Ciclo di seminari

Università degli studi di Roma "Tor Vergata"

May, 2013

Introduction to fundamentals of Turbulence, Large Eddy Simulation and subgrid-scale modeling

Charles Meneveau

Johns Hopkins University

Baltimore, USA

Additional reference:

http://www.scholarpedia.org/article/Turbulence:_Subgrid-Scale_Modeling

Institute for Data Intensive Engineering and Science

JOHNS HOPKINS UNIVERSITY

JHU Mechanical Engineering

Center for Environmental & Applied Fluid Mechanics

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OVERVIEW:

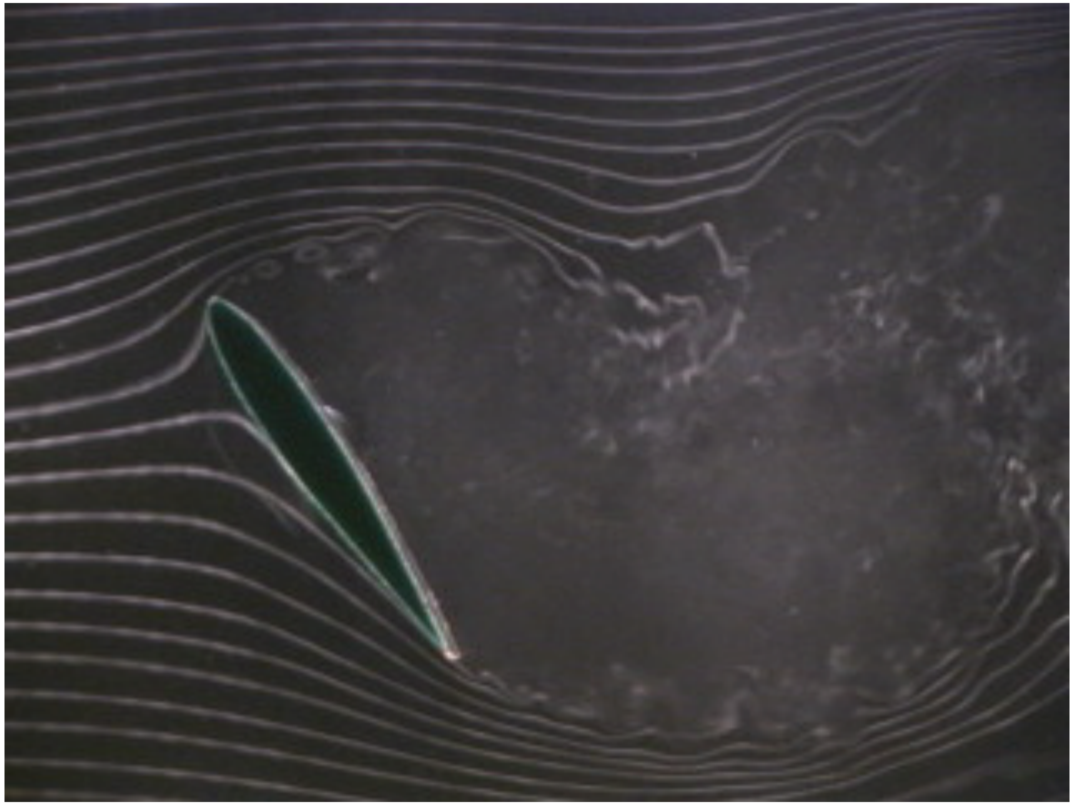
Mercoledì:

- Introduction to fundamentals of Turbulence
- Intro to Large Eddy Simulation (LES)
- Intro to Subgrid-scale (SGS) modeling
- The dynamic SGS model
- Some sample applications from our group

Venerdì:

- Dynamic model for LES over rough multiscale surfaces
- LES studies of large wind farms

Turbulence = eddies of many sizes



From: Multimedia Fluid Mechanics, Cambridge Univ. Press

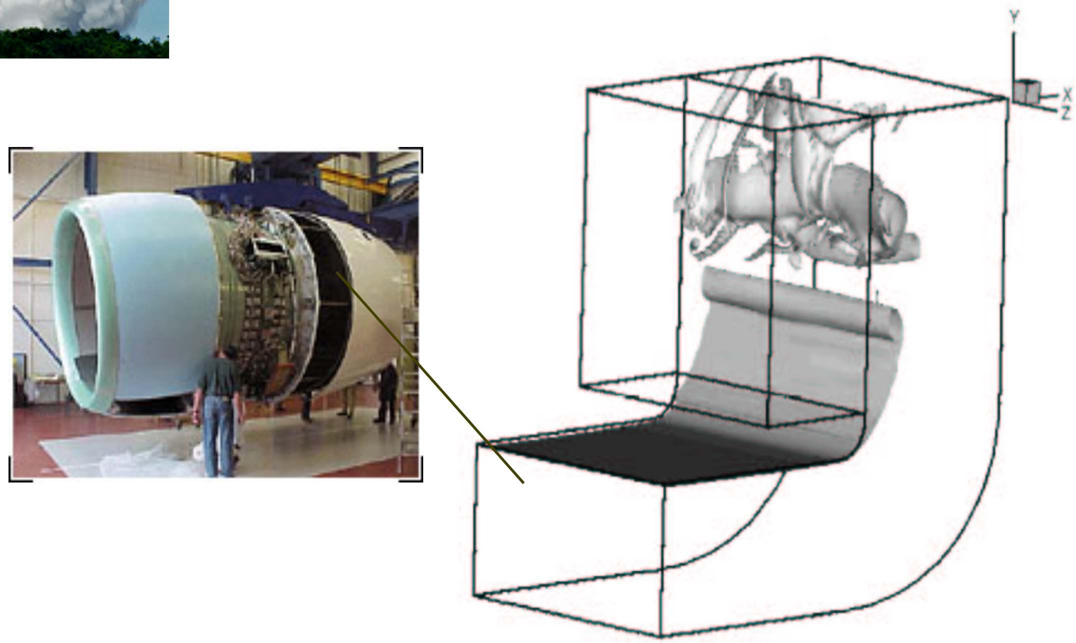
Turbulence = eddies of many sizes
+ large-scale vortices

From: Multimedia Fluid Mechanics, Cambridge Univ. Press

Turbulence in aerospace systems:

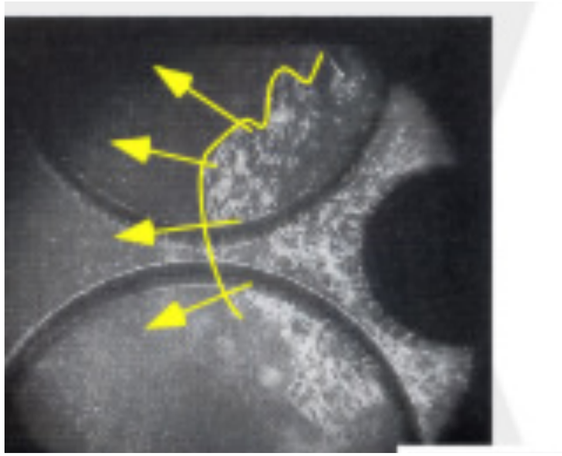
Jets and eddies during shuttle launch

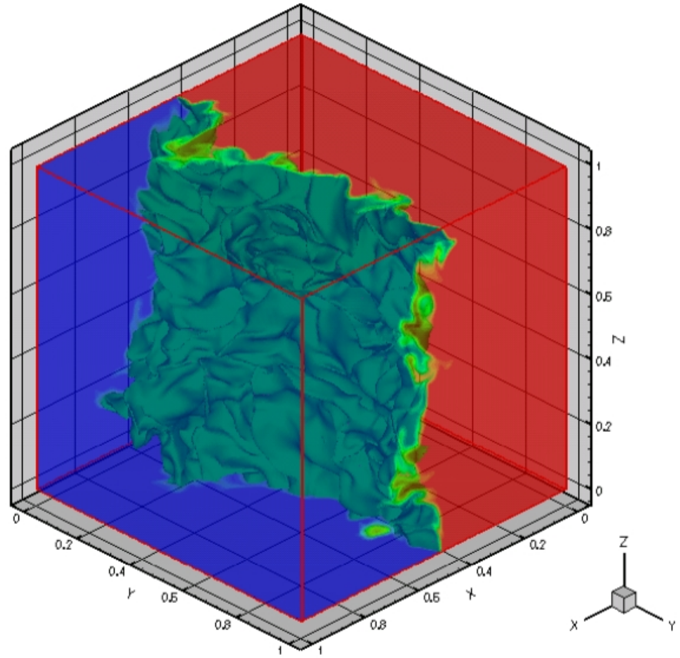
Large Eddy Simulation of flow in thrust-reversers
Blin, Hadjadi & Vervisch (2002)
J. of Turbulence.



Turbulence in reacting flows:

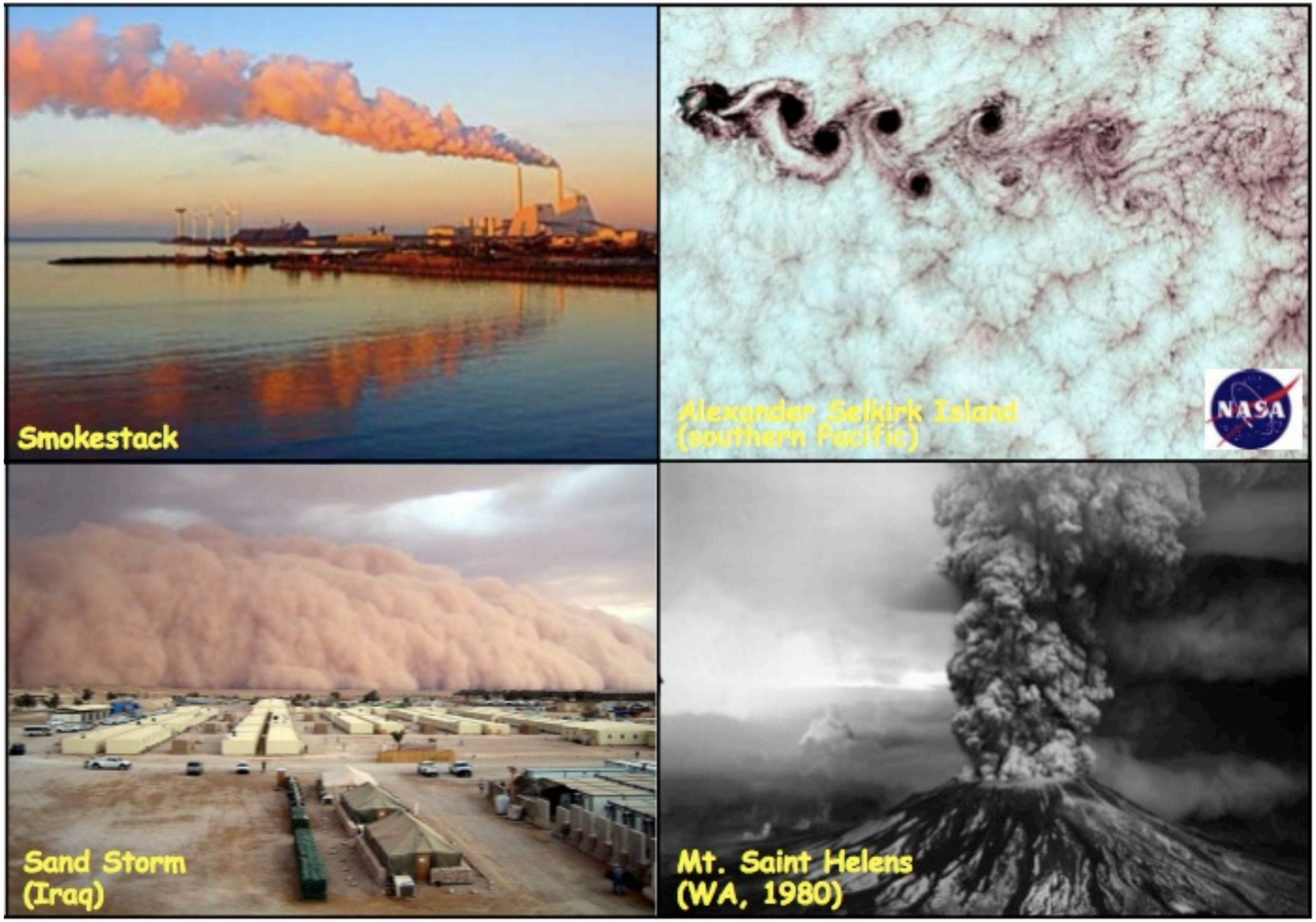
Premixed flame in I.C. engine, combustion





Numerical simulation of flame propagation in decaying isotropic turbulence

Turbulence in environment and geophysics



Turbulence in renewable energy

From J.N. Sørensen, Annual Rev. Fluid Mech. 2011:

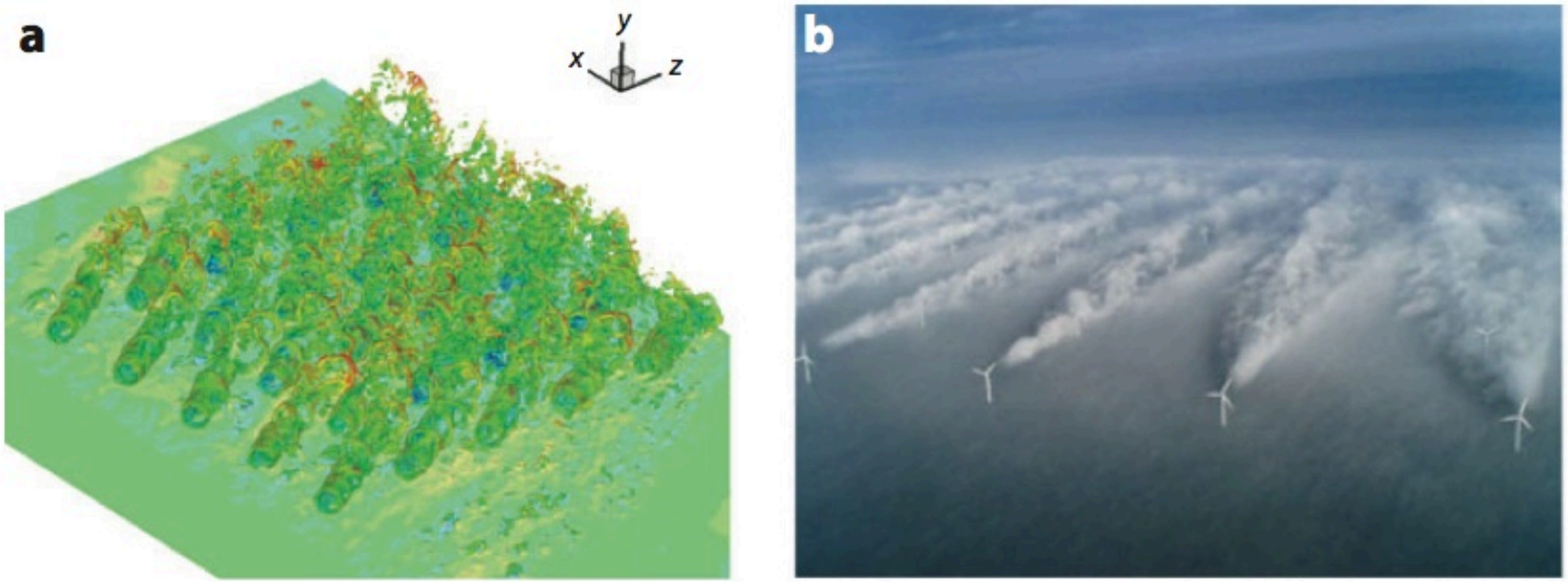
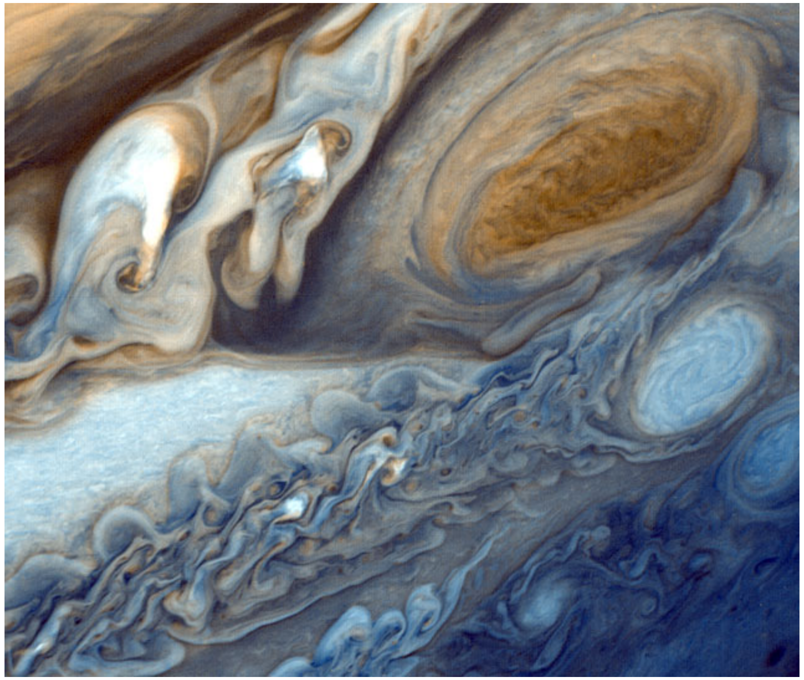
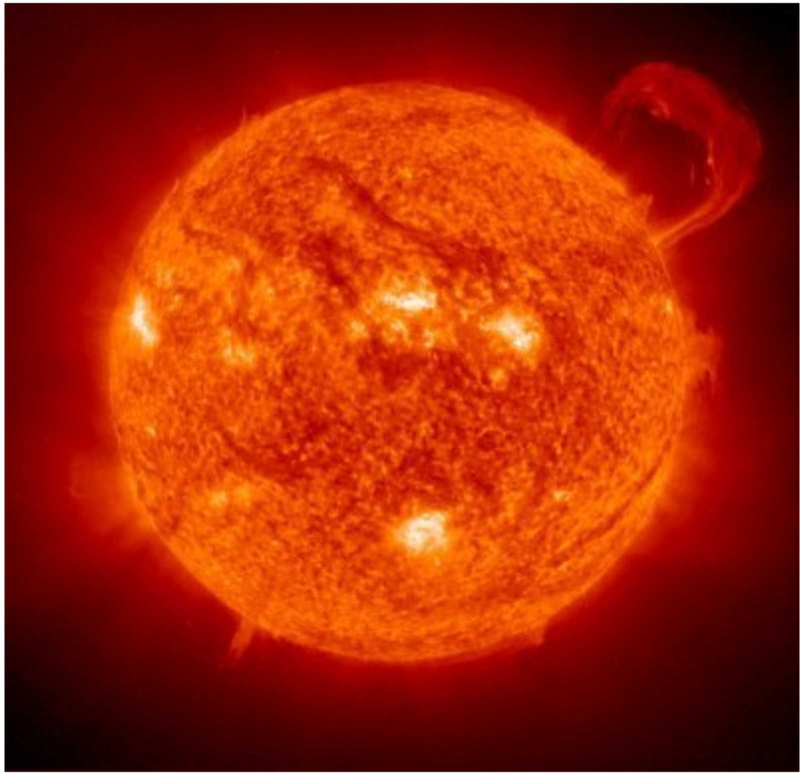


