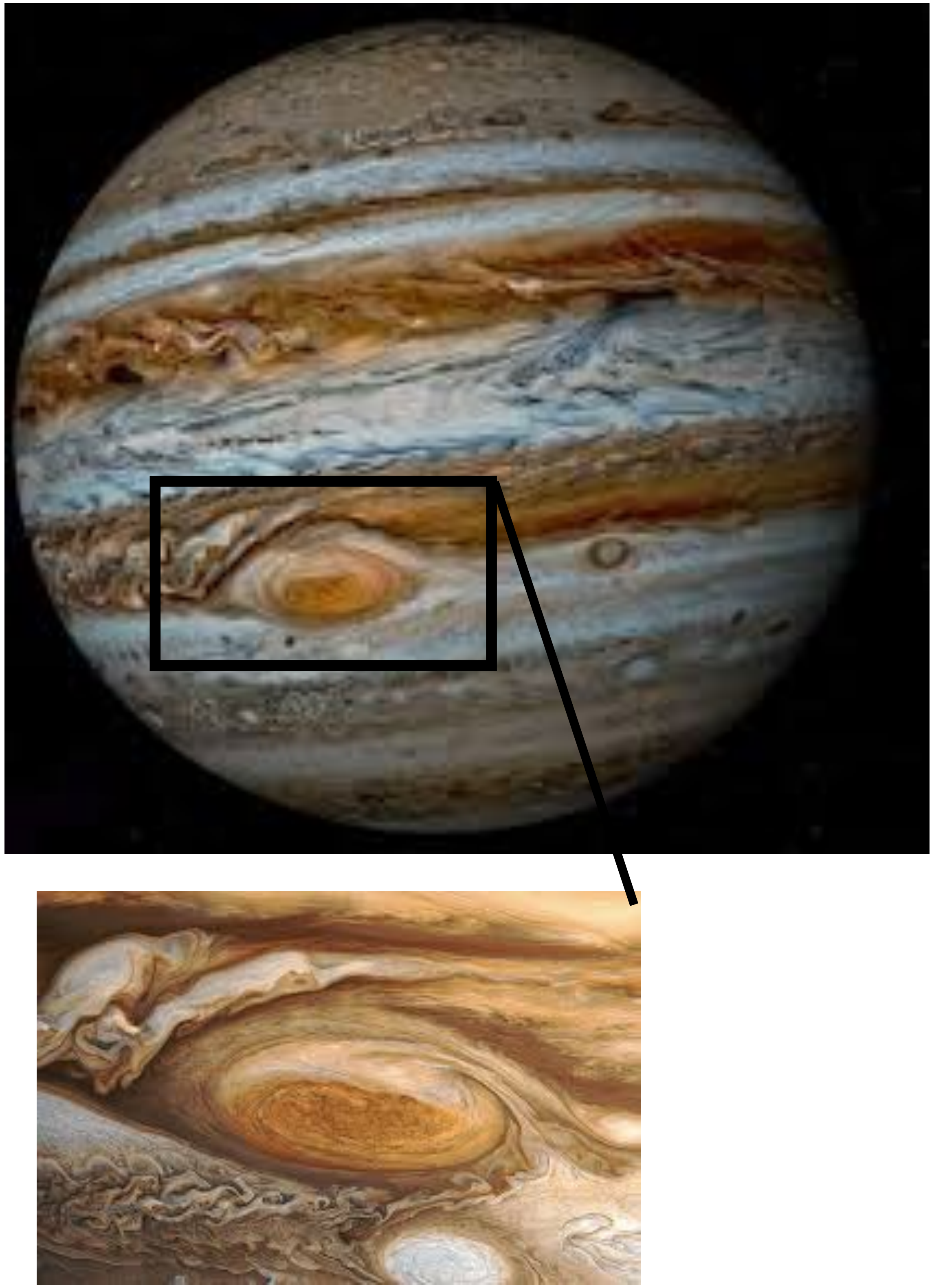
The Navier-Stokes equations in *Sports*

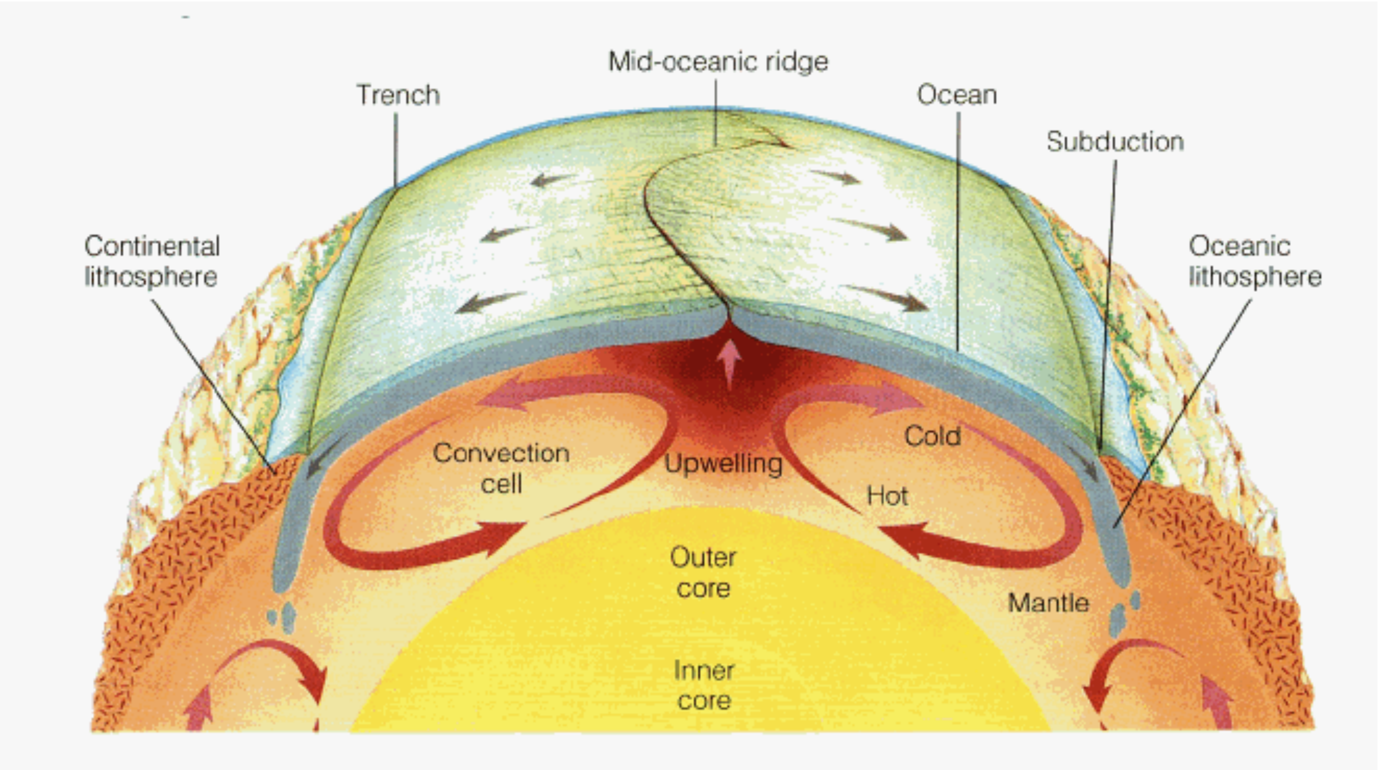




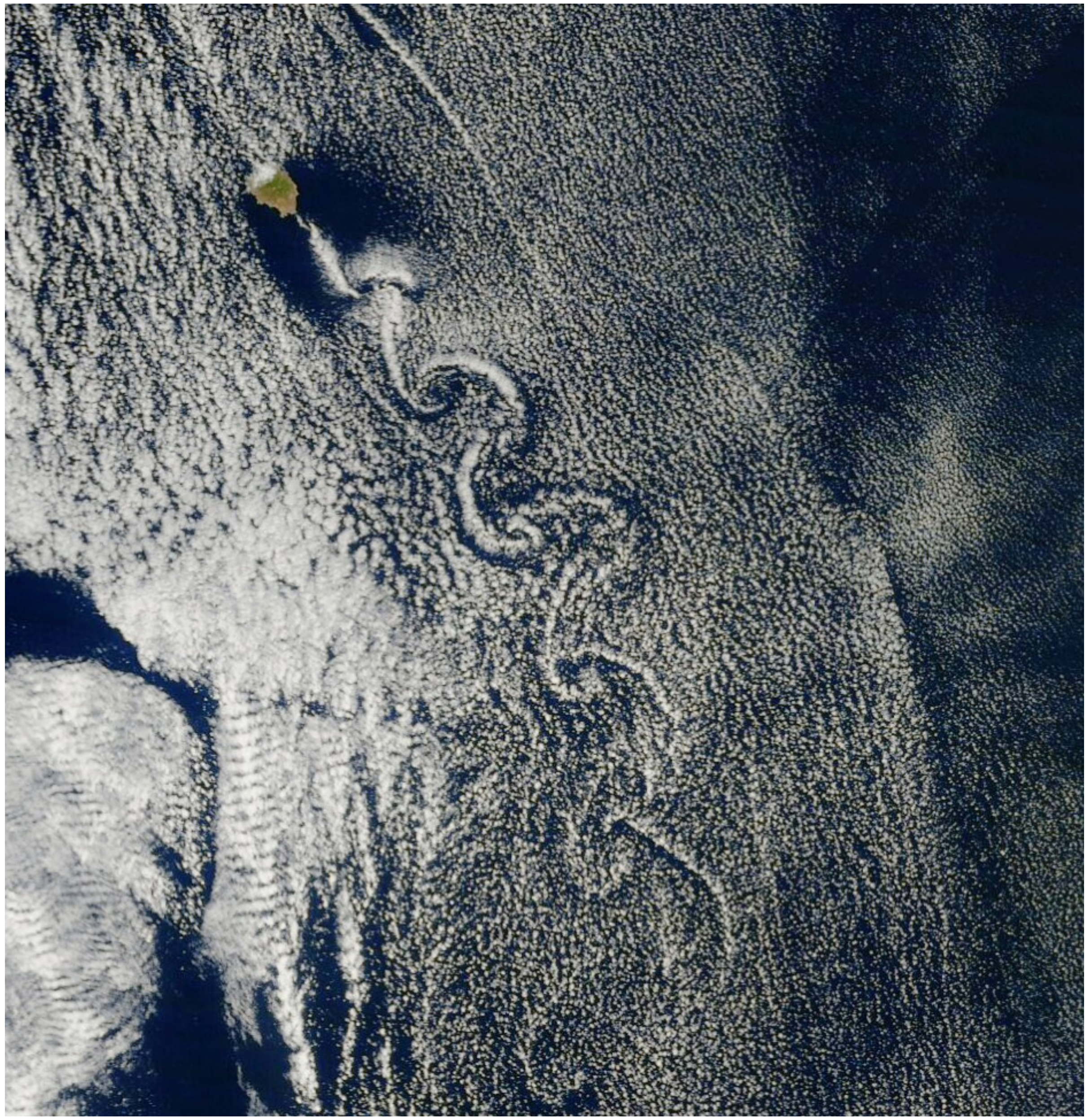


The Navier-Stokes equations in *Geophysics*

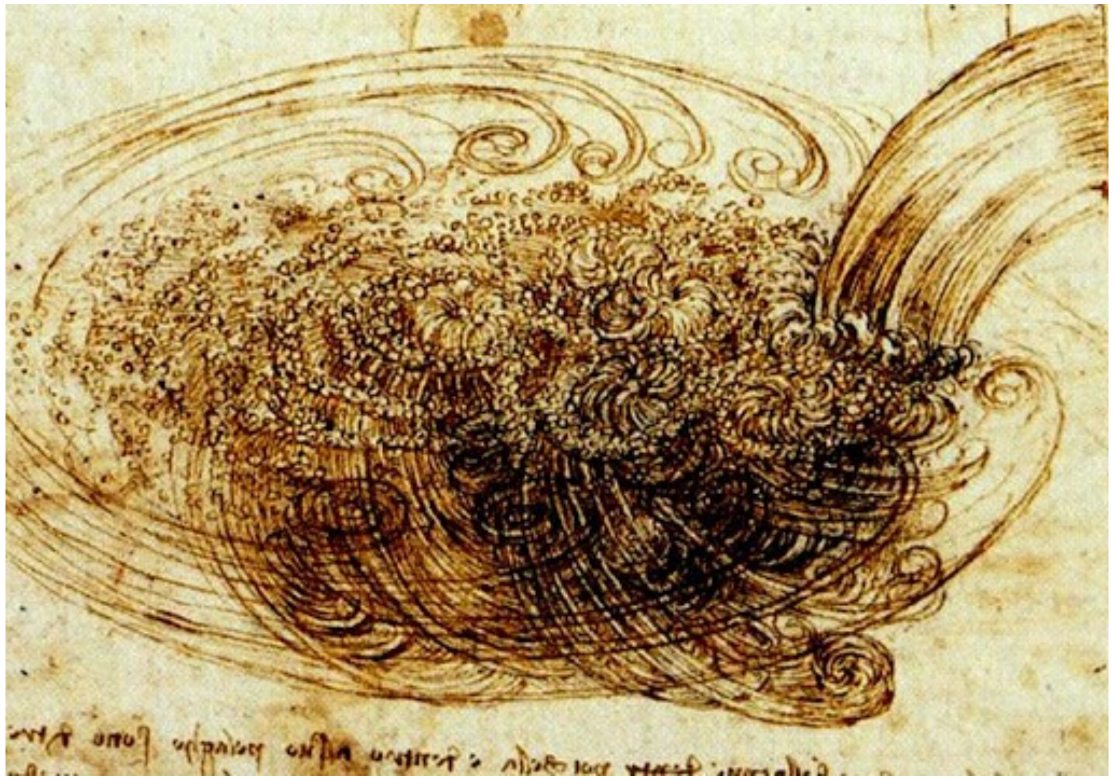


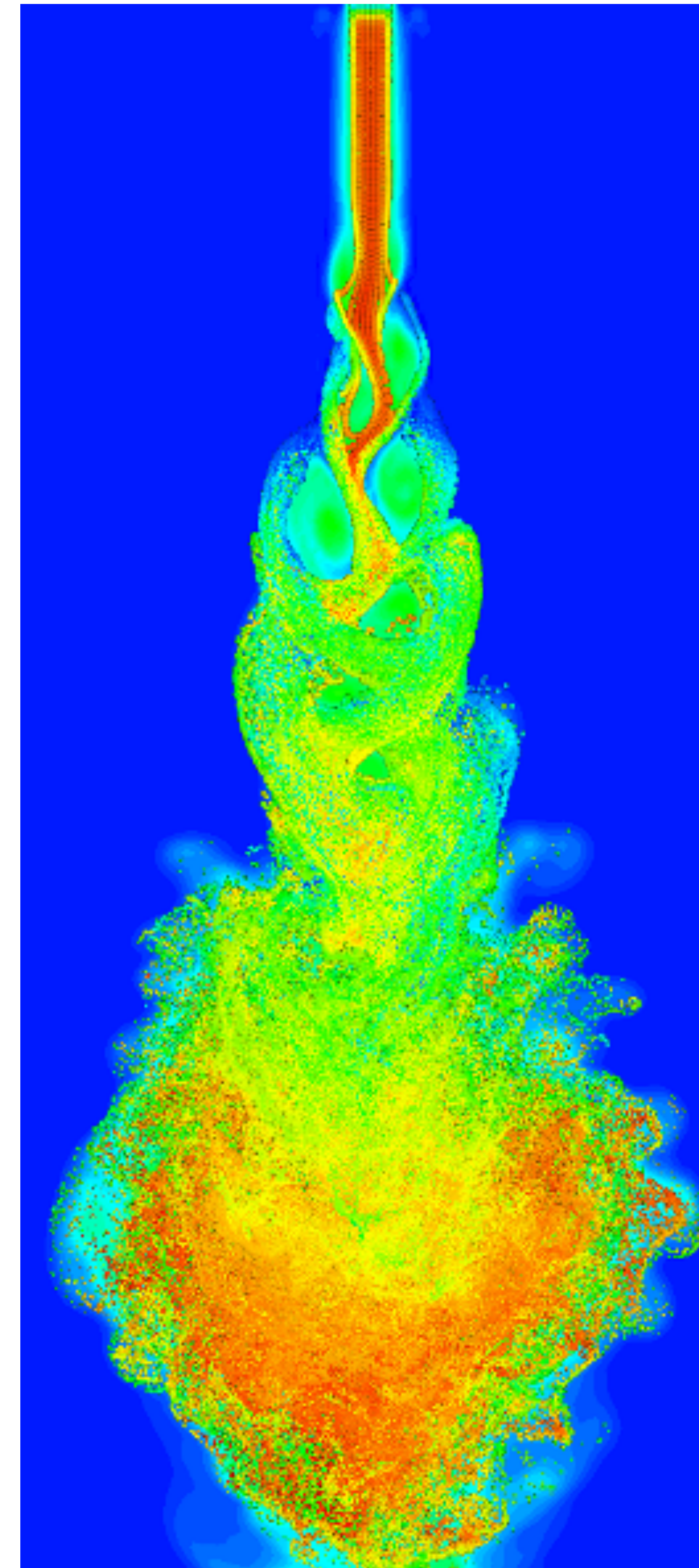


The Navier-Stokes equations in *Atmospheric flows*



Navier-Stokes equations govern *turbulence* as well



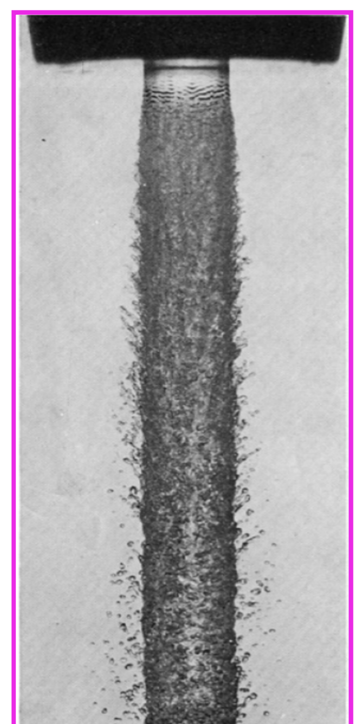