Figure 6

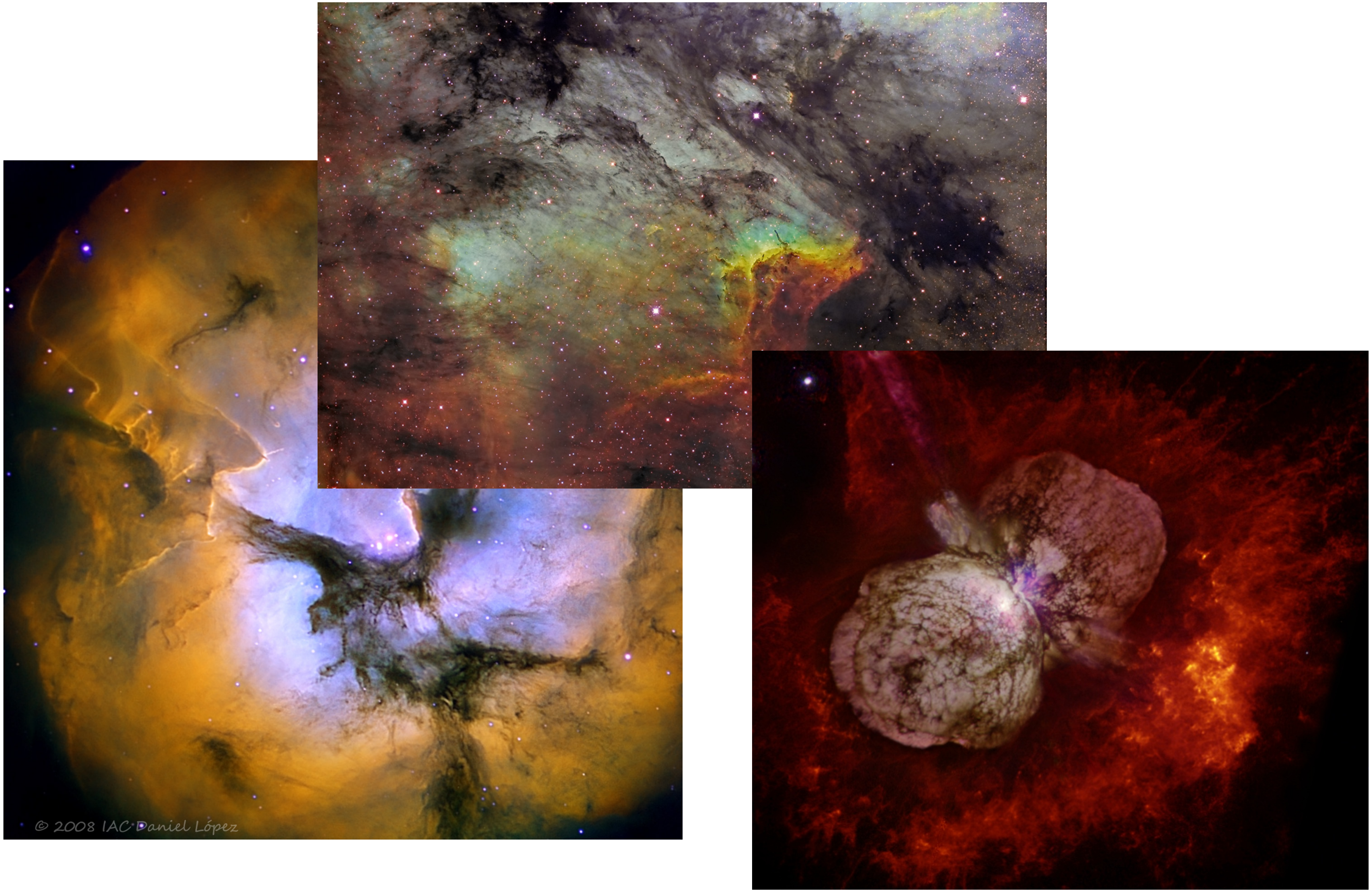
(*a*) Actuator disc computation of a wind farm consisting of 5 × 5 wind turbines. (*b*) Photograph showing the flow field around the Horns Rev wind farm.

Turbulence in astrophysics

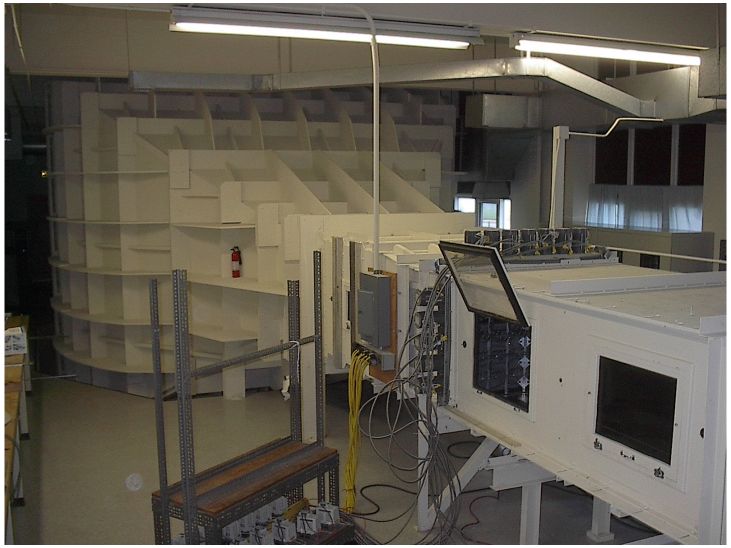




Turbulence in astrophysics

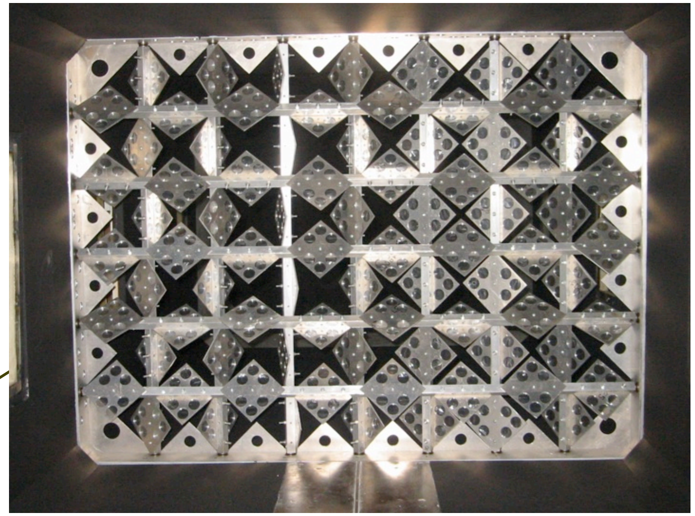


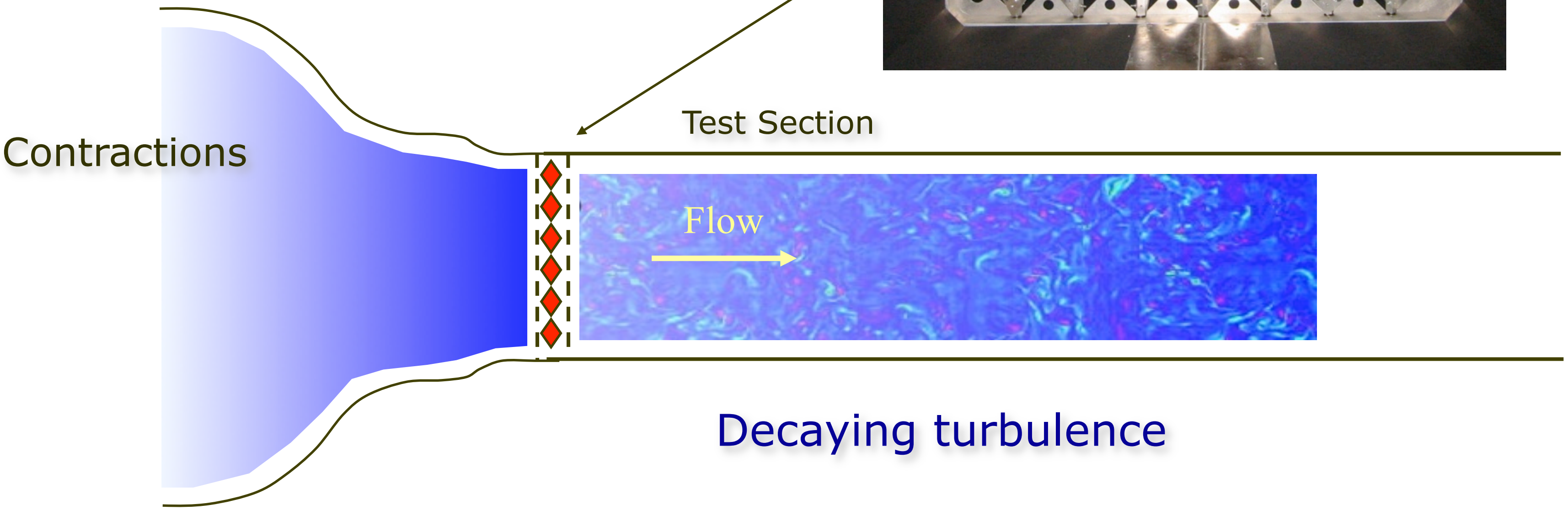
Simplest turbulence: Isotropic turbulence



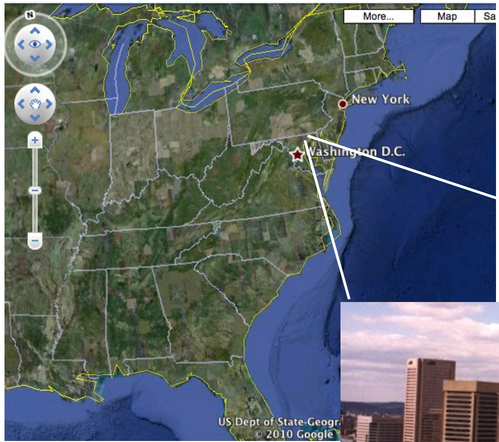
Corrsin wind tunnel
at the Johns Hopkins University

Active Grid M = 6 "



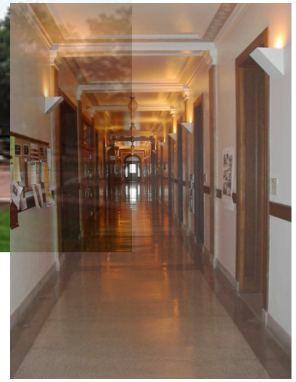


Johns Hopkins University: Baltimore, Maryland USA



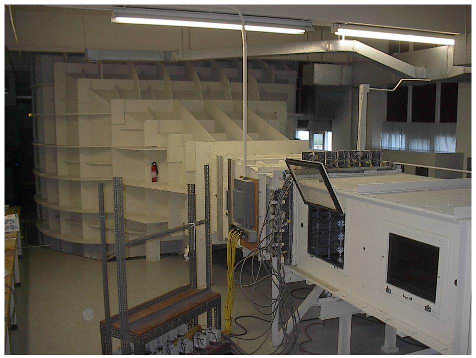
Downtown Baltimore

JHU: Latrobe Hall (Mechanical Engineering)

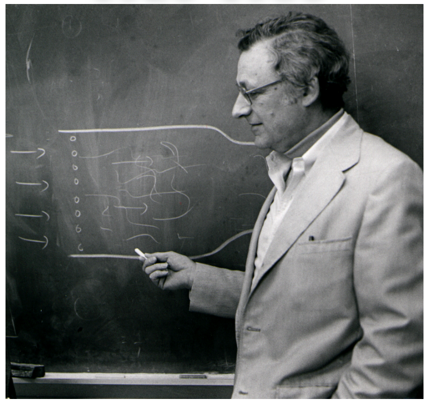


Maryland Hall

Corrsin wind tunnel at JHU

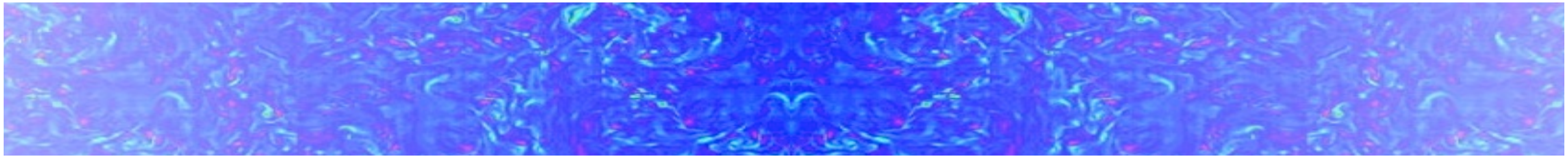


Stan Corrsin 1920-1986



Turbulence is:

- multiscale,
- mixing,
- dissipative,
- chaotic,
- vortical
- well-defined statistics,
- important in practice



Physical quantities describing fluid flow

- Density field
- Velocity vector field
- Pressure field
- Temperature field (or internal energy, or enthalpy etc..)

Physical laws governing fluid flow

- Conservation of mass
- Newton's second law (linear momentum)
- First law of thermodynamics (energy)
- Equation of state
- Some constraints in closure relations from second law of TD

↓

Navier Stokes equations for a Newtonian, incompressible fluid

Navier-Stokes equations, incompressible, Newtonian

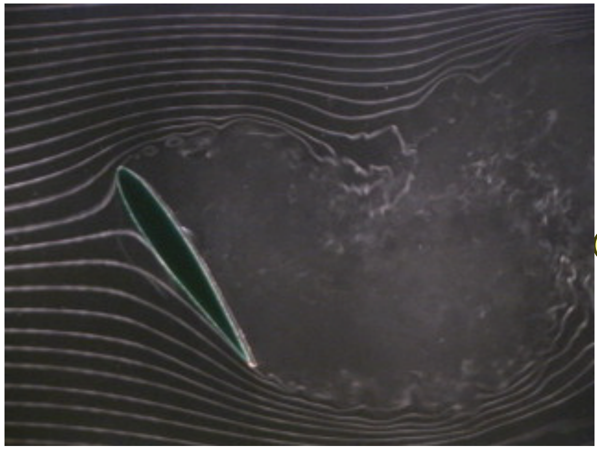
$$\begin{cases} \dfrac{\partial u_j}{\partial x_j} = 0 \\ \dfrac{\partial u_j}{\partial t} + \dfrac{\partial u_k u_j}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x_j} + \nu \nabla^2 u_j + g_j \end{cases}$$

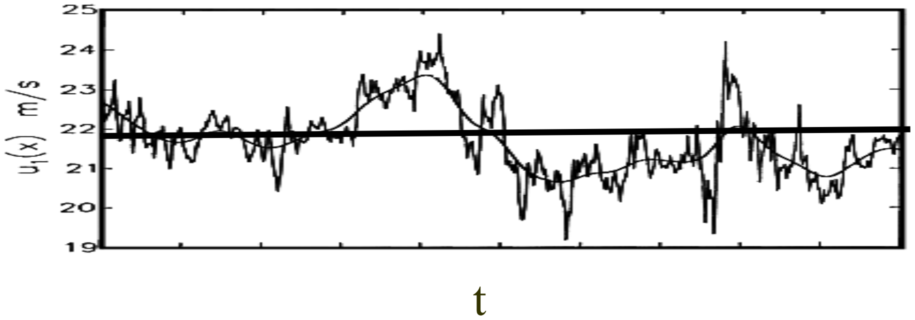
$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$a_j = \frac{F_j}{m}$$

Traditional approach: Reynolds decomposition

$$\begin{cases} \dfrac{\partial u_j}{\partial x_j} = 0 \\ \dfrac{\partial u_j}{\partial t} + \dfrac{\partial u_k u_j}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x_j} + \nu \nabla^2 u_j + g_j \end{cases}$$

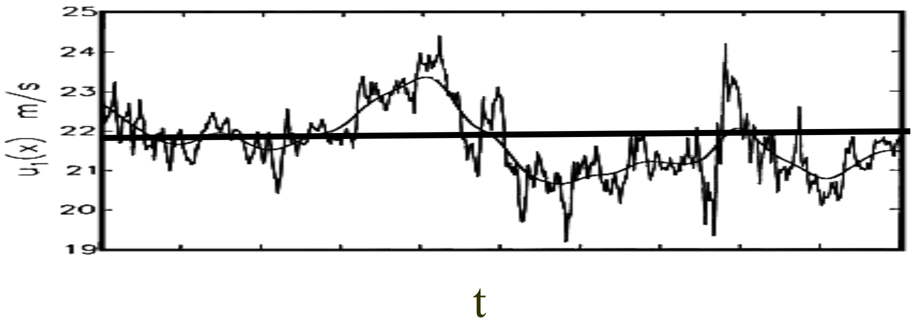




Traditional approach: Reynolds decomposition

$$\begin{cases} \dfrac{\partial u_j}{\partial x_j} = 0 \\[2ex] \dfrac{\partial u_j}{\partial t} + \dfrac{\partial u_k u_j}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x_j} + \nu \nabla^2 u_j + g_j \end{cases}$$

Reynolds' equations:



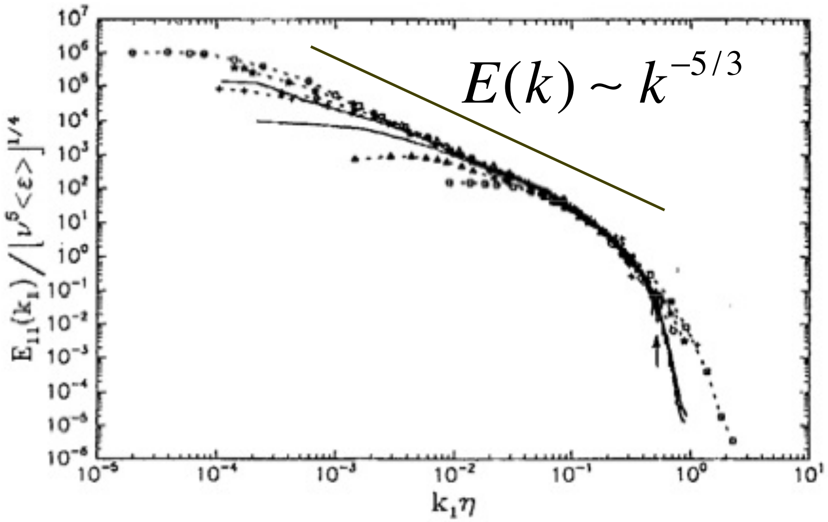
$$\begin{cases} \dfrac{\partial \overline{u}_j}{\partial x_j} = 0 \\[2ex] \dfrac{\partial \overline{u}_j}{\partial t} + \dfrac{\partial \overline{u}_j \overline{u}_k}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial \overline{p}}{\partial x_j} + \nu \nabla^2 \overline{u}_j + g_j - \dfrac{\partial}{\partial x_k}\left(\underbrace{\overline{u_j u_k} - \overline{u}_j \overline{u}_k} \right) \end{cases}$$

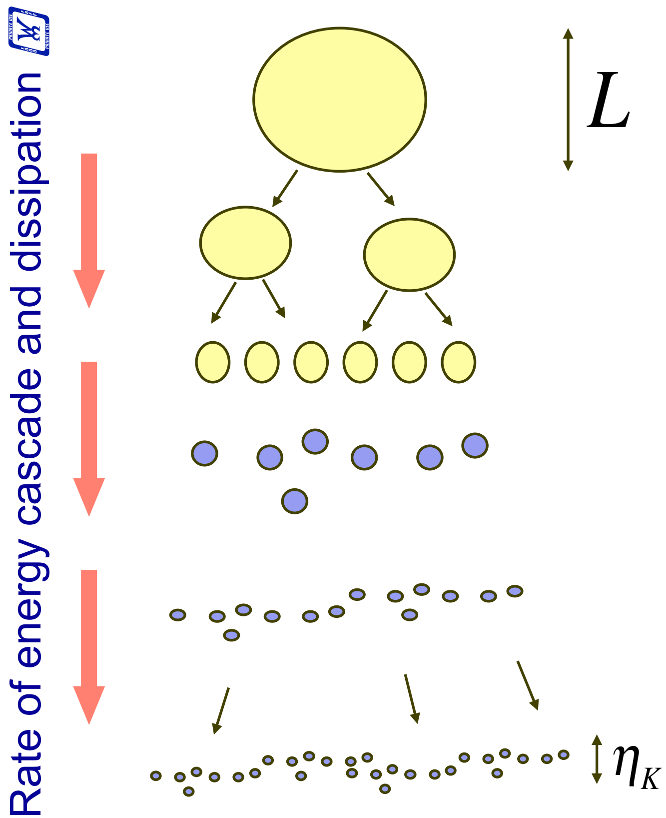
Reynolds' stress tensor: $\overline{u'_j u'_k}$

Requires closure ("turbulence problem")

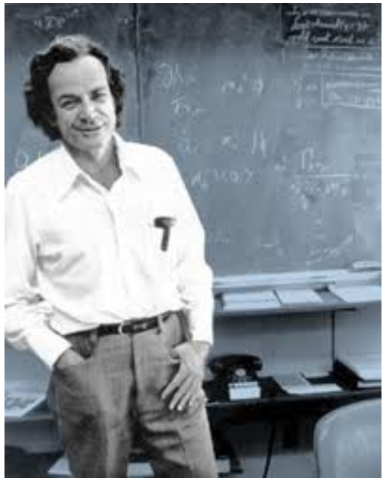
Turbulence physics: the energy cascade

(Richardson 1922, Kolmogorov 1941)





Turbulence problem:
"last unsolved problem in classical physics"
(e.g. Feynman 1979)

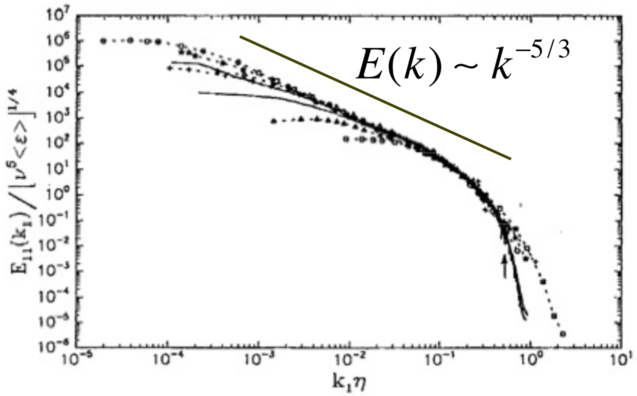


"With turbulence, it's not just a case of physical theory being able to handle only simple cases—we can't do any. We have no good fundamental theory at all." (Feynman, 1979, Omni Magazine, Vol. 1, No.8).

Sample challenges (unsolved)

Reynolds' stress tensor: $\overline{u'_j u'_k}$
Requires closure ("turbulence problem")

First-principles derivation of *Kolmogorov's* $k^{-5/3}$ spectrum

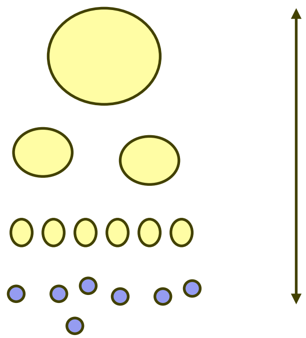


Direct Numerical Simulation (DNS):

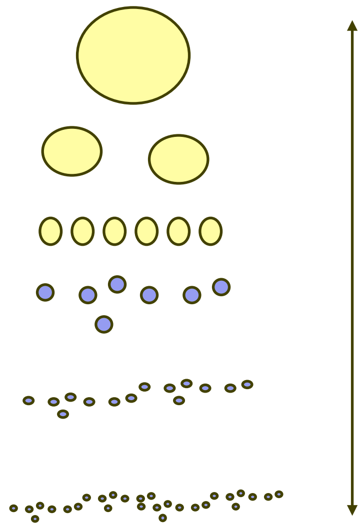
N-S equations:

$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j + g_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$

Moderate Re (~ 10^3),
DNS possible



High Re (~ 10^7),
DNS impossible



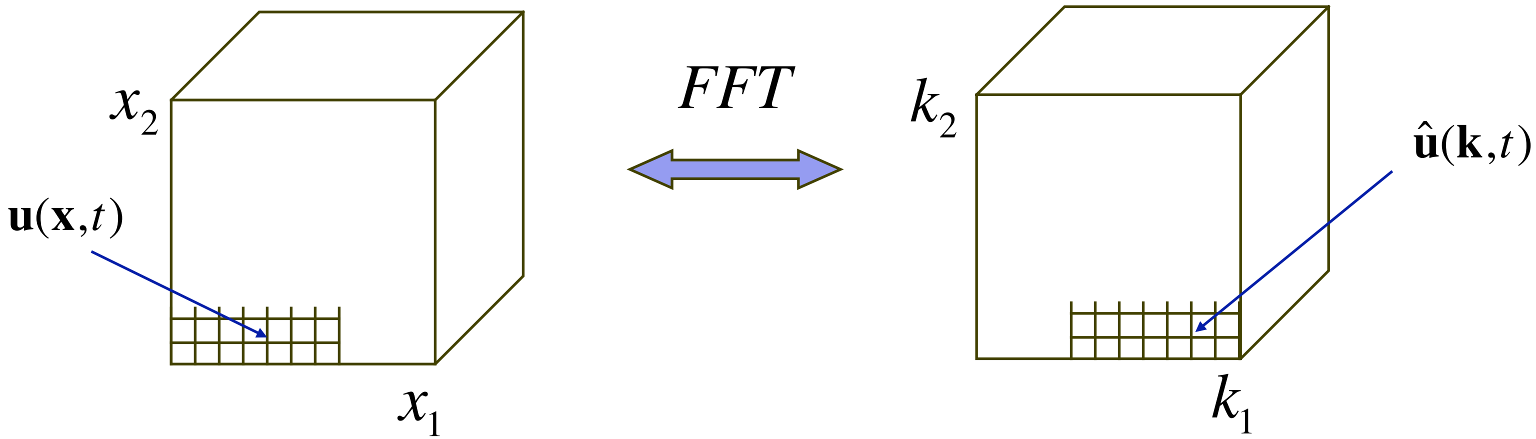
DNS - pseudo-spectral calculation method

(Orszag 1971: for isotropic turbulence - triply periodic boundary conditions)

$$\hat{\mathbf{u}}(\mathbf{k},t) = FFT[\mathbf{u}(\mathbf{x},t)]$$

$$\mathbf{k}\cdot\hat{\mathbf{u}}(\mathbf{k},t) = 0, \qquad \frac{\partial \hat{\mathbf{u}}(\mathbf{k},t)}{\partial t} = \mathbf{P}(\mathbf{k})\cdot(\widehat{\mathbf{u}\times\boldsymbol{\omega}})(\mathbf{k}) - \nu k^2 \hat{\mathbf{u}}(\mathbf{k},t) + \mathbf{P}(\mathbf{k})\cdot\hat{\mathbf{f}}(\mathbf{k},t)$$

$$(\widehat{\mathbf{u}\times\boldsymbol{\omega}})(\mathbf{k}) = FFT[\mathbf{u}(\mathbf{x},t)\times\nabla\times\mathbf{u}(\mathbf{x},t)]$$



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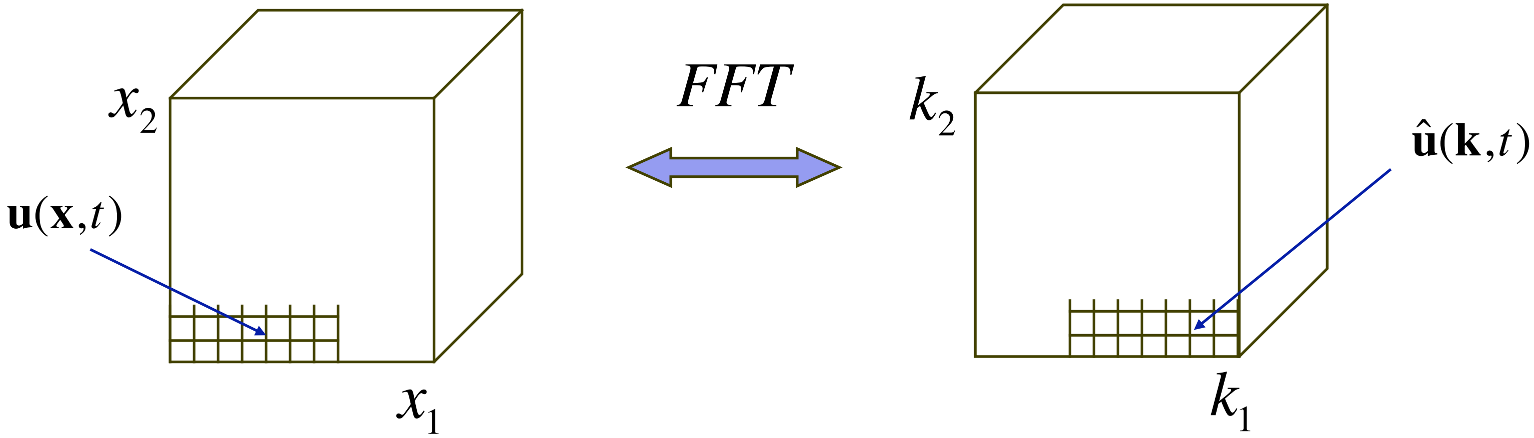


Computational state-of-the-art:

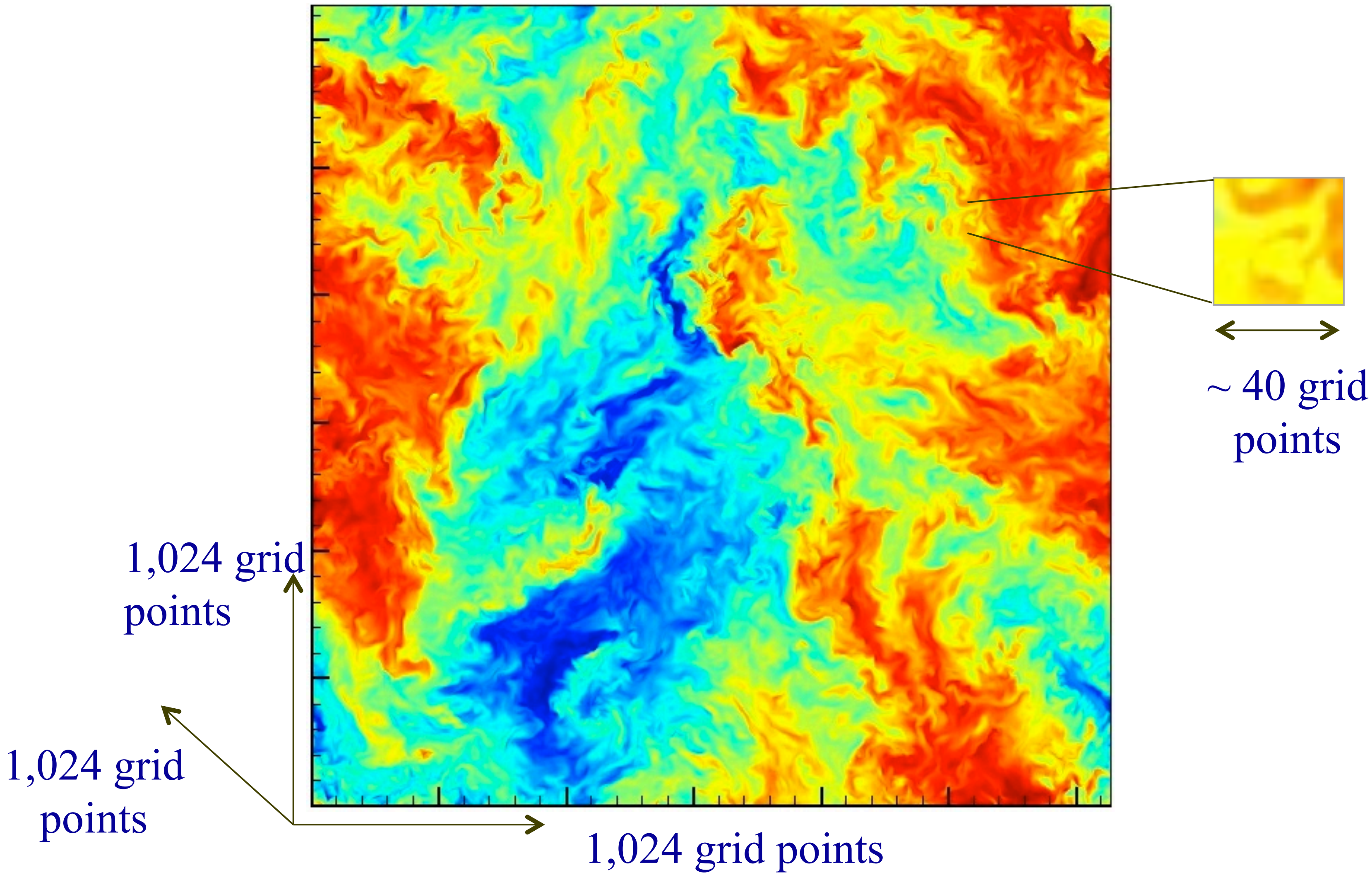
128^3 - 256^3: Routine, can be run on small
PC with (256^3 x 12 x 4) Bytes = 100 MB of RAM
1024^3 : needs cluster with O(100) nodes and O(64 GB) RAM
4096^3 : world record (Earth Simulator, Japan, 2003, 30 - TFlops), 4 TB RAM