Laminar ⇨ **Instability** ⇨ **Pattern** ⇨ **Turbulence**

Marmottant and Villermaux (2004)

Rayleigh (1891)

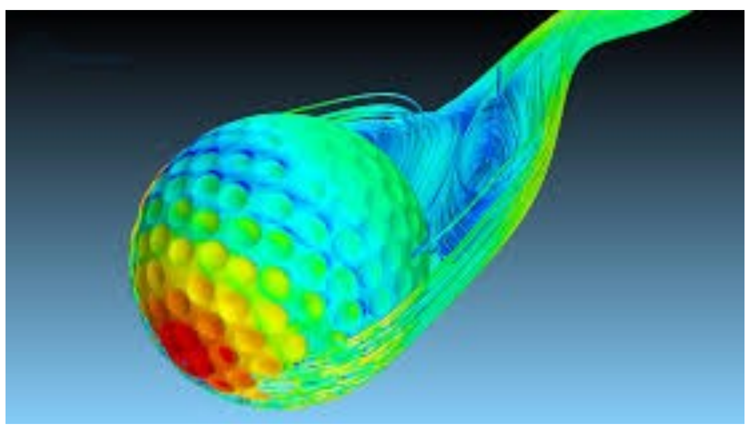
Marmottant and Villermaux (2004)



Hoyt and Taylor (1977)

steady ⇨ **ω** ⇨ **ω_1, ω_2** ⇨ **full frequency spectrum**

Solution of the Navier-Stokes equations find several applications



$$\nabla \cdot \mathbf{u} = 0$$

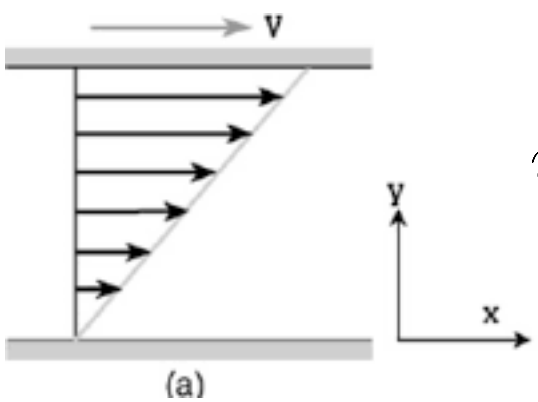
$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

…however only few analytical solutions are known

$$\nabla \cdot \mathbf{u} = 0$$

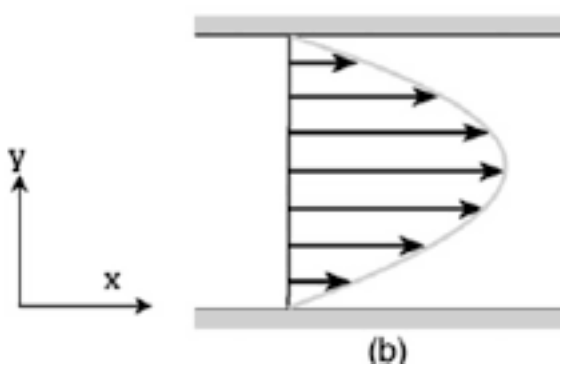
$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

Plane Couette



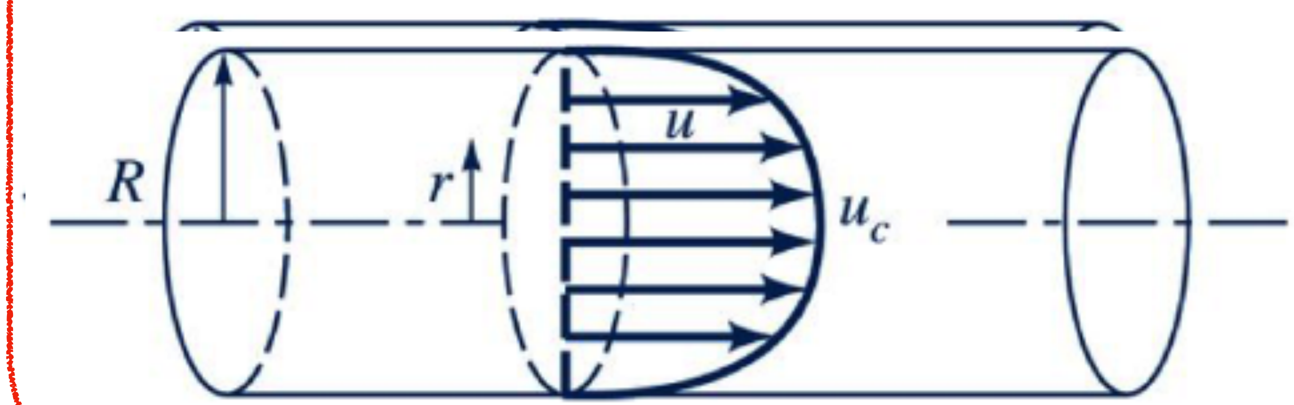
$$u_x(y) = U \frac{y}{h}$$

Plane Poiseuille



$$u_x(y) = -\frac{1}{2\mu} \frac{\partial p}{\partial x} (h^2 - y^2)$$

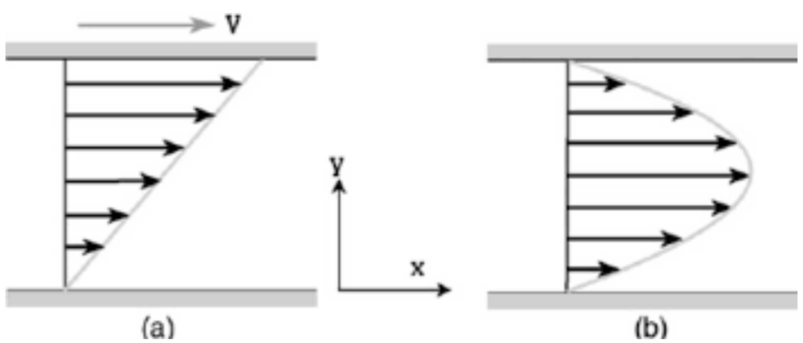
Hagen-Poiseuille



$$u_x(r) = -\frac{1}{4\mu} \frac{\partial p}{\partial x} (R^2 - r^2)$$

Many flows of interest can not be solved analytically

Ideal flows



$\neq$

Real flows

COMPUTATIONAL FLUID DYNAMICS (CFD)

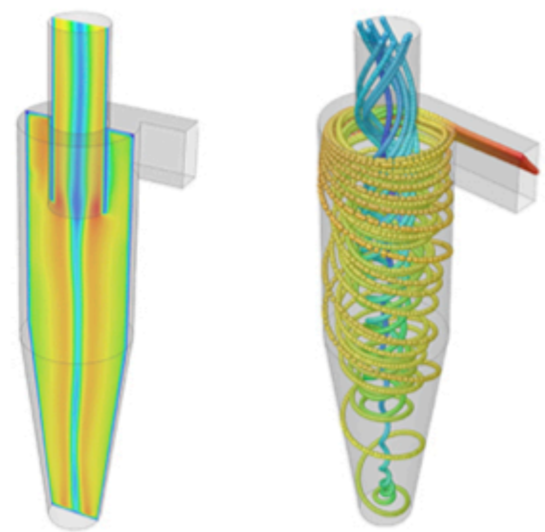
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$
$$\nabla \cdot \mathbf{u} = 0$$

$\mathbf{u}_d, \; p_d$

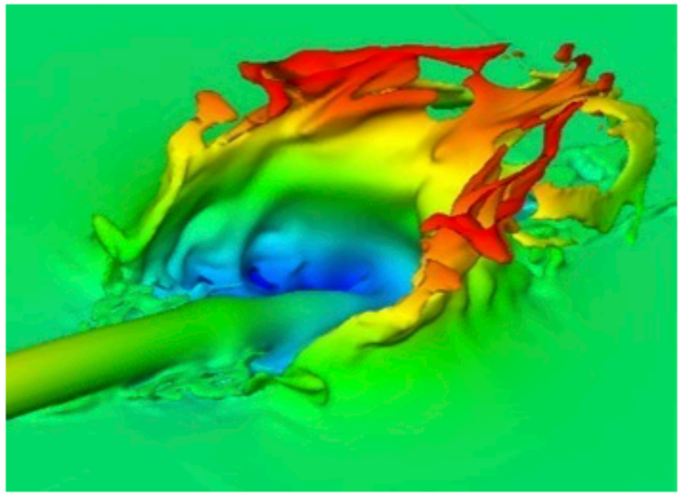
Applications of CFD

Method	Advantages	Disadvantages
Experimental	1. More realistic 2. Allows "complex" problems	1. Need for instrumentation 2. Scale effects 3. Difficulty in measurements & perturbations 4. Operational costs
Theoretical	1. Simple information 2. General validity 3. Understanding and interpretation of phenomena	1. Limited to simple cases 2. Typically linear problems
CFD	1. Not limited to linear cases 2. Allows "complex" problems 3. Stationary and non-stationary 4. Relatively affordable cost 5. Integration in the project chain	1. Errors: discretization, truncation 2. Difficulty in boundary conditions 3. Simplifications needed 4. Time for set-up & run 5. Time for post-processing 6. Difficult interpretation