$1{,}024^3$ DNS: iso-velocity filled contours

(data from: JHU public database cluster, Claire Verhulst & Jason Graham Matlab visualization)

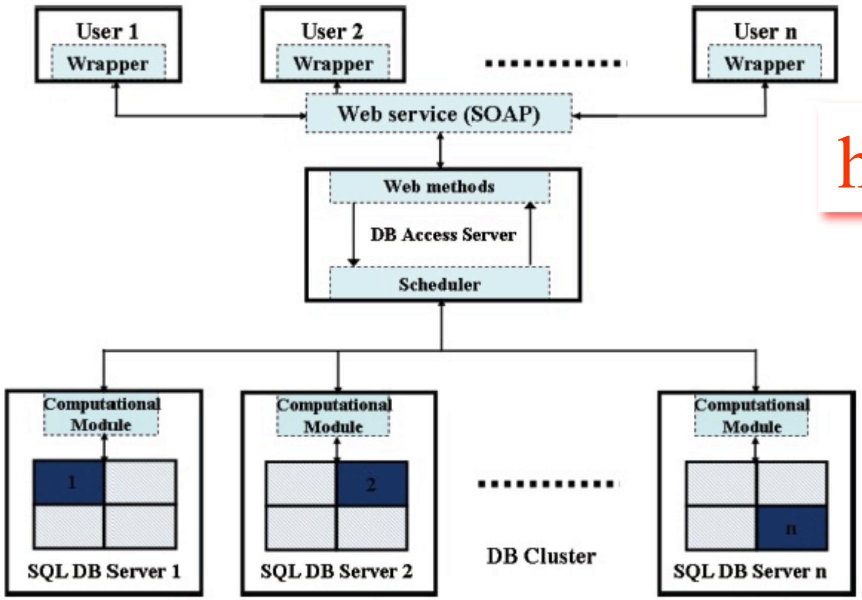
Energetic signature of the large, intermediate (and some smaller) turbulent eddies:

$$u_1(x, y, z_0, t_0)$$



Take 1024^3 DNS of forced isotropic turbulence

(standard pseudo-spectral Navier-Stokes simulation, dealiased):



1024^4 space-time history

27 Tbytes, $Re_\lambda \sim 430$

http://turbulence.pha.jhu.edu

Y. Li, E. Perlman, M. Wan, Y. Yang, R. Burns, C.M., R. Burns, S. Chen, A. Szalay & G. Eyink:
"A public turbulence database cluster and applications to study Lagrangian evolution of velocity increments in turbulence". Journal of Turbulence 9, No 31, 2008.

So far 12 papers published using data from public database

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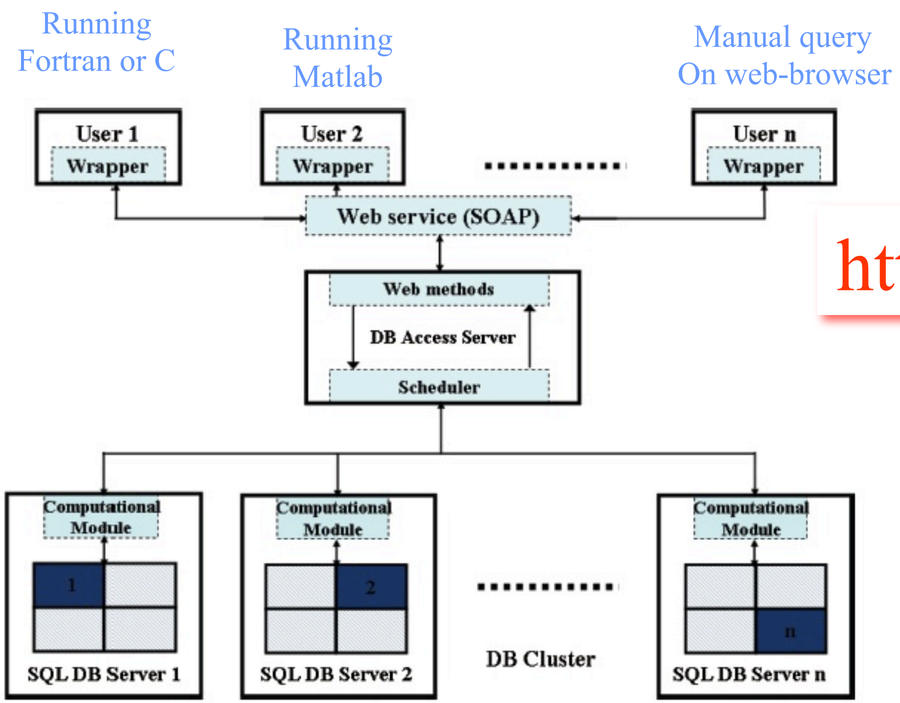

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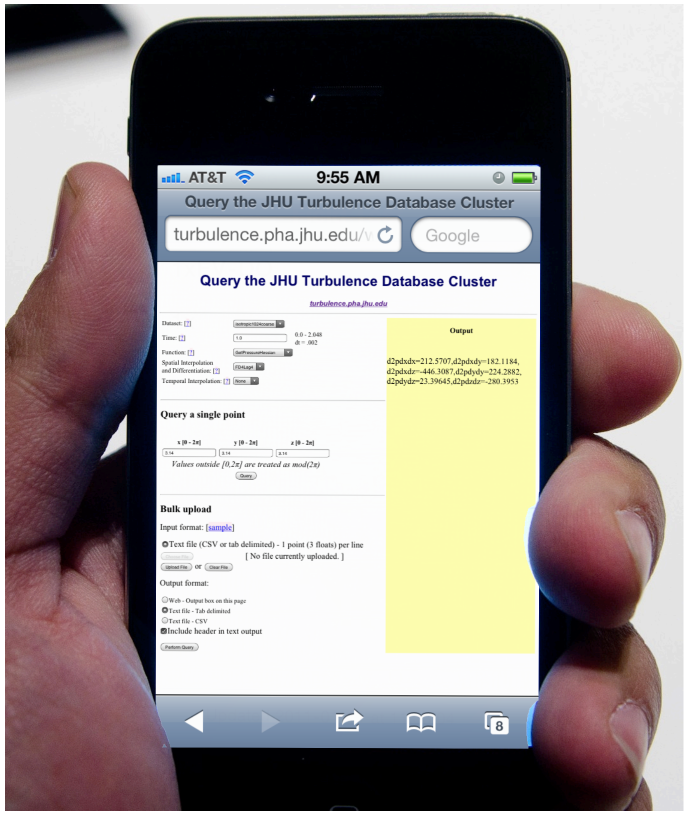

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So far 12 papers published using data from public database

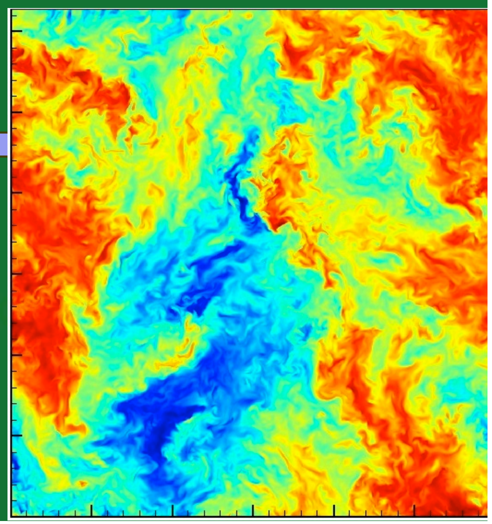
New paradigm:

Client computer (e.g. my laptop) runs the analysis using Fortran, C, Matlab codes, fetching data as needed from databases through a web-service

we adapted Fortran, C and Matlab to "surf the web" for data

Y. Li, E. Perlman, M. Wan, Y. Yang, R. Burns, C. Meneveau, R. Burns, S. Chen, A. Szalay & G. Eyink: Journal of Turbulence 9, No 31, 2008.

```
  ! This is required before any WebService routines are called.
  !
  CALL soapinit()

  ! Enable exit on error.  See README for details.
  CALL turblibSetExitOnError(1)

  dx = 2*3.1415926535/1024.
  ind = 0
  do i = 1, 1024
   do j = 1, 1024
    ind = ind+1
    points(1, ind) = i*dx
    points(2, ind) = j*dx
    points(3, ind) = 3.1415926535
  end do
  end do
  write(*,*)
  write(*,*) 'Requesting velocity at 1024x1024 points...'

  rc = getvelocity(authkey, dataset, 1.00, 0, 0, 1048576, points, dataout3)

  ind=0
  do i = 1, 1024
   do j = 1, 1024
    ind = ind+1
    u(i,j) = dataout3(1,ind)
  end do
  end do
  !
  ! Destroy the gSOAP runtime.
  ! No more WebService routines may be called.
  !
  CALL soapdestroy()

end program TurbTest

CMMacBookPro-2:turblib-20111031 meneveau$
```

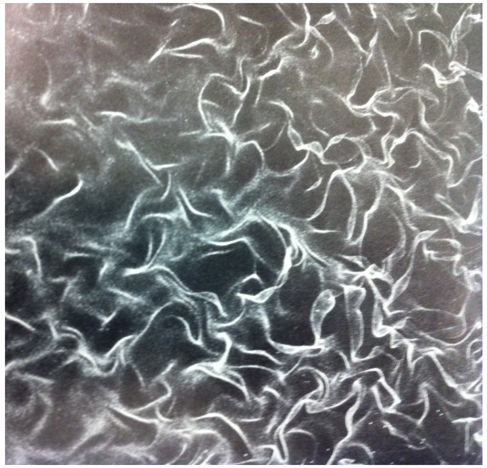


Coupling GPU's with databases:

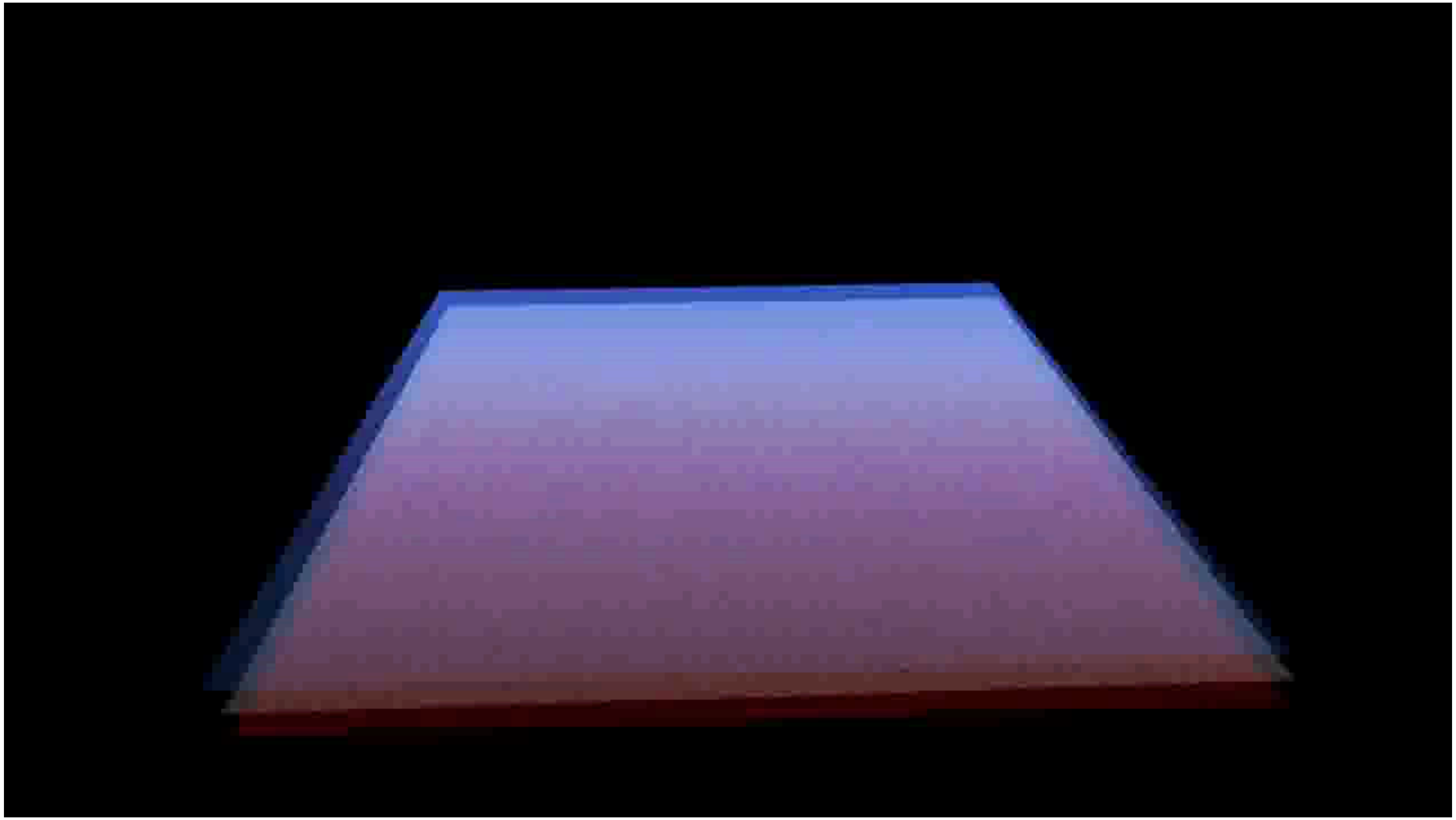
material surfaces (24 million particles) advected in turbulent $1{,}024^3$ field

Real-time flow-viz (download 1 snapshot at fixed t, animated on GPU)

Dr. Kai Buerger (TUM, summer 2011)



M. Karweit (MS Thesis, & Album of Fluid Motion, CUP)

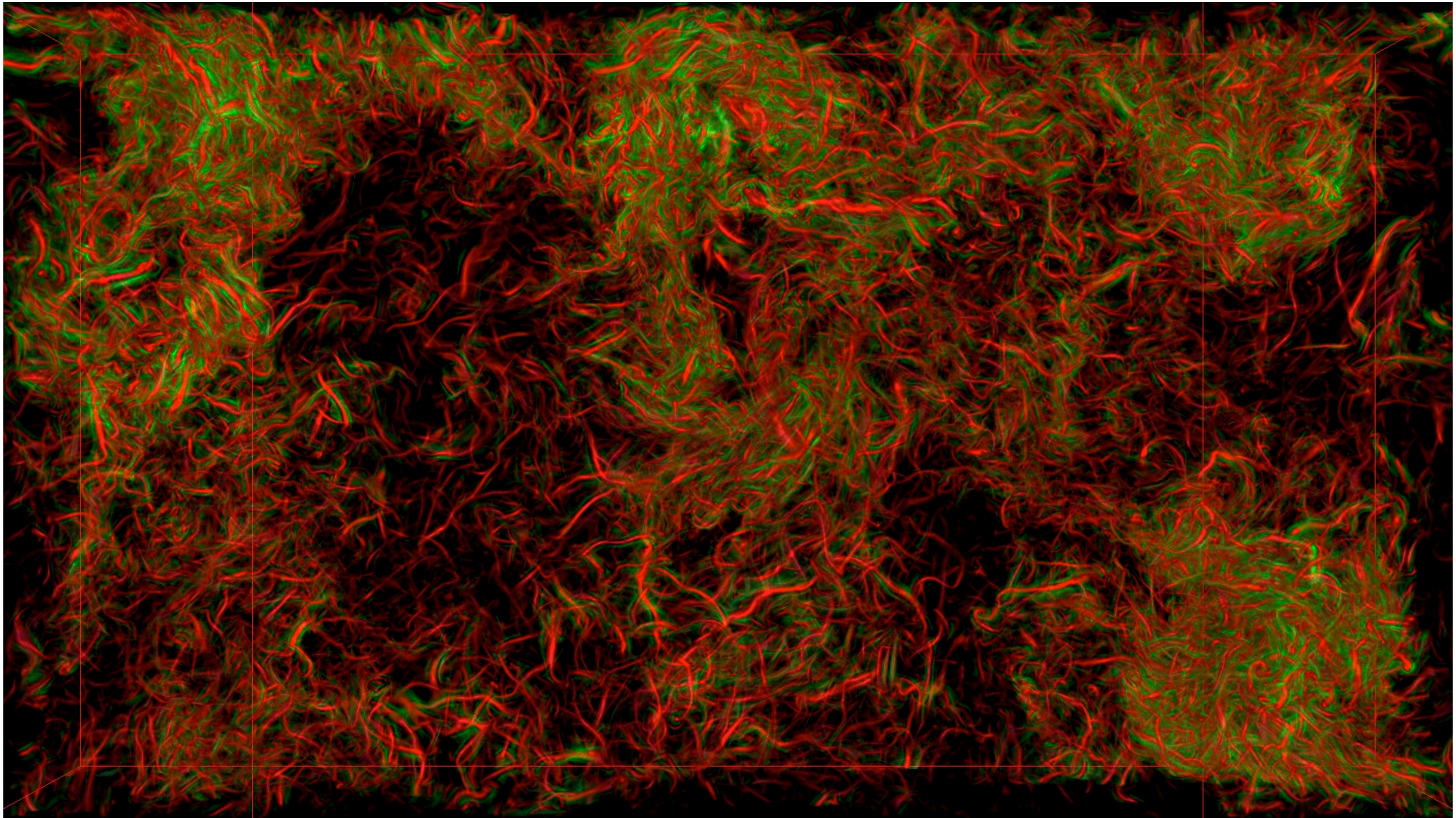


iso-vorticity surfaces

(JHU database, Dr. Kai Buerger visualization)