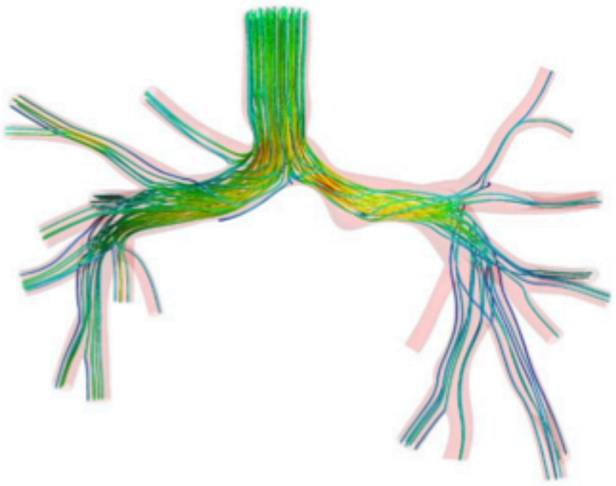
Applications of CFD



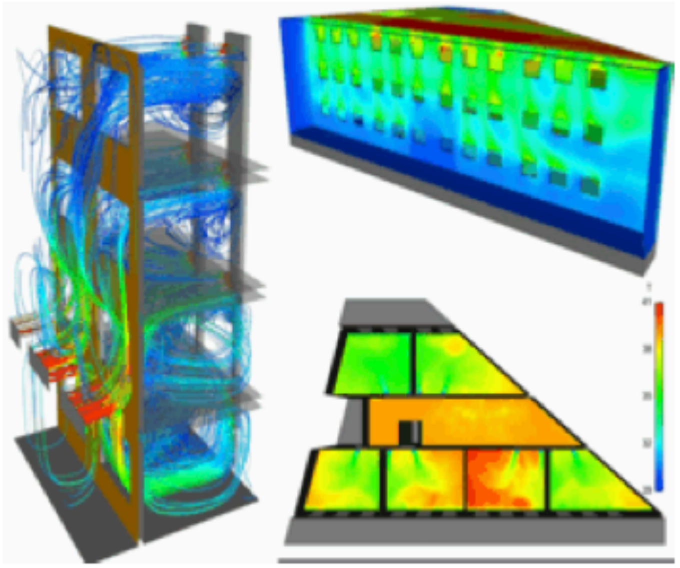
Industry



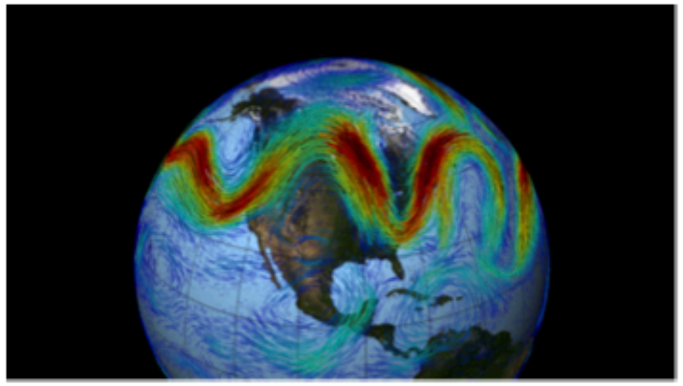
Physics



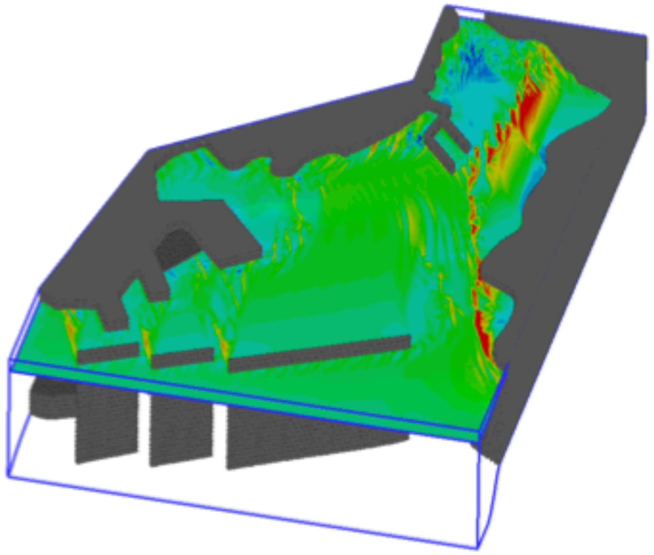
Medicine



Architecture

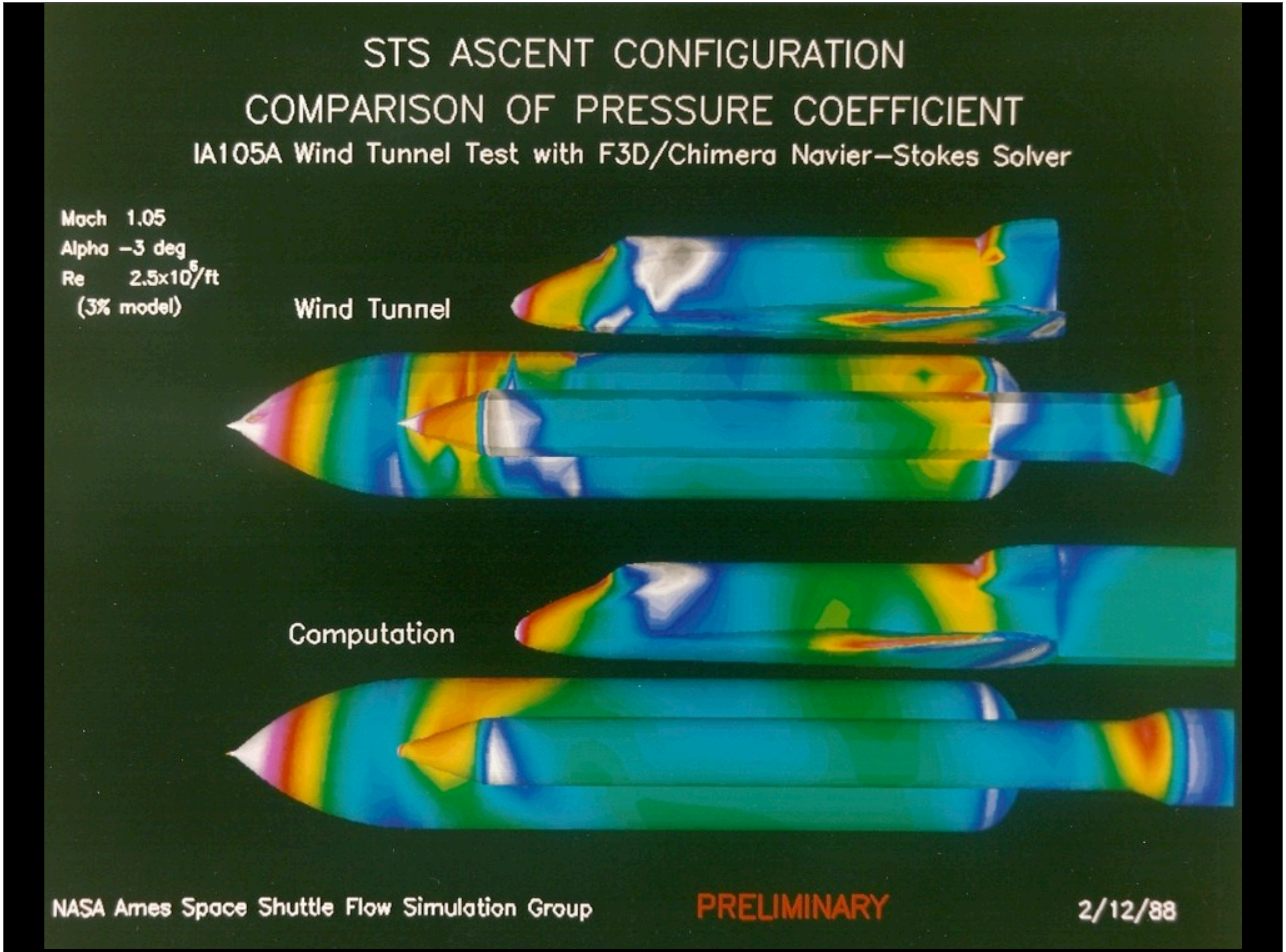


Meteorology



Environment

CFD vs experiments: the Space Shuttle

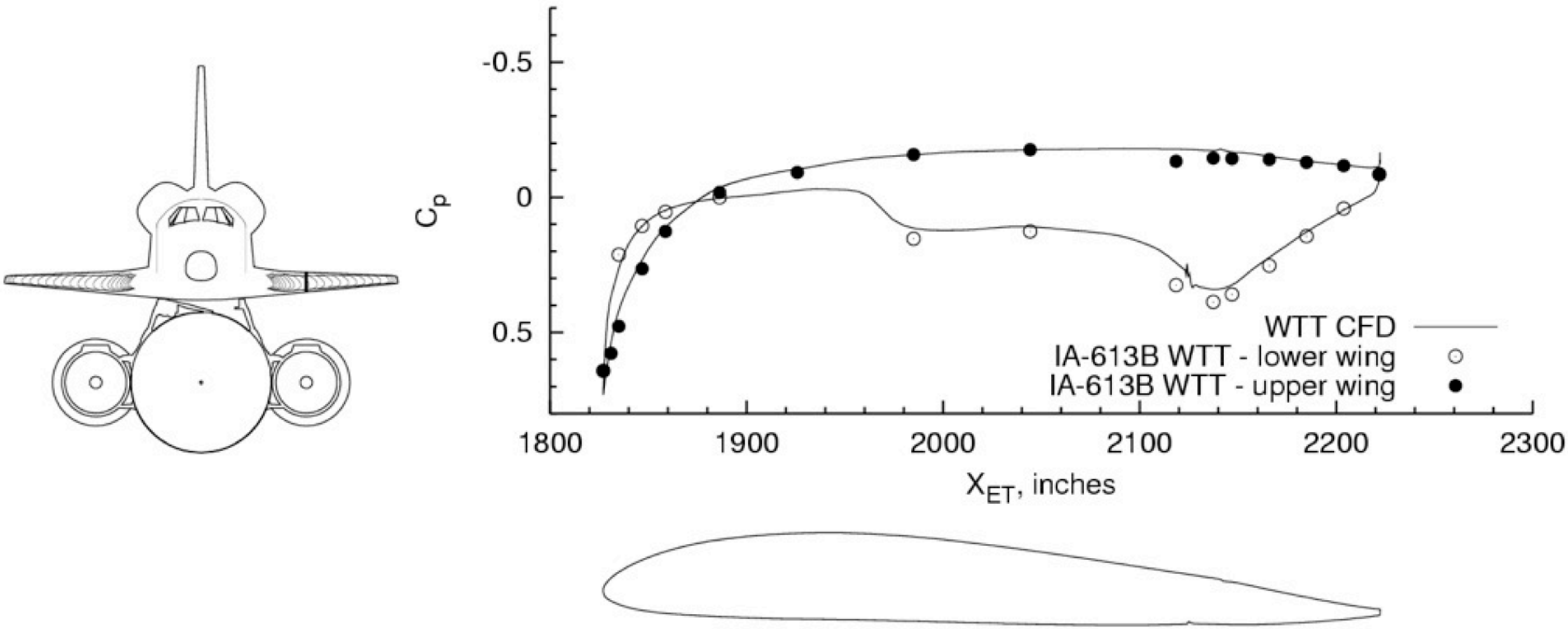


from Reynaldo J. Gomez III, NASA

CFD vs experiments: the Space Shuttle

CFD conditions: M_∞ = 2.50, α = 2.03°, β = 0.00°, Reynolds # = 2.50 x10^6/ft, IB elevon = 4.07°, OB elevon = -4.39°

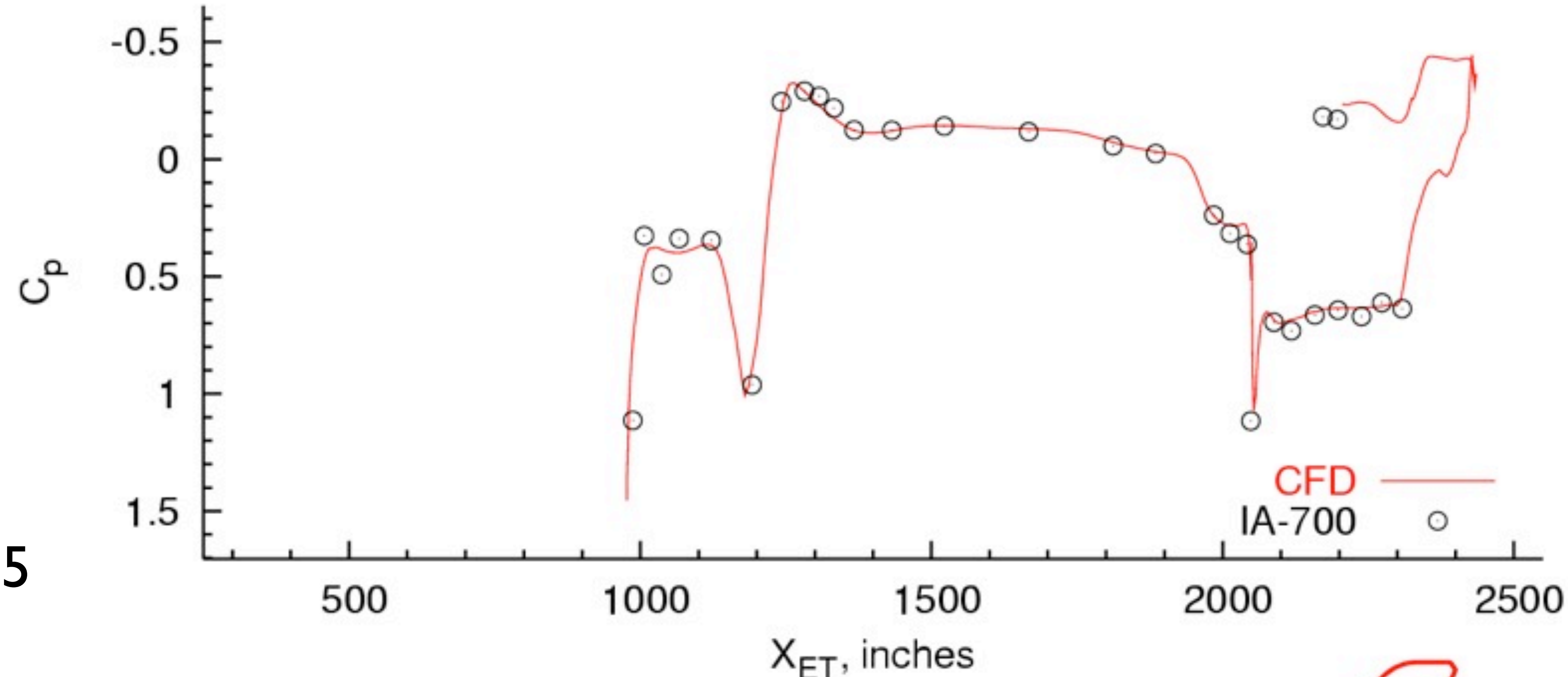
WTT conditions: M_∞ = 2.50, α = 2.03°, β = 0.00°, Reynolds # = 2.50 x10^6/ft, IB elevon = 4.07°, OB elevon = -4.39°