Vortical signature of the smallest turbulent eddies: $\left\|(\nabla \times \mathbf{u})^2\right\|$

$$a_i = \frac{F_i}{m}$$



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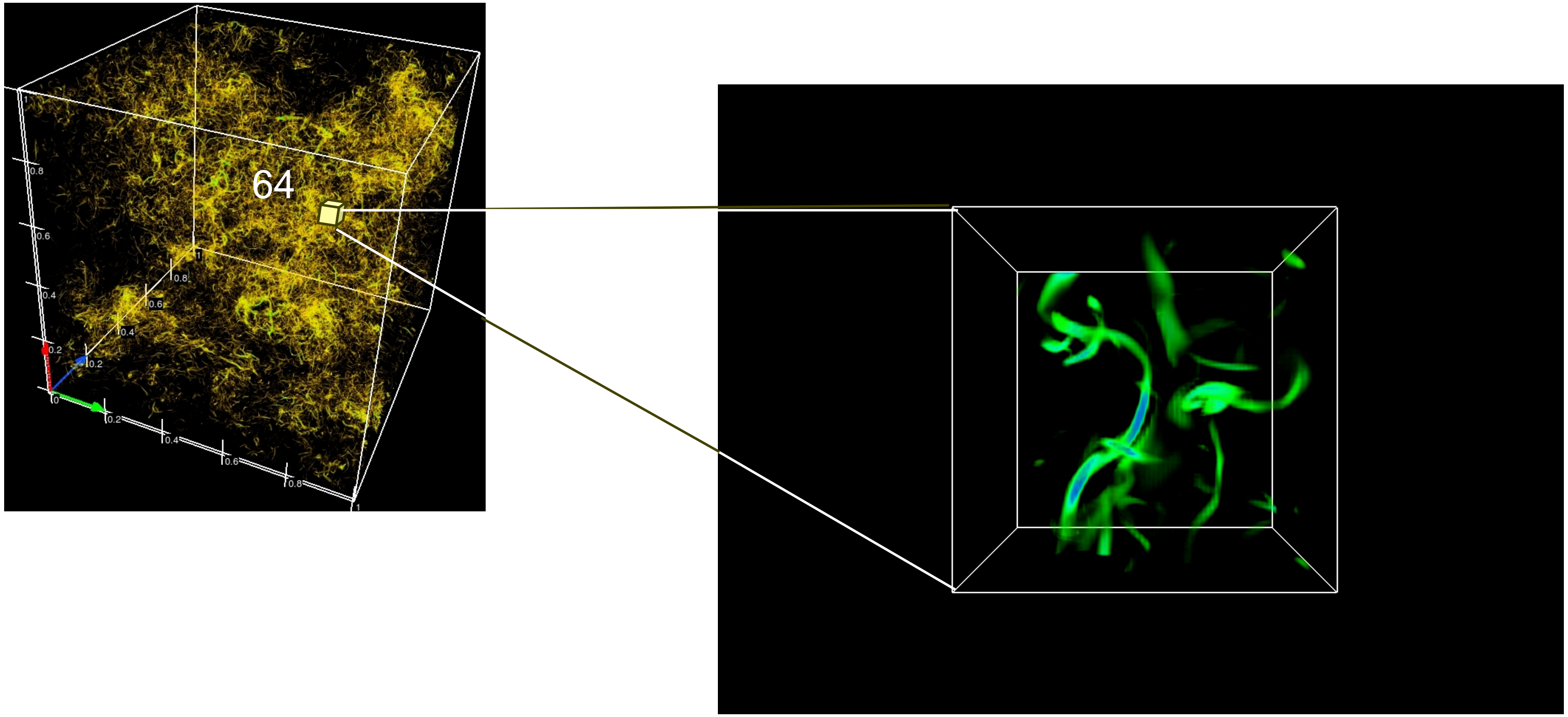
Data mining: high-intensity vorticity event associated with vortex reconnections

Colors: surfaces of iso-vorticity magnitude

1024



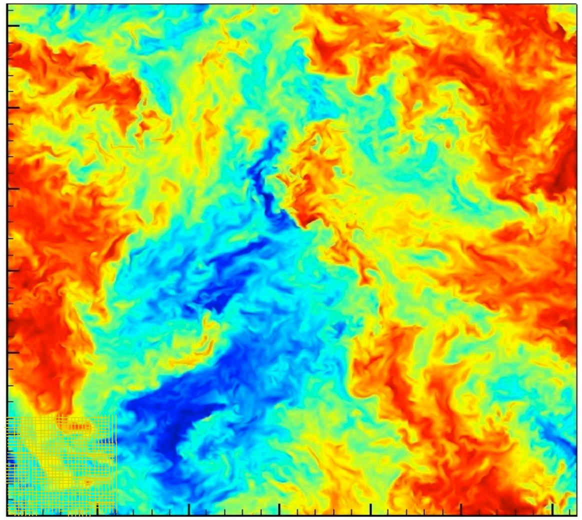
Video generated by Dr. Jonathan P. Graham
using VAPOR software (NCAR)

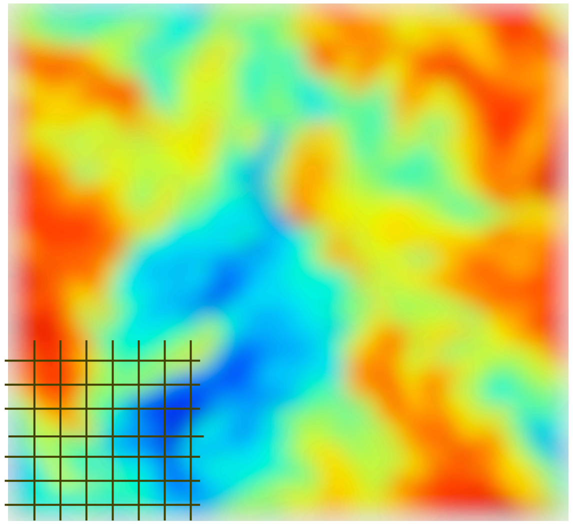
Basic (and applied) research challenge:
Coarse-graining - Large-Eddy-Simulation (LES):

Coarse-graining for more affordable simulations

$u_1(x,y,z_0,t_0)$ $\tilde{u}_1(x,y,z_0,t_0)$

$4\text{x}10^9$ d.o.f.





10^5 d.o.f.

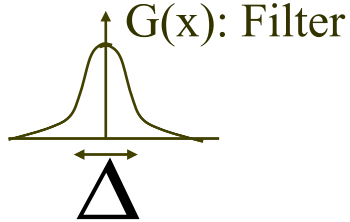
Ongoing Research Questions:

- How do small-scales affect large scale motions (and vice-versa)?
- How can we replace the effects of small scales on large scales (SGS modeling)?

Large-eddy-simulation (LES) and filtering:

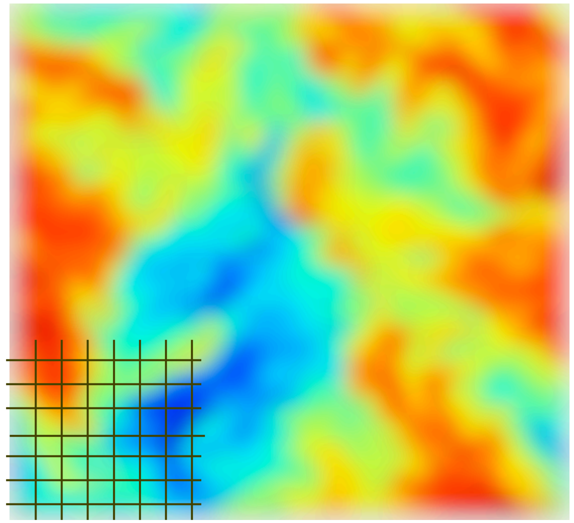
N-S equations:

$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$



$$\tilde{u}_i(\mathbf{x},t) = G_\Delta * u_i = \int G_\Delta(\mathbf{x}-\mathbf{x}')u_i(\mathbf{x}')d^3\mathbf{x}'$$

$\tilde{u}_1(x,y,z_0,t_0)$



Large-eddy-simulation (LES) and filtering:

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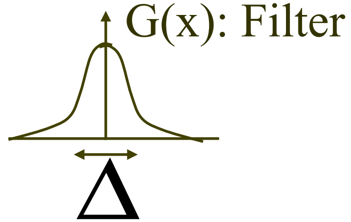
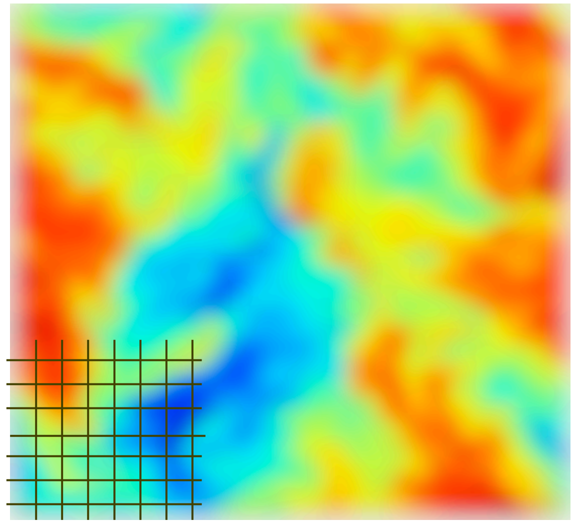


$$\tilde{u}_1(x, y, z_0, t_0)$$



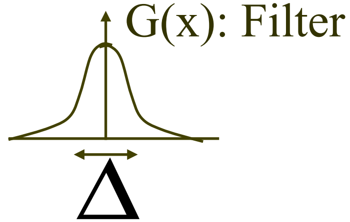
Filtered N-S equations:

$$\frac{\partial \tilde{u}_j}{\partial t} + \frac{\partial \widetilde{u_k u_j}}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j$$

Large-eddy-simulation (LES) and filtering:

N-S equations:

$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$

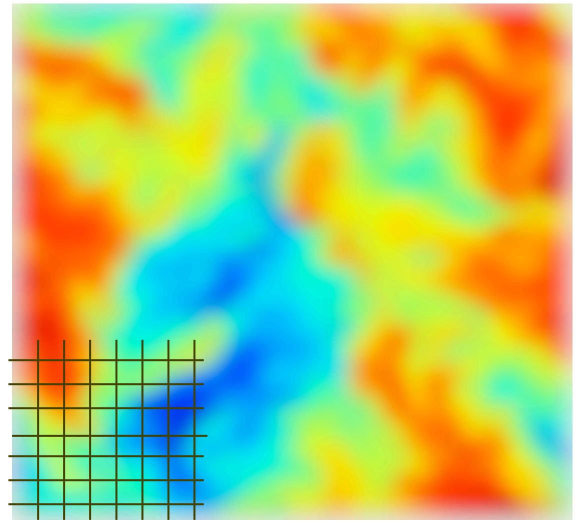


Filtered N-S equations:

$$\frac{\partial \tilde{u}_j}{\partial t} + \frac{\partial \widetilde{u_k u_j}}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j$$

$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k} \tau_{jk}$$

$\tilde{u}_1(x, y, z_0, t_0)$

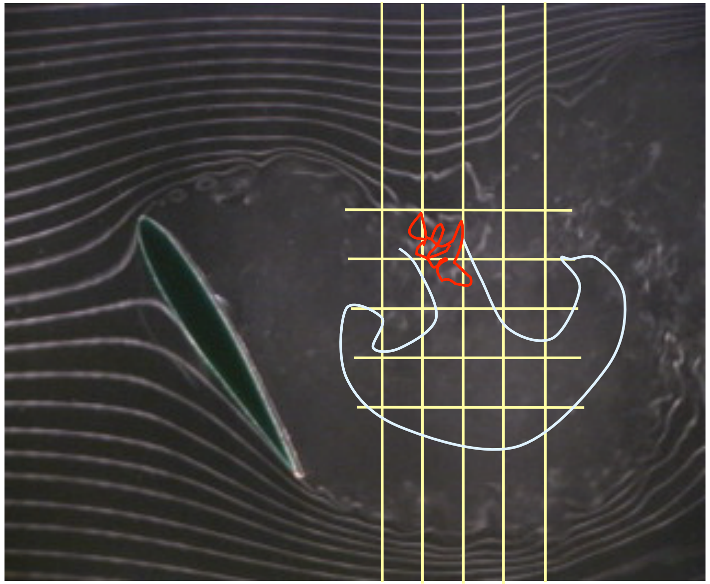


where SGS stress tensor is:

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

Most common modeling approach: eddy-viscosity

$$\tau_{ij}^{d} = -\nu_{sgs}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -2\nu_{sgs}\tilde{S}_{ij}$$



Functional form in analogy to kinetic theory of gases (Chapman-Enskog expansions, etc..)

"Eddies ~ molecules" (???)

$$\nu_{sgs} = (c_s\Delta)^2 \,|\tilde{S}|$$

c_s: *"Smagorinsky coefficient"*

Detour: some remarks on eddy-viscosity

$$\tau_{ij}^d = -\nu_{sgs}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -2\nu_{sgs}\tilde{S}_{ij}$$

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Limitations of basic eddy-viscosity:

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Turbulence is not like a “can of sand”

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Turbulence is not like a “can of sand”

but more like a
“can of worms”

Visualizations of multi-scale vortices in turbulence

Entry #: 84174

Vortices within vortices: hierarchical nature of vortex tubes in turbulence

Kai Bürger[1], Marc Treib[1], Rüdiger Westermann[1],
Suzanne Werner[2], Cristian C Lalescu[3],
Alexander Szalay[2], Charles Meneveau[4], Gregory L Eyink[2,3,4]

[1] Informatik 15 (Computer Graphik & Visualisierung), Technische Universität München
[2] Department of Physics & Astronomy, The Johns Hopkins University
[3] Department of Applied Mathematics & Statistics, The Johns Hopkins University
[4] Department of Mechanical Engineering, The Johns Hopkins University

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$$\tau_{ij}^{d} = -\nu_{sgs}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -2\nu_{sgs}\tilde{S}_{ij}$$

Turbulence is not like a "can of sand"

but more like a "can of worms"

Still, in LES eddy-viscosity seems to work "better than it should"
Also, many models need eddy-viscosity additions in ad-hoc "regularizations"
Next slides: an "excuse" for eddy-viscosity via "fluid dynamics" arguments

A "fluid-mechanical" rationale for basic eddy-viscosity:

$$\tau_{ij}^{d} \equiv \left(\widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j\right)^{d} \approx \left(\widetilde{u'_i u'_j}\right)^{d}$$

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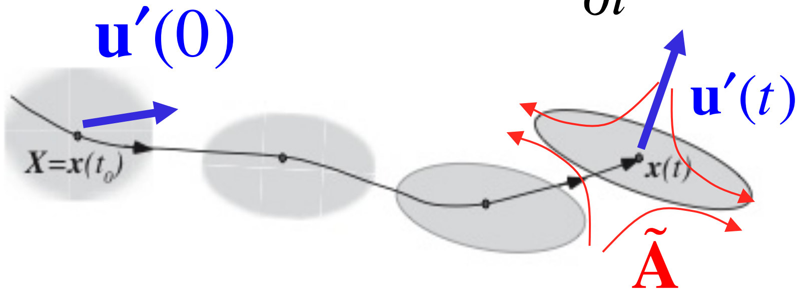
$$\tau_{ij}^{d} \equiv \left(\widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j\right)^{d} \approx \left(\widetilde{u'_i u'}\right)^{d}$$

$$\frac{\partial(\tilde{u}_i + u'_i)}{\partial t} + (\tilde{u}_k + u'_k)\frac{\partial(\tilde{u}_i + u'_i)}{\partial x_k} = forces$$



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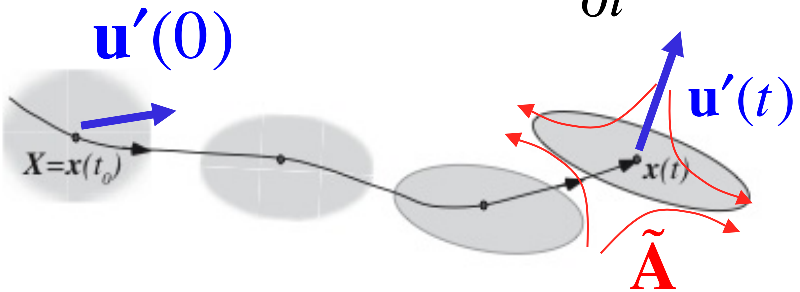
$$\frac{\partial(\tilde{u}_i + u_i')}{\partial t} + (\tilde{u}_k + u_k')\frac{\partial(\tilde{u}_i + u_i')}{\partial x_k} = forces$$



$$\frac{du_i'}{dt} = -\frac{\partial \tilde{u}_i}{\partial x_k} u_k' + \left[\, forces' - \nabla(u'u')\right]$$

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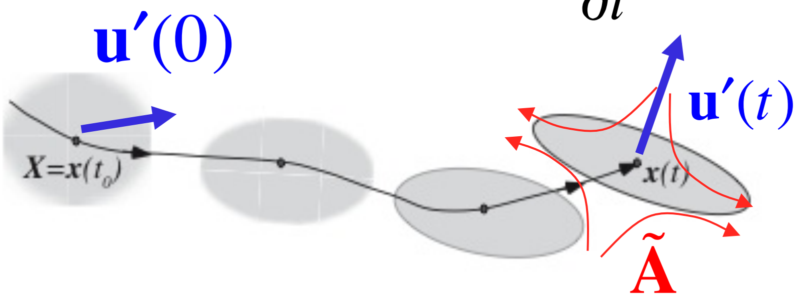


$$\frac{du_i'}{dt} = -\frac{\partial \tilde{u}_i}{\partial x_k} u_k' + \left[forces' - \nabla(u'u') \right]$$

"Production-only" approximation: $$\frac{d\mathbf{u}'}{dt} \approx -\tilde{\mathbf{A}} \bullet \mathbf{u}',$$

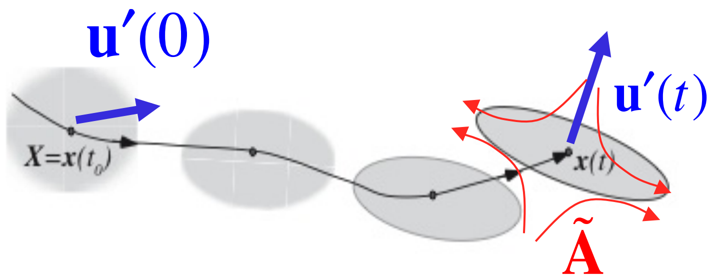
Li, Chevillard, Eyink & CM
Phys Rev. E, 2009.