AIAA 2004-2226

from Reynaldo J. Gomez III, NASA

CFD vs experiments: the Space Shuttle



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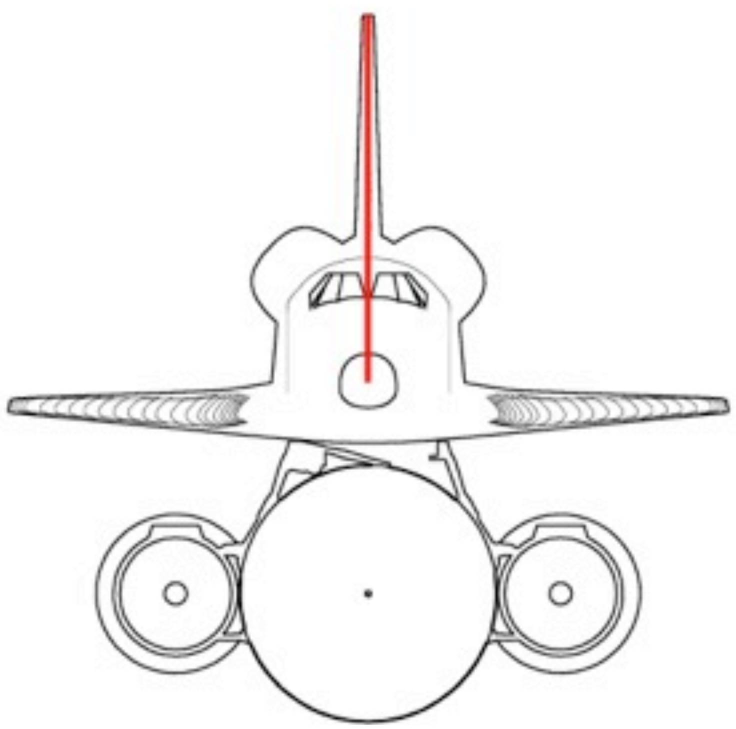

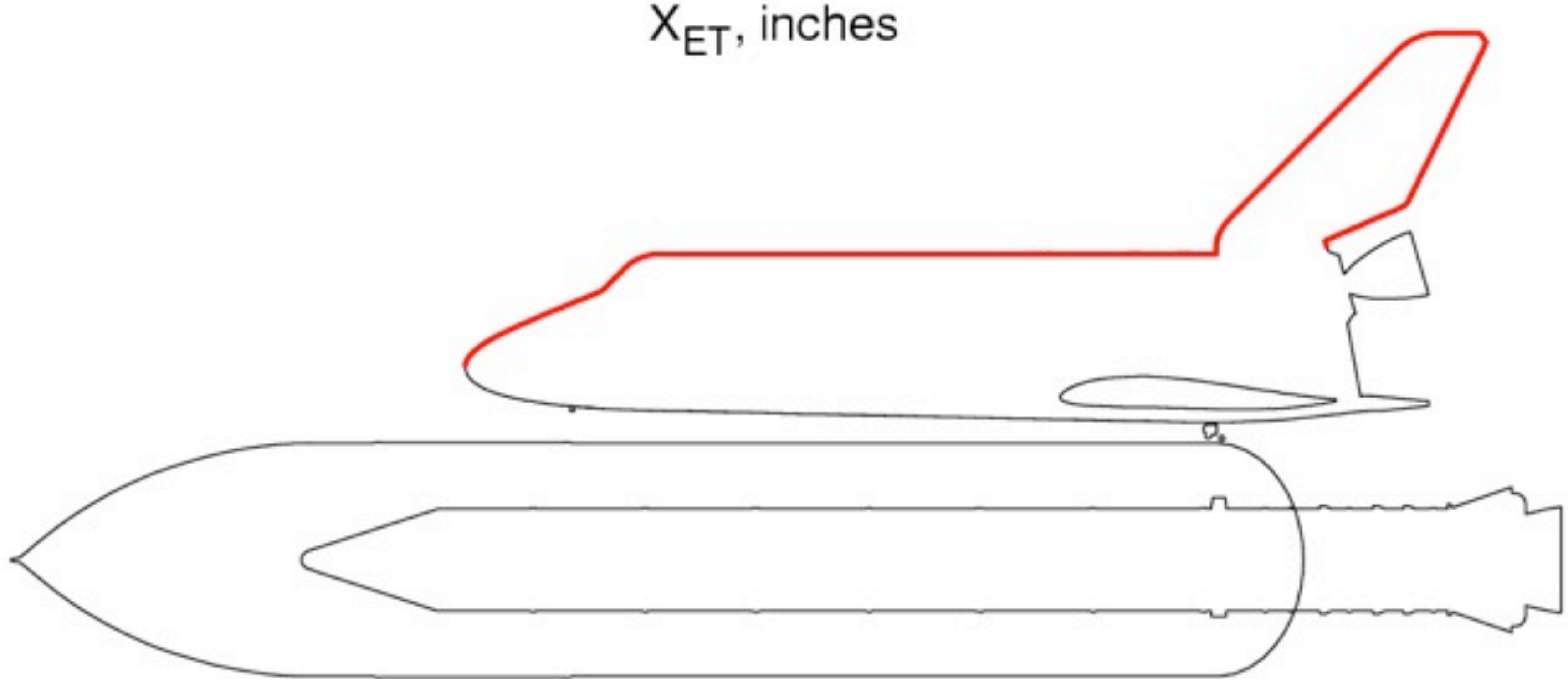

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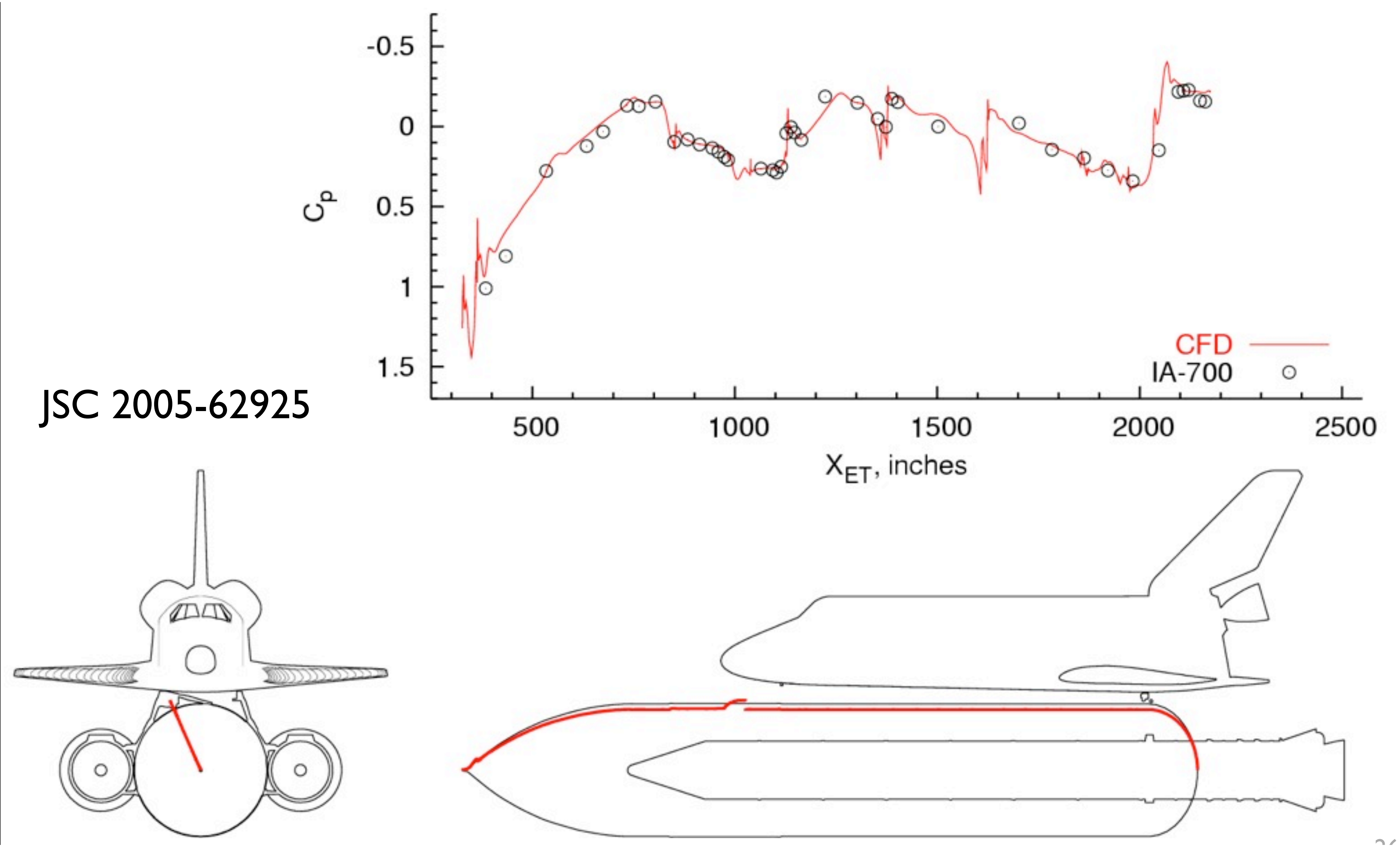

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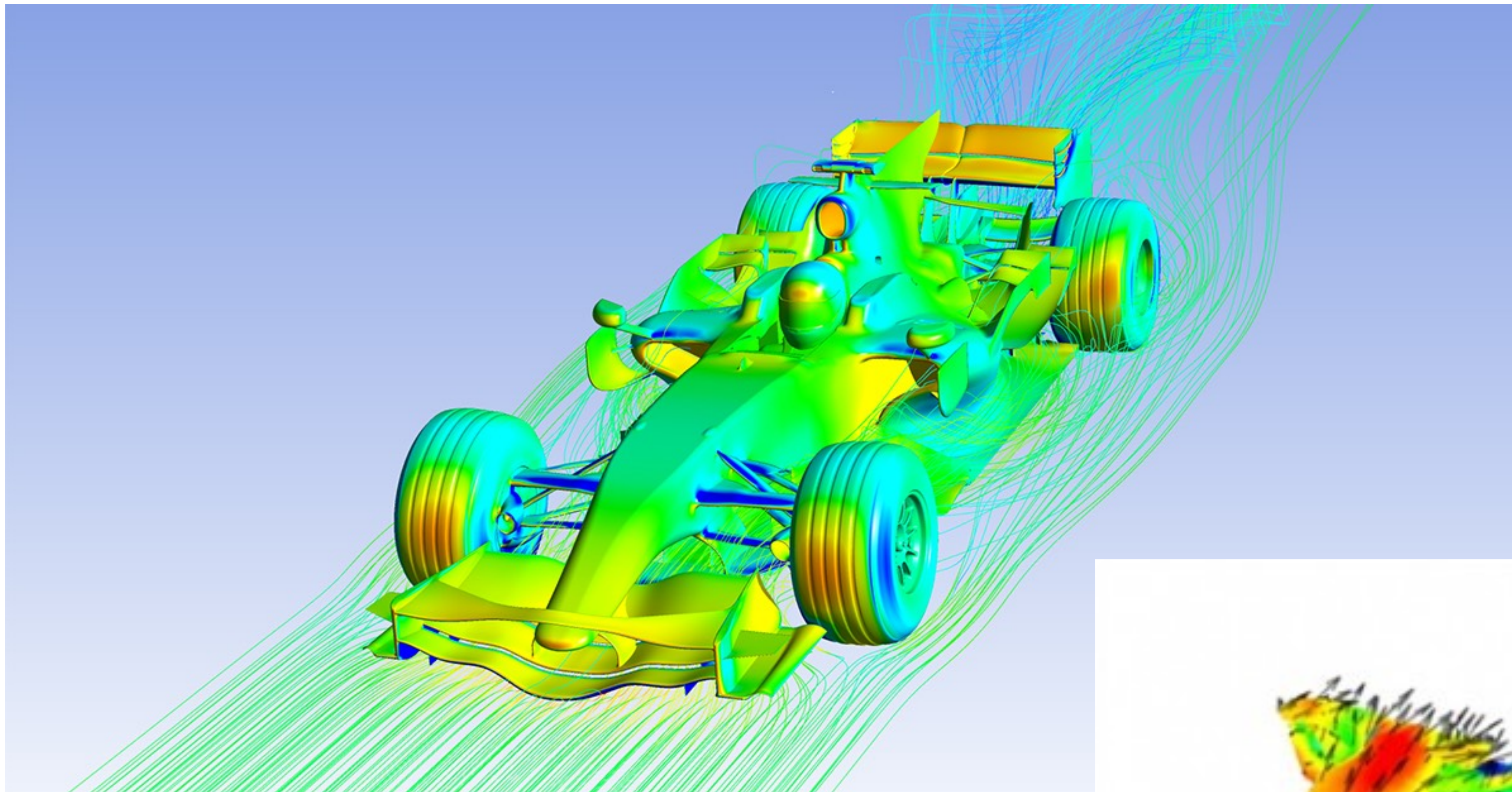
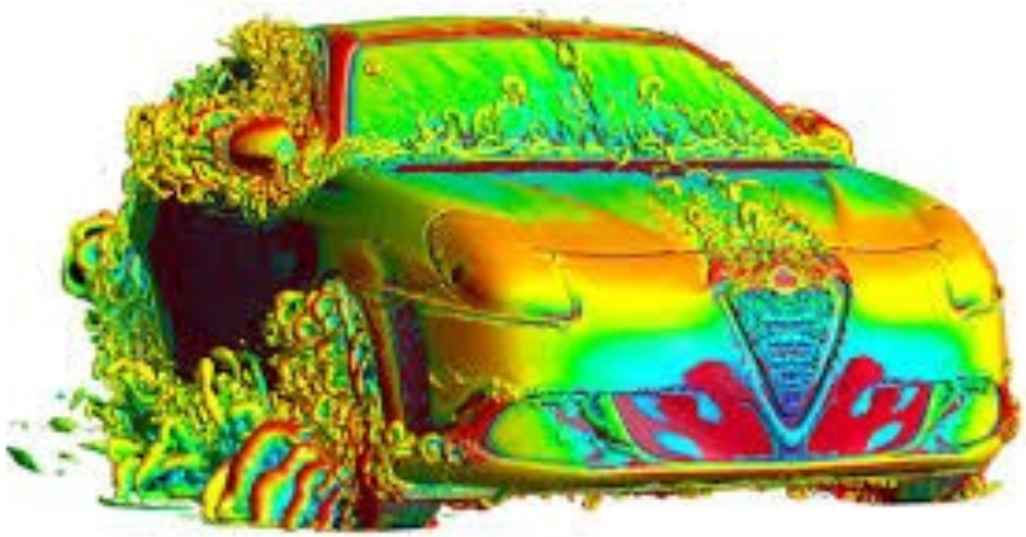
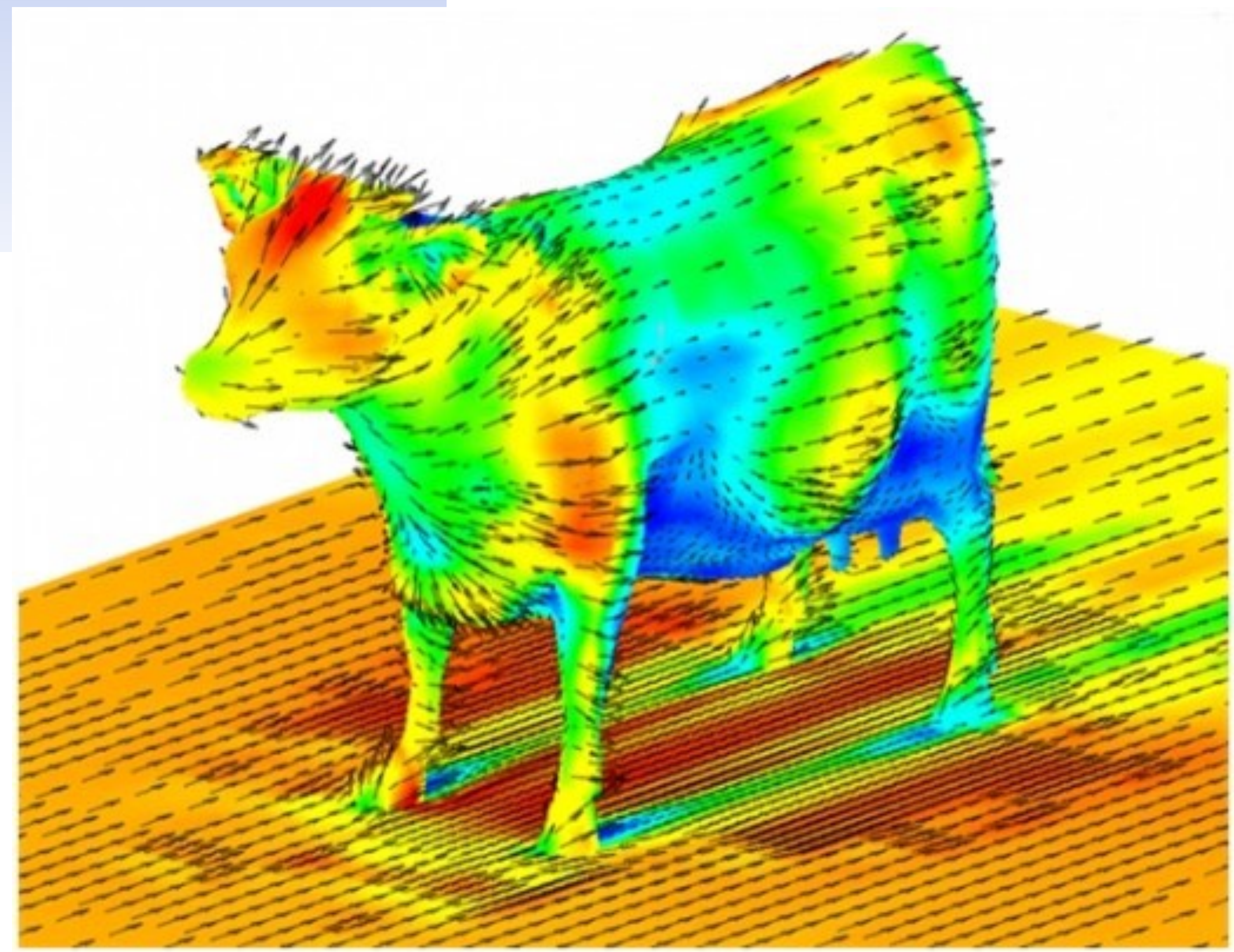
Is CFD always reliable?







Is CFD always reliable?