(stretching and tilting of vel.
fluctuation by large-scale velocity gradients
- consistent with vortex stretching)

$$\tilde{A}_{ik} = \frac{\partial \tilde{u}_i}{\partial x_k}$$

A "fluid-mechanical" rationale for basic eddy-viscosity:

$$\frac{d\mathbf{u}'}{dt} \approx -\tilde{\mathbf{A}} \bullet \mathbf{u}'$$



A "fluid-mechanical" rationale for basic eddy-viscosity:

$$\mathbf{u}'(t) = \mathbf{exp}(-\tilde{\mathbf{A}}t) \bullet \mathbf{u}'(0)$$

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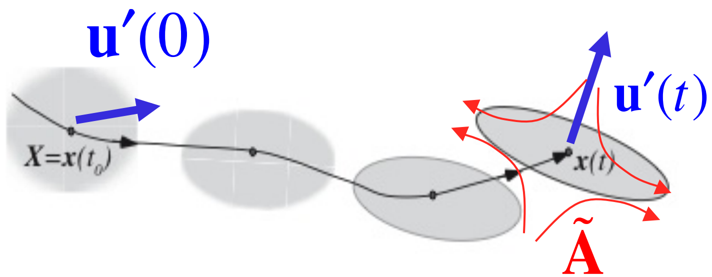

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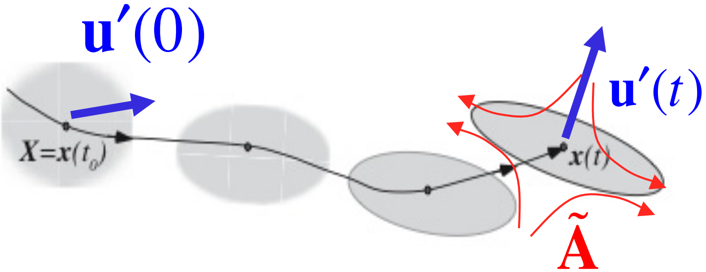
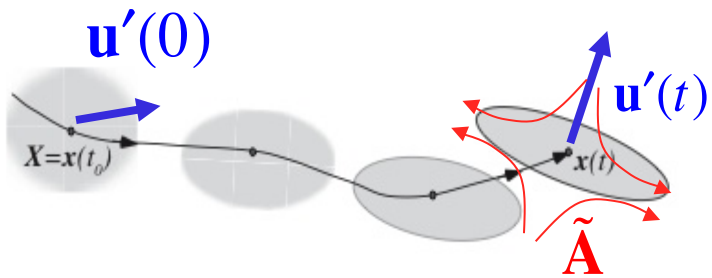
$$\mathbf{u}'(t) \approx \left(\mathbf{I} - \tilde{\mathbf{A}}t + \frac{1}{2}(\tilde{\mathbf{A}}t)^2 - \ldots \right) \bullet \mathbf{u}'(0)$$



A "fluid-mechanical" rationale for basic eddy-viscosity:

$$\frac{d\mathbf{u}'}{dt} \approx -\tilde{\mathbf{A}} \bullet \mathbf{u}' \longrightarrow \mathbf{u}'(t) \approx \left(\mathbf{I} - \tilde{\mathbf{A}}t + \frac{1}{2}(\tilde{\mathbf{A}}t)^2 - \ldots \right) \bullet \mathbf{u}'(0)$$

$$\mathbf{u}' \otimes \mathbf{u}' = \mathbf{u}'\mathbf{u}'^T \approx \left[\left(\mathbf{I} - \tilde{\mathbf{A}}t \right) \bullet \mathbf{u}'(0) \right] \left[\mathbf{u}'^T(0) \bullet \left(\mathbf{I} - \tilde{\mathbf{A}}^T t \right) \right]$$



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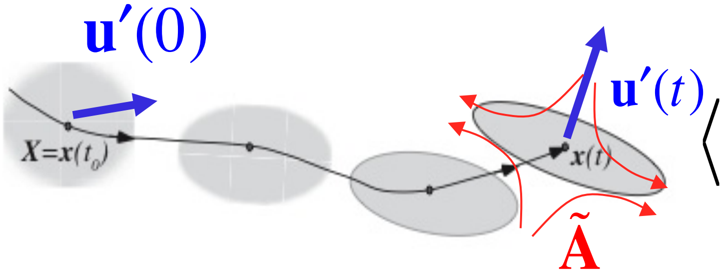

$$\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle \approx \left(\mathbf{I} - \tilde{\mathbf{A}}t \right) \bullet \left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle_{t=0} \bullet \left(\mathbf{I} - \tilde{\mathbf{A}}^T t \right)$$

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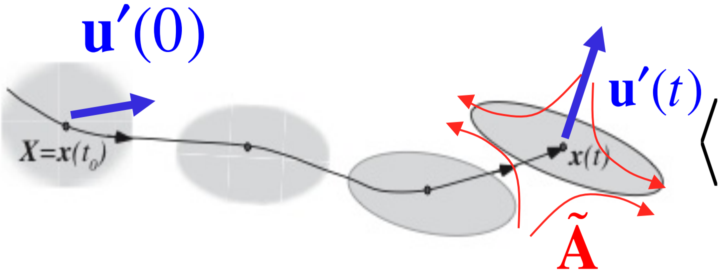

$$\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle \approx \left(\mathbf{I} - \tilde{\mathbf{A}}t \right) \bullet \underbrace{\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle_{t=0}}_{\textit{isotropy}:\ \left(c_e \Delta |\tilde{S}|\right)^2 \mathbf{I}} \bullet \left(\mathbf{I} - \tilde{\mathbf{A}}^T t \right)$$

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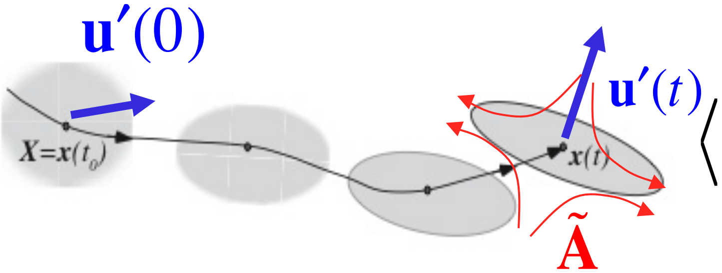

$$\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle \approx \left(\mathbf{I} - \tilde{\mathbf{A}}t\right) \bullet \underbrace{\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle_{t=0}}_{} \bullet \left(\mathbf{I} - \tilde{\mathbf{A}}^T t\right)$$

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$$\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle_{t_A} \approx \left(c_e \Delta \left|\tilde{S}\right|\right)^2 \left[(\mathbf{I} - \tilde{\mathbf{A}}t_A)\bullet(\mathbf{I} - \tilde{\mathbf{A}}^T t_A)\right] \approx \left(c_e \Delta \left|\tilde{S}\right|\right)^2 \left[\mathbf{I} - \underbrace{(\tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T)}_{2\,\tilde{\mathbf{S}}}\, t_A + O(t^2)\right]$$

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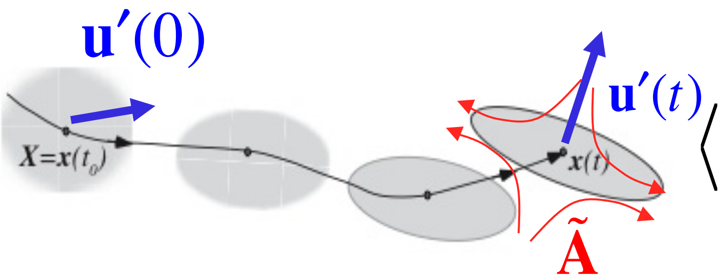

$$\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle \approx \left(\mathbf{I} - \tilde{\mathbf{A}}t\right) \bullet \underbrace{\left\langle \mathbf{u}'\mathbf{u}'^T \middle| \tilde{\mathbf{A}} \right\rangle_{t=0}}_{} \bullet \left(\mathbf{I} - \tilde{\mathbf{A}}^T t\right)$$

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A "fluid-mechanical" rationale for basic eddy-viscosity:

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choosing $\quad t_A \propto \dfrac{1}{\left|\tilde{S}\right|}$

$$\nu_{sgs} = (c_s \Delta)^2 \, |\tilde{S}|$$

c_s: "Smagorinsky coefficient"

Effects of t_{ij} upon resolved motions: Energetics (kinetic energy):

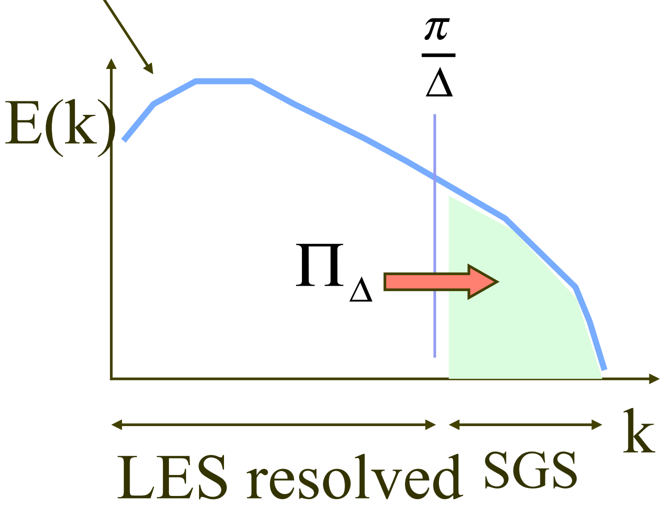
$$\frac{\partial \frac{1}{2}\tilde{u}_j\tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \frac{1}{2}\tilde{u}_j\tilde{u}_j}{\partial x_k} = -\frac{\partial}{\partial x_j}(\ldots.) - 2\nu \tilde{S}_{jk}\tilde{S}_{jk} - \left(-\tau_{jk}\tilde{S}_{jk}\right)$$

$$\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$$

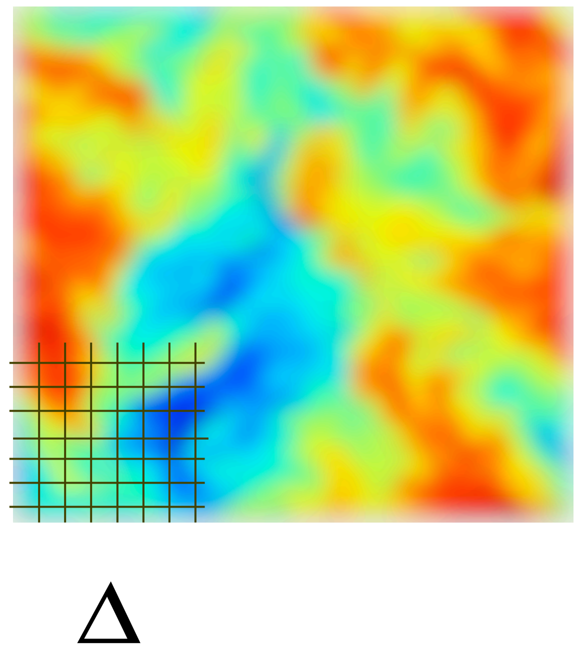
$$\Pi_\Delta = -\left\langle \tau_{jk}\tilde{S}_{jk} \right\rangle$$

Inertial-range flux

$$\varepsilon = \frac{u'^3}{L}$$



$$\tilde{u}_1(x, y, z_0, t_0)$$

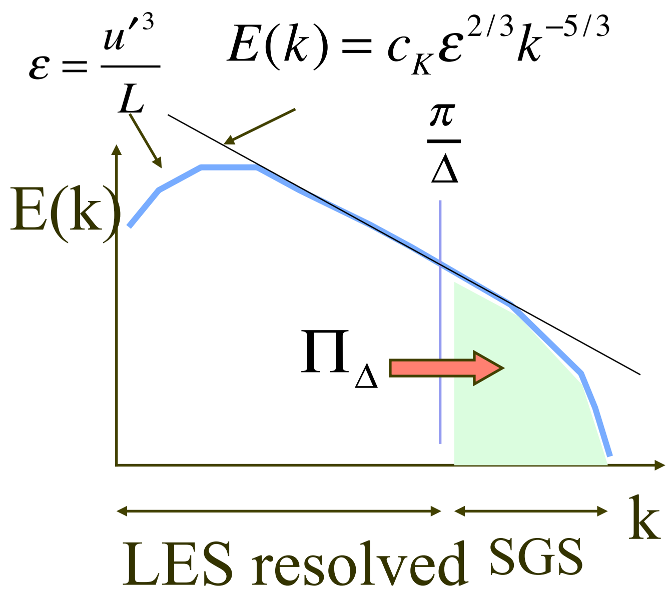


Theoretical calibration of c_s (D.K. Lilly, 1967):

$$\Pi_\Delta = \varepsilon = -\left\langle \tau_{ij}\tilde{S}_{ij} \right\rangle \qquad \tau_{ij} = -2(c_s\Delta)^2 |\tilde{S}|\tilde{S}_{ij}$$

$$\varepsilon = c_s^2\Delta^2 2\left\langle |\tilde{S}|\tilde{S}_{ij}\tilde{S}_{ij} \right\rangle$$

$$\varepsilon \approx c_s^2\Delta^2 2^{3/2}\left\langle \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle^{3/2}$$



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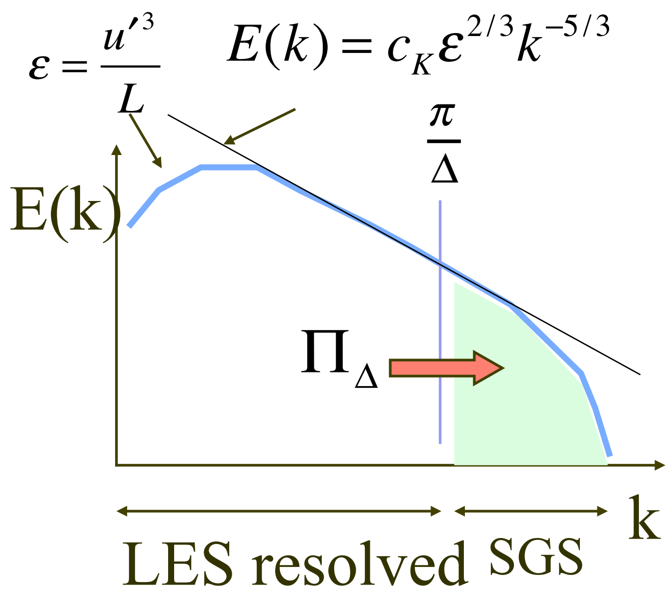
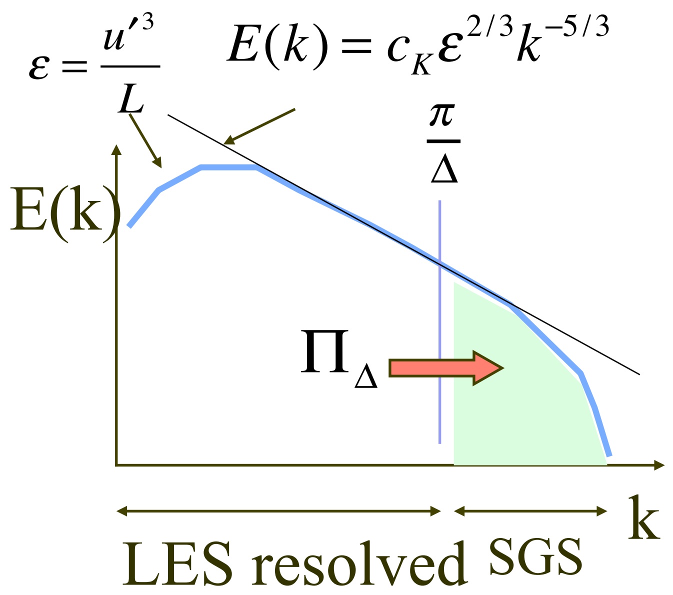


$$\left\langle \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle = \frac{1}{2}\left\langle \frac{\partial \tilde{u}_i}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \right\rangle =$$

$$= \frac{1}{2}\iiint_{|\mathbf{k}|<\pi/\Delta} [k_j{}^2\Theta_{ii}(\mathbf{k}) + k_i k_j \Theta_{ij}(\mathbf{k})]d^3\mathbf{k} = \frac{1}{2}\iiint_{|\mathbf{k}|<\pi/\Delta} [k^2(\frac{E(k)}{4\pi k^2}(\delta_{ii} - \frac{k^2}{k^2})) + 0]d^3\mathbf{k}$$

$$= c_K\varepsilon^{2/3}\frac{1}{2}\int_0^{\pi/\Delta} k^{-5/3+2}\frac{3-1}{4\pi k^2}4\pi k^2 dk = c_K\varepsilon^{2/3}\int_0^{\pi/\Delta} k^{1/3}dk = c_K\varepsilon^{2/3}\frac{3}{4}\left(\frac{\pi}{\Delta}\right)^{4/3}$$