Three types of systematic errors:

1. **Model error: difference between the real problem and the chosen equations**

2. **Discretization error: difference between the exact solution of the model equations and the exact solution of the discretized system**

3. **Convergence error: difference between the exact solution of the discretized system and the solution obtained with a given mesh**

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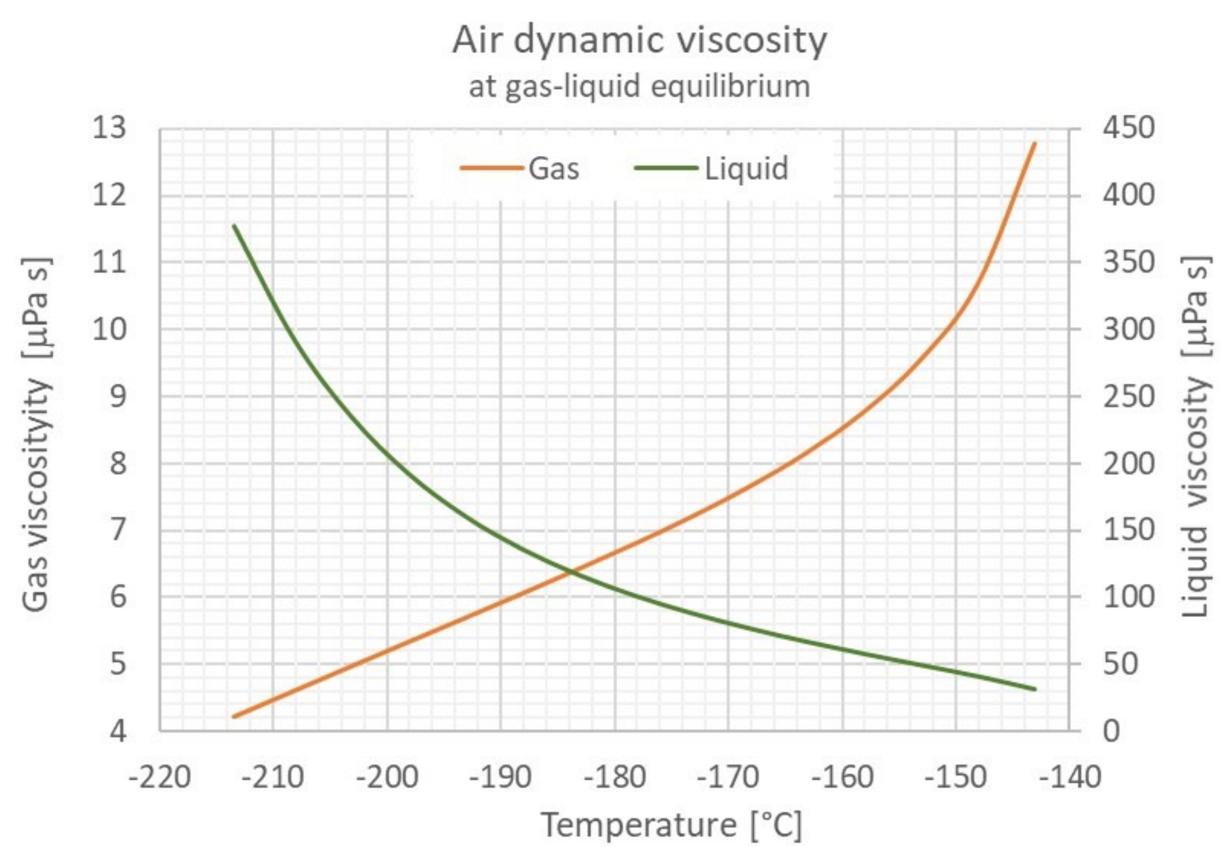
air dynamic viscosity



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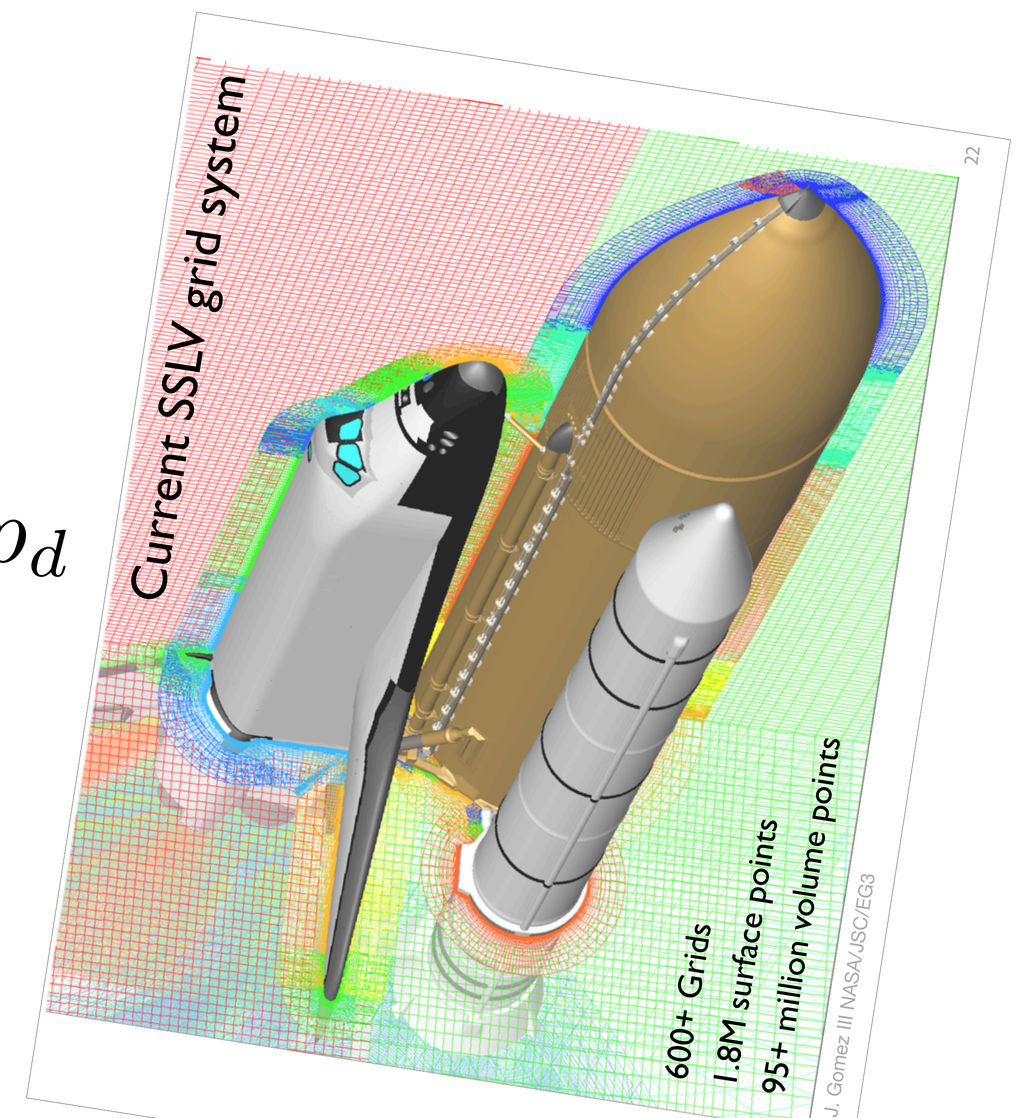
$$\mathbf{u},\ p \longrightarrow \mathbf{u}_d,\ p_d$$