Theoretical calibration of c_s (D.K. Lilly, 1967):



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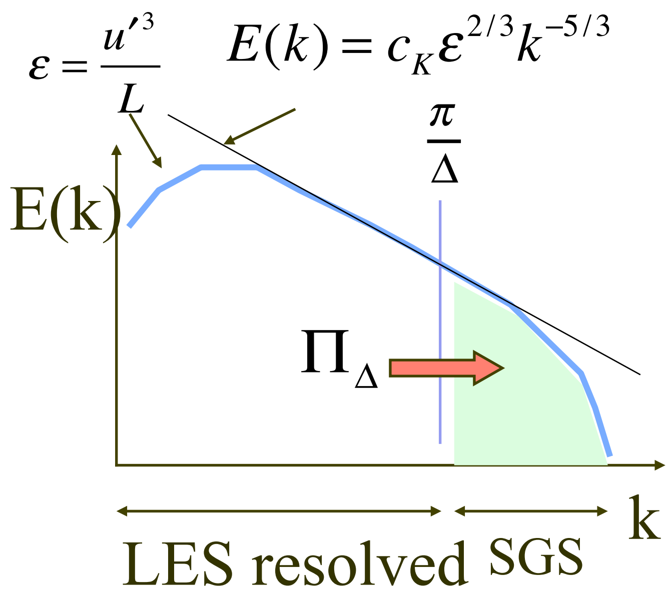
$$\varepsilon \approx c_s^2 \Delta^2 2^{3/2} \left(c_K \varepsilon^{2/3} \frac{3}{4} \left(\frac{\pi}{\Delta} \right)^{4/3} \right)^{3/2}$$

Theoretical calibration of c_s (D.K. Lilly, 1967):



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$$\left\langle \tilde{S}_{ij}\tilde{S}_{ij} \right\rangle = \frac{1}{2}\left\langle \frac{\partial \tilde{u}_i}{\partial x_j}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \right\rangle =$$

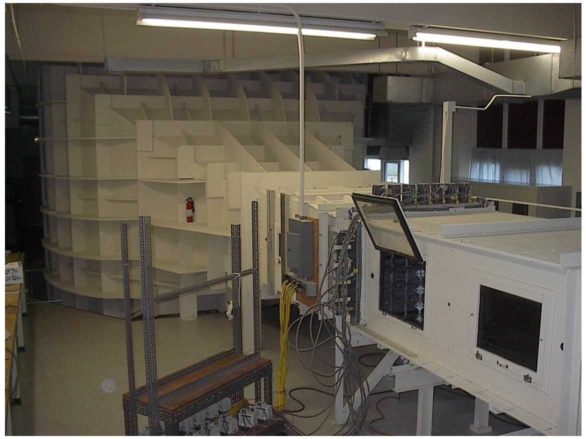
$$= \frac{1}{2} \iiint_{|\mathbf{k}|<\pi/\Delta} [k_j^{\,2}\Theta_{ii}(\mathbf{k}) + k_i k_j \Theta_{ij}(\mathbf{k})] d^3\mathbf{k} = \frac{1}{2} \iiint_{|\mathbf{k}|<\pi/\Delta} [k^2 (\frac{E(k)}{4\pi k^2}(\delta_{ii} - \frac{k^2}{k^2})) + 0] d^3\mathbf{k}$$

$$= c_K \varepsilon^{2/3} \frac{1}{2} \int_0^{\pi/\Delta} k^{-5/3+2} \frac{3-1}{4\pi k^2} 4\pi k^2 dk = c_K \varepsilon^{2/3} \int_0^{\pi/\Delta} k^{1/3} dk = c_K \varepsilon^{2/3} \frac{3}{4}\left(\frac{\pi}{\Delta}\right)^{4/3}$$

$$\varepsilon \approx c_s^2 \Delta^2 2^{3/2} \left(c_K \varepsilon^{2/3} \frac{3}{4} \left(\frac{\pi}{\Delta}\right)^{4/3} \right)^{3/2}$$

$$\Rightarrow 1 \approx c_s^2 \pi^2 \left(\frac{3c_K}{2}\right)^{3/2} \Rightarrow c_s = \left(\frac{3c_K}{2}\right)^{-3/4} \pi^{-1}$$

$$c_K = 1.6 \Rightarrow c_s \approx 0.16$$



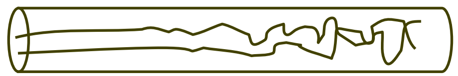
c_s=0.16 works well for isotropic, high Reynolds number turbulence

But in practice (complex flows)

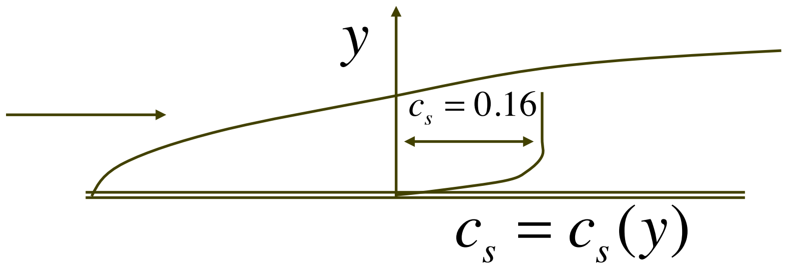
$$c_s = c_s(\mathbf{x}, t)$$

Ad-hoc tuning?

Examples: Transitional pipe flow: from 0 to 0.16



Near wall damping for wall boundary layers (Piomelli et al 1989)



How does c_s vary under realistic conditions? Interrogate data:

Measure: $$\Pi_\Delta = -\left\langle \tau_{jk} \tilde{S}_{jk} \right\rangle$$

Measure: $$\frac{\Pi_\Delta^{Smag}}{c_s^2} = 2\Delta^2 \left\langle |\tilde{S}| \tilde{S}_{ij} \tilde{S}_{ij} \right\rangle$$

Obtain "empirical" Smagorinsky coefficient = *f(x,conditions…):*

$$c_s = \left(\frac{-\left\langle \tau_{jk} \tilde{S}_{jk} \right\rangle}{2\Delta^2 \left\langle |\tilde{S}| \tilde{S}_{ij} \tilde{S}_{ij} \right\rangle} \right)^{1/2}$$

An example result from atmospheric turbulence…:

Measure "empirical" Smagorinsky coefficient for atmospheric surface layer as function of height and stability (thermal forcing or damping):

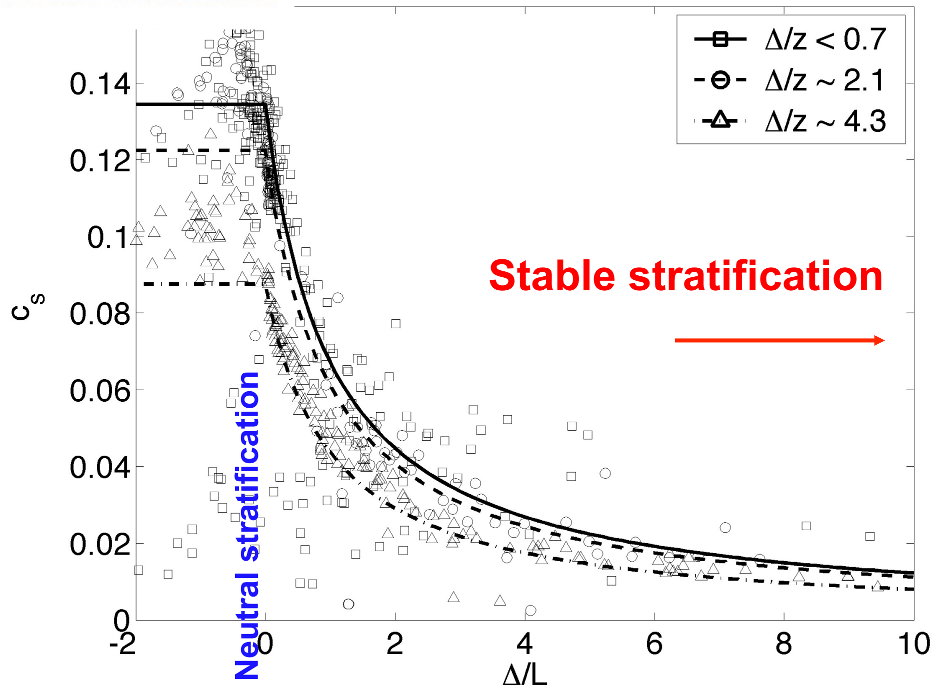
HATS - 2000

(with NCAR researchers: Horst, Sullivan)

Kettleman City (Central Valley, CA)

$$c_s = \left(\frac{-\langle \tau_{jk} \tilde{S}_{jk} \rangle}{2\Delta^2 \langle |\tilde{S}| \tilde{S}_{ij} \tilde{S}_{ij} \rangle} \right)^{1/2}$$

Example result: **effect of atmospheric stability on coefficient from sonic anemometer measurements in atmospheric surface layer (Kleissl et al., J. Atmos. Sci. 2003)**



$$c_s = c_s(\mathbf{x}, t)$$

How to avoid "tuning" and case-by-case adjustments of model coefficient in LES?

The Dynamic Model

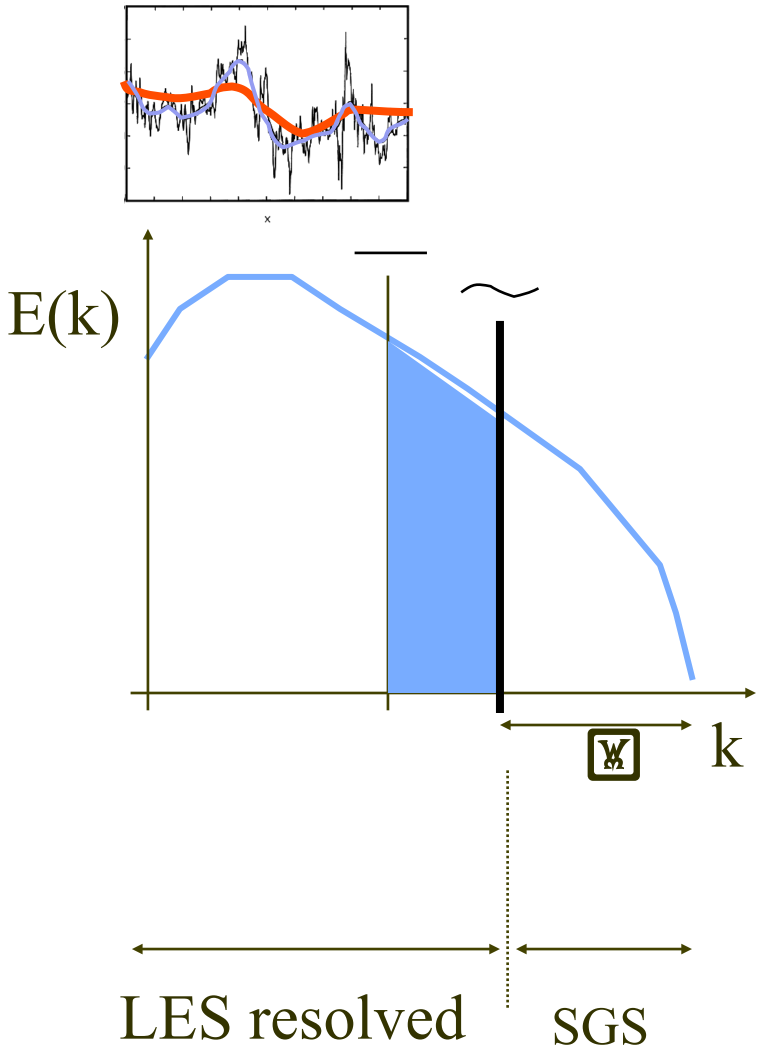
(Germano et al. Physics of Fluids, 1991)

Germano identity and dynamic model

(Germano et al. 1991):

Exact ("rare" in turbulence):

$$\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = \overline{\widetilde{u_i u_j}} \qquad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$



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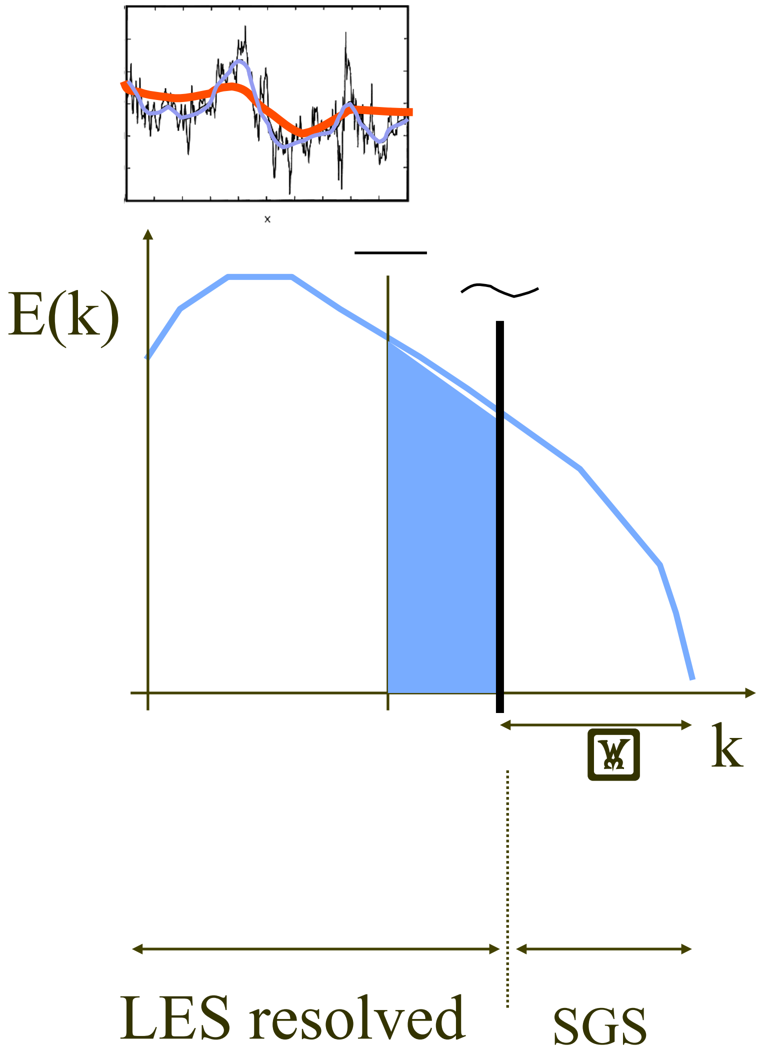