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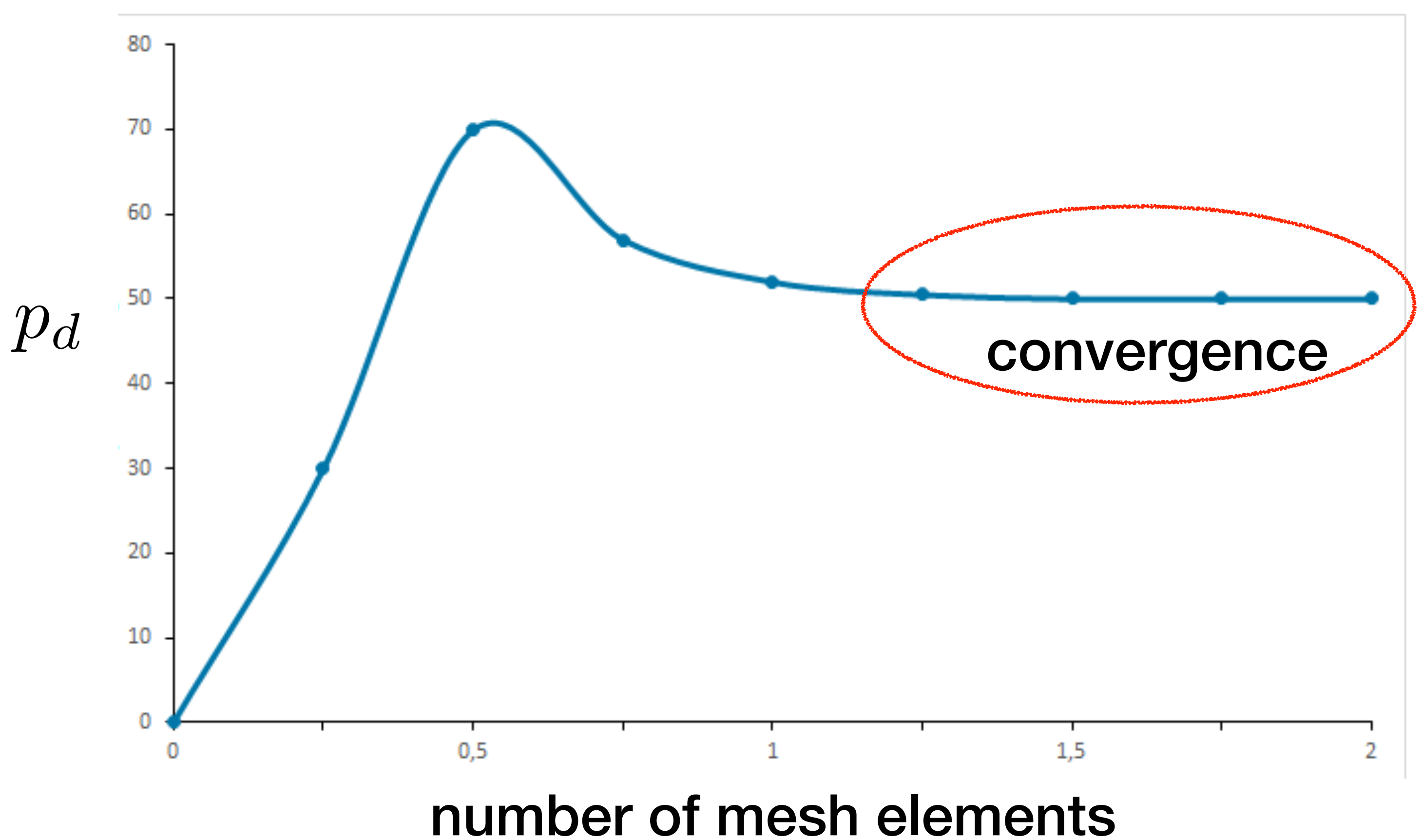
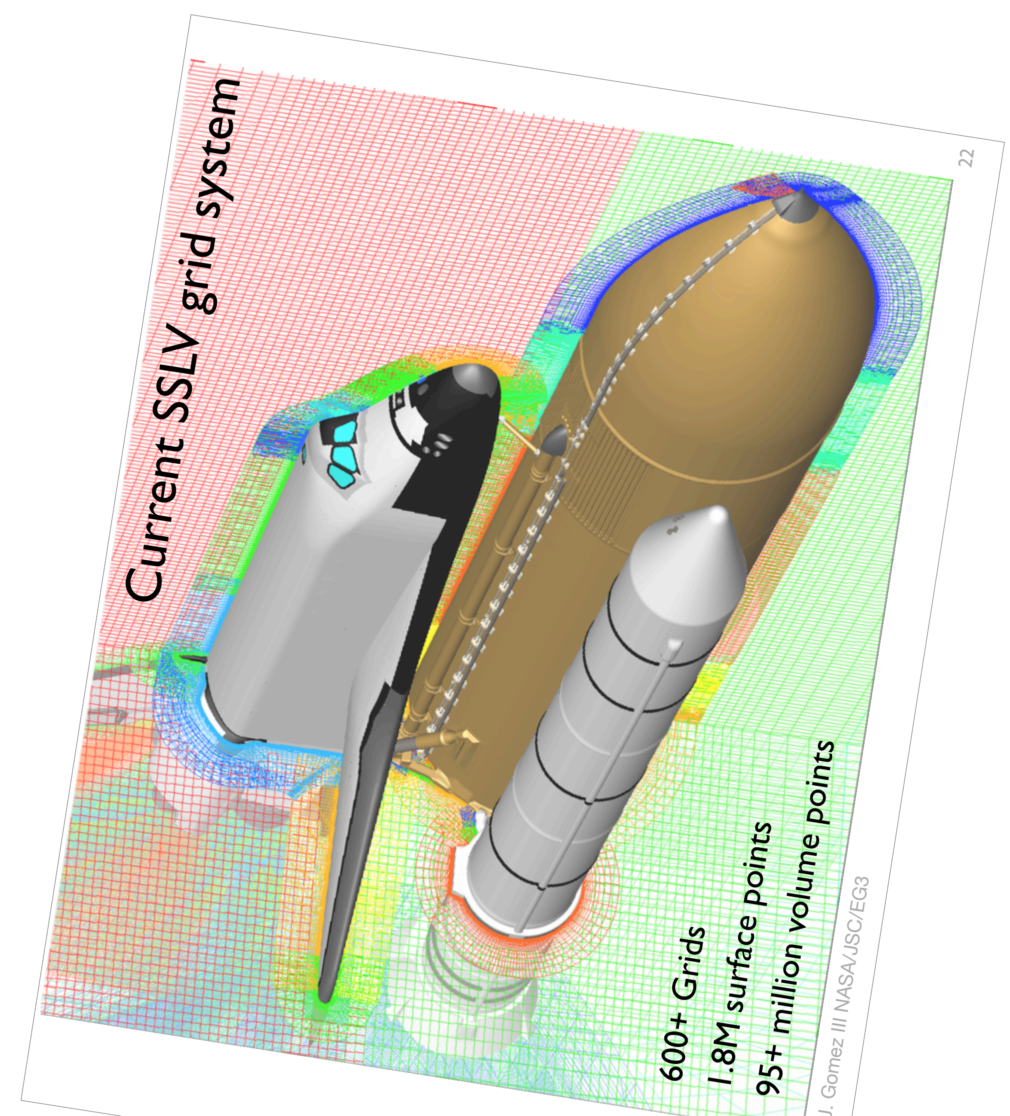
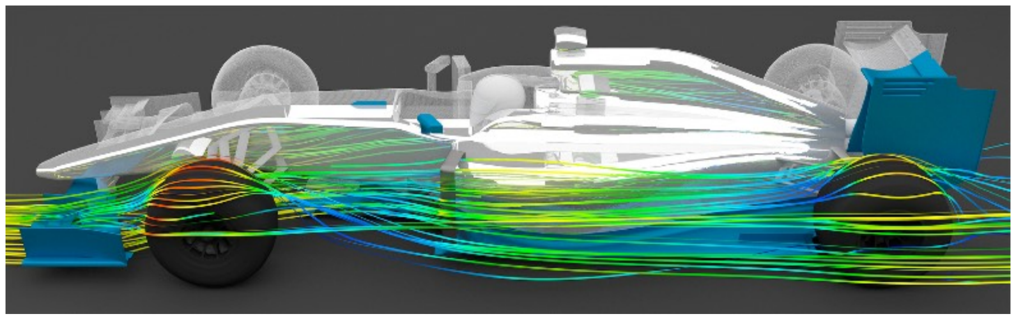
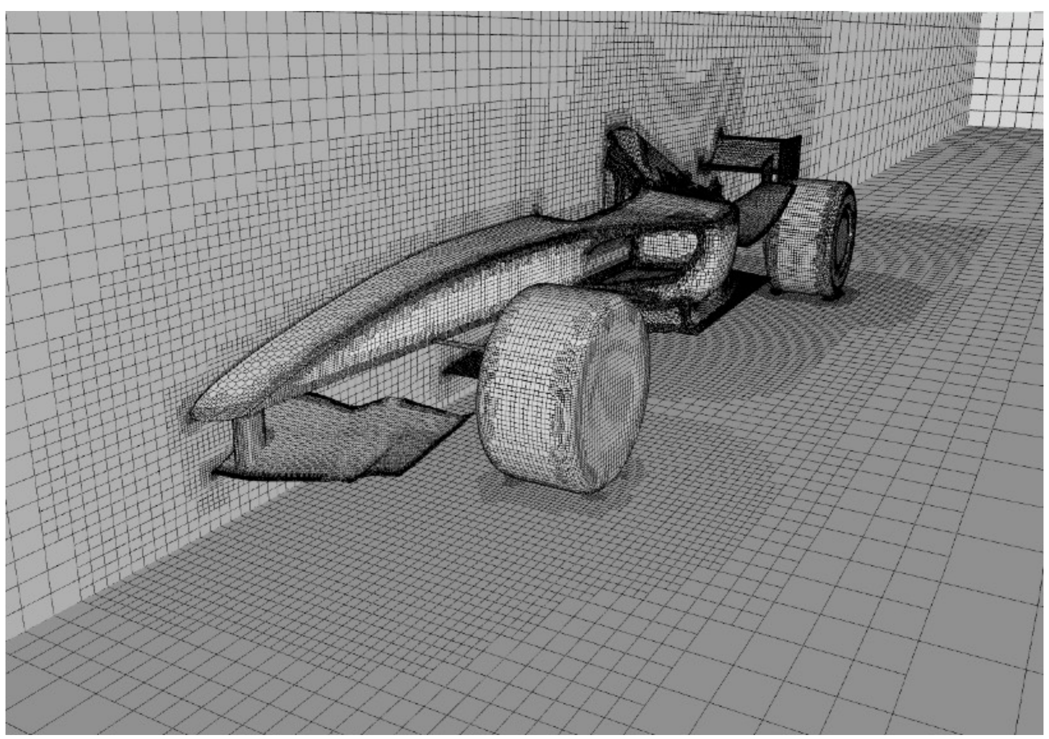




CFD: key ingredients





- **Target problem**
  (flow around a racing car)
- **Mathematical model**
  (Navier-Stokes equations, RANS, heat equation)
- **Discretization method**
  (finite differences, finite elements, finite volumes)
- **Computational mesh**
  (structured, unstructured, curvilinear)
- **Time integration method**
  (implicit, explicit)
- **Solve linear systems of equations**
- **Solve nonlinear systems of equations**
- **Post-processing**
  (e.g. find Cx—> integration)

This course

Numerical mathematics

- Number system and errors
- Roots of equations
- System of linear equations
- Interpolation
- The method of least squares
- Integration
- Time integration of ODEs

Numerical solution of PDEs

- The finite difference method FD
- Solution of the steady heat equations (linear and nonlinear)
- Solution of the unsteady heat equations
- Solution of the Navier-Stokes eq.

CFD for applications

- The finite element method (FEM)
- The finite volume method (FVM)
- Introduction to RANS and LES
- Commercial software using finite differences
- Library for FEM

Hand-on sessions

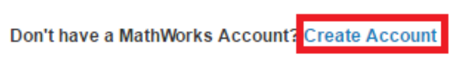
https://docs.ccd.uniroma2.it/matlab/

Per poter utilizzare i servizi messi a disposizione degli studenti di Tor Vergata relativi a MatLab è necessario aver già attivato l'indirizzo di posta elettronica fornito dall'Ateneo.

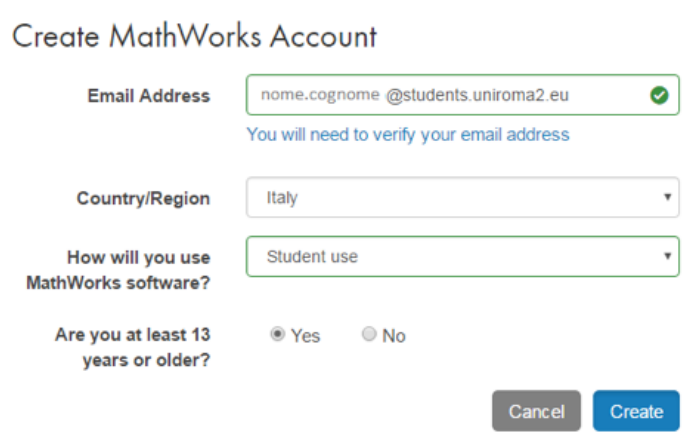
Se non si sa come attivare il proprio indirizzo mail guardare la **guida rela tiva**.

L'Ateneo mette a disposizione di tutti gli studenti e di tutto il personale la possibilità di installare il software MatLab, per fini didattici e di ricerca. Ogni docente, ricercatore e studente può iscriversi ai corsi online della M TLAB Academy, attraverso la **pagina dedicata** all'Università degli Studi d Roma "Tor Vergata".

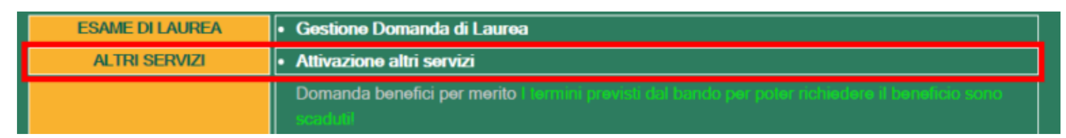
Per attivare il servizio è sufficiente collegarsi al sito **//it.mathworks.com/mwaccount/** e creare il proprio Account.



Passo 2



A questo punto basterà seguire le istruzioni e, quando richiesto, inserire il codice di attivazione presente sulla pagina personale **Delphi**.



Per altre informazioni, documentazione Mathworks, risorse gratuite Mathworks e ulteriori guide visitare il sito dell'Università nella **pagina dedicata**.

Per problemi, assistenza tecnica o domande rivolgersi a: *support@mathworks.it*.

Hand-on sessions

Matlab

python

NumPy

IPython

SciPy

matplotlib

SymPy

Octave

Fortran 77/90

C/C++

or others…

Esame

1. Svolgimento progetto

+

2. Orale:

Domande sul programma

Presentazione del progetto