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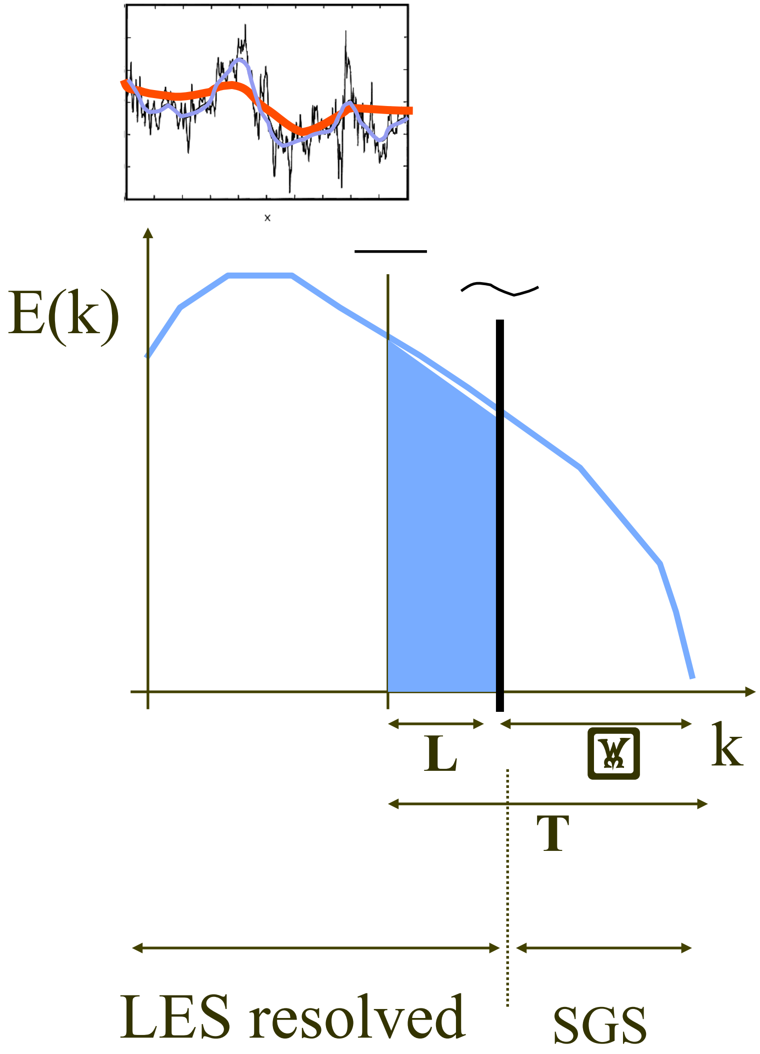
$$T_{ij} = \bar{\tau}_{ij} + L_{ij}$$

$$L_{ij} - (T_{ij} - \bar{\tau}_{ij}) = 0$$



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$$\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j = \overline{\widetilde{u_i u_j}} \quad - \overline{\tilde{u}_i \tilde{u}_j} \quad + \overline{\tilde{u}_i \tilde{u}_j} \quad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j$$

$$T_{ij} = \quad \bar{\tau}_{ij} \quad + \quad L_{ij}$$

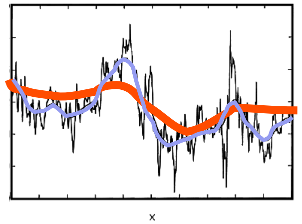
$$L_{ij} - (T_{ij} - \bar{\tau}_{ij}) = 0$$

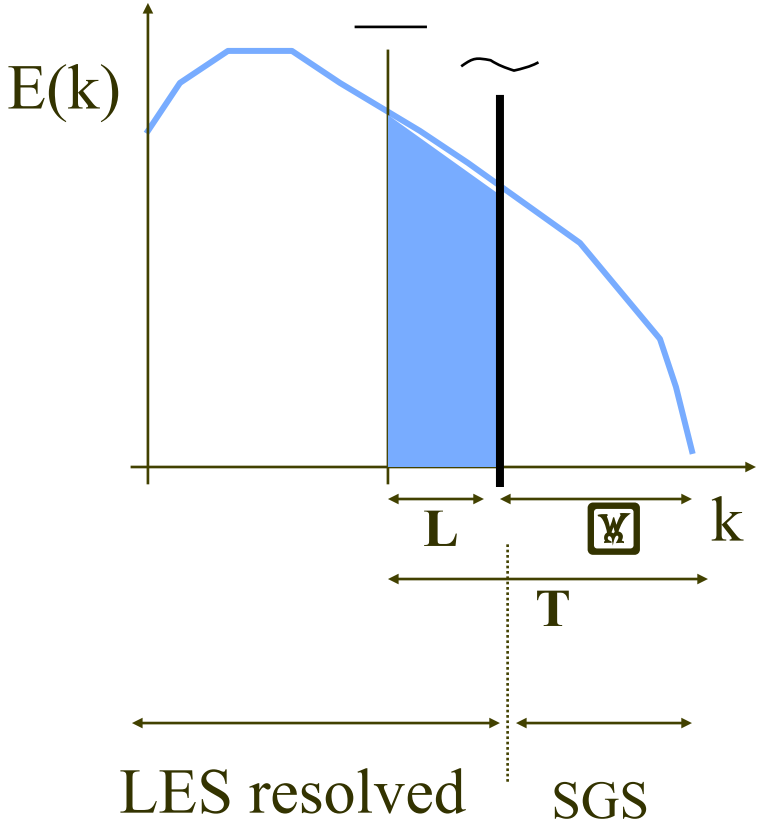
$$-2(c_s 2\Delta)^2 \, |\bar{\tilde{S}}| \bar{\tilde{S}}_{ij} \qquad -2(c_s \Delta)^2 \, \overline{|\tilde{S}| \tilde{S}_{ij}}$$

Assumes scale-invariance:

$$L_{ij} - c_s^2 M_{ij} = 0$$

where $M_{ij} = 2\Delta^2 \left(\overline{|\tilde{S}| \tilde{S}_{ij}} - 4 |\bar{\tilde{S}}| \bar{\tilde{S}}_{ij} \right)$





Germano identity and dynamic model

(Germano et al. 1991):

$$L_{ij} - c_s^2 M_{ij} = 0$$

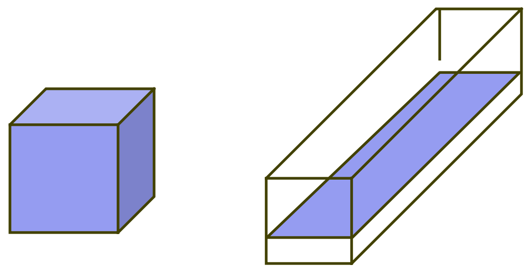
Over-determined system:
solve in "some average sense"
(minimize error, Lilly 1992):

$$\mathrm{E} = \left\langle \left(L_{ij} - c_s^2 M_{ij} \right)^2 \right\rangle$$

Minimized when:

$$c_s^2 = \frac{\langle L_{ij} M_{ij} \rangle}{\langle M_{ij} M_{ij} \rangle}$$

Averaging over regions of
statistical homogeneity
or fluid trajectories



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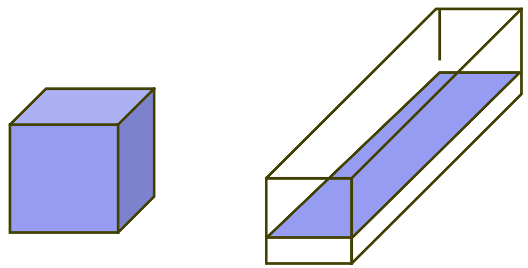


Homework:

(a) Prove the above equation, and

(b) repeat entire formulation for the SGS heat flux vector

modeled using an SGS diffusivity (find C_{scalar})

$$q_i = \widetilde{u_i T} - \tilde{u}_i \tilde{T}$$

$$q_i = C_{scalar} \Delta^2 |\tilde{S}| \frac{\partial \tilde{T}}{\partial x_i}$$

Similarity, tensor eddy-viscosity, and mixed models

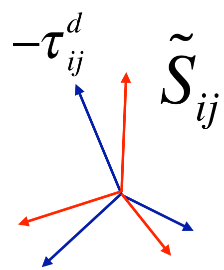
$$\tau_{ij}^{mnl} = C_{nl}\Delta^2 \frac{\partial \tilde{u}_i}{\partial x_k}\frac{\partial \tilde{u}_j}{\partial x_k} - 2(C_S\Delta)^2 |\tilde{S}|\tilde{S}_{ij}$$

Two-parameter dynamic mixed model

$$L_{ij} \equiv T_{ij} - \bar{\tau}_{ij} = \overline{\tilde{u}_i\tilde{u}_j} - \bar{\tilde{u}}_i\bar{\tilde{u}}_j$$

$$T_{ij} = -2(C_S 2\Delta)^2 |\bar{\tilde{S}}|\bar{\tilde{S}}_{ij} + C_{nl}(2\Delta)^2 \frac{\partial \bar{\tilde{u}}_i}{\partial x_k}\frac{\partial \bar{\tilde{u}}_j}{\partial x_k}$$

- Mixed tensor Eddy Viscosity Model:
  - *Taylor-series expansion of similarity (Bardina 1980) model* (Clark 1980, Liu, Katz & Meneveau (1994), …)
  - *Deconvolution:* (Leonard 1997, Geurts et al, Stolz & Adams, Winckelmans etc..)
  - Significant d*irect empirical evidence, experiments:*
    - Liu et al. (JFM 1999, 2-D PIV)
    - Tao, Katz & CM (J. Fluid Mech. 2002): **tensor alignments** from 3-D HPIV data
    - Higgins, Parlange & CM (Bound Layer Met. 2003): **tensor alignments** from ABL data
    - From DNS: Horiuti 2002, Vreman et al (LES), etc…



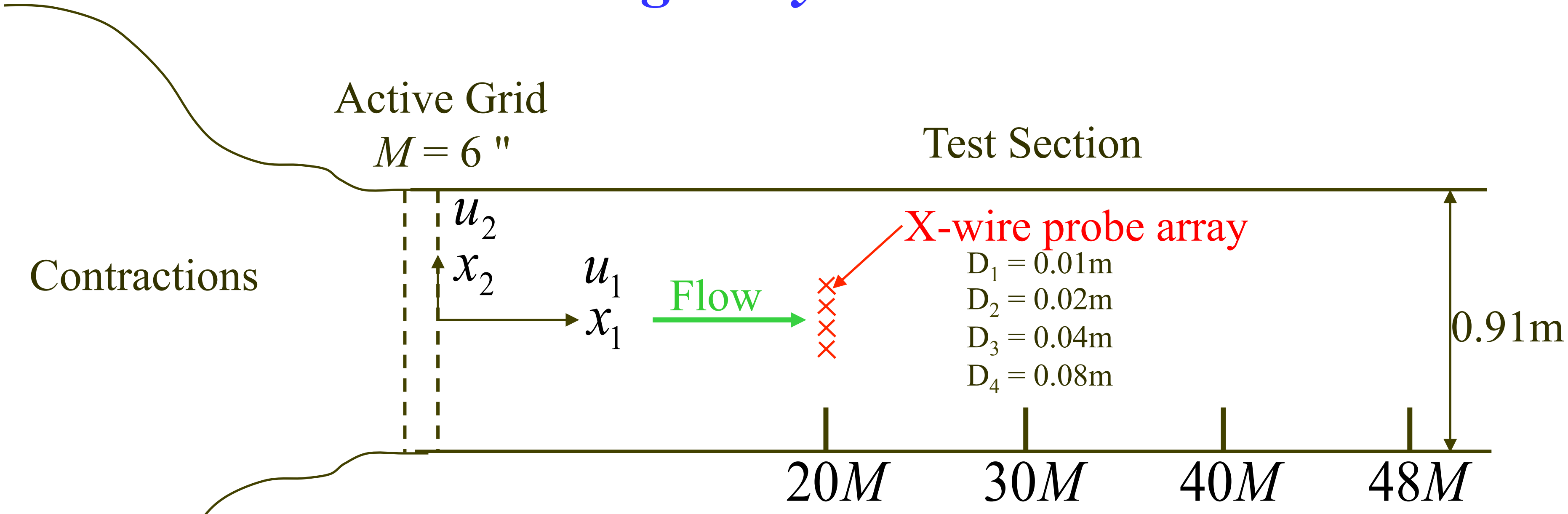
Reality check:

Do simulations with these closures produce realistic statistics of $\widetilde{u}_i(x,t)$?

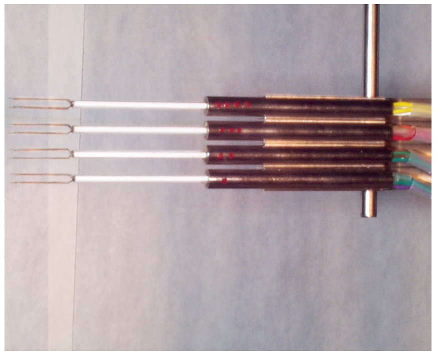
- *Need good data*
- *Need good simulations*

  - *Next: Summary of results from Kang et al. (JFM 2003)*
    - *Smagorinsky model,*
    - *Dynamic Smagorinsky model,*
    - *Dynamic 2-parameter mixed model*

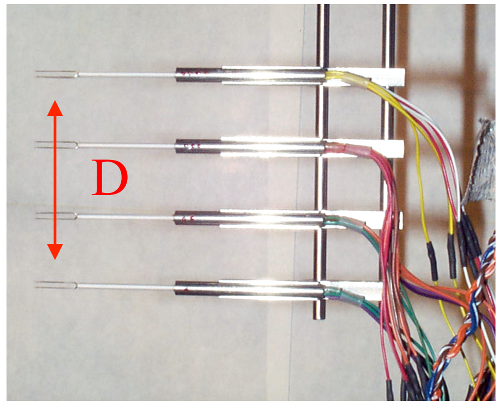
Remake of Comte-Bellot & Corrsin (1967) decaying isotropic turbulence experiment at high Reynolds number



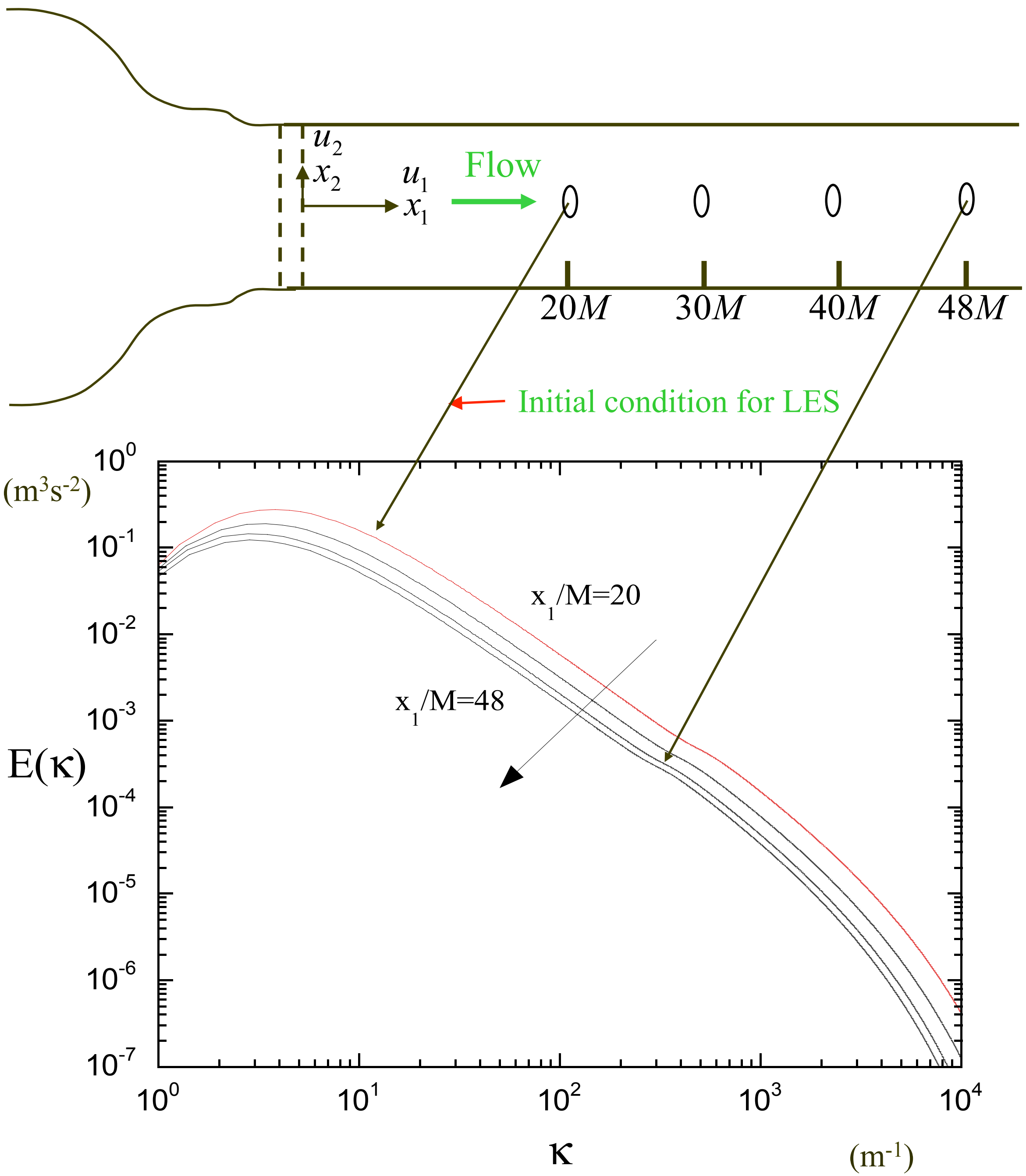
$h_1 = 0.005\text{m}$
$D_1 = 0.01\text{m}$



$h_3 = 0.02\text{m}$
$D_3 = 0.04\text{m}$



Results



u_2
x_2
u_1
x_1
Flow
20M
30M
40M
48M
Initial condition for LES
(m^3s^{-2})
E(κ)
$x_1/M=20$
$x_1/M=48$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^0
10^1
10^2
10^3
10^4
κ
(m^{-1})

LES of Temporally Decaying Turbulence

Experiment	Taylor Hypothesis	LES
Spatially Decaying Grid Turbulence	→	Temporally Decaying Turbulence
2-D Box Filter	$x_1 = \langle u_1 \rangle t$	3-D Filter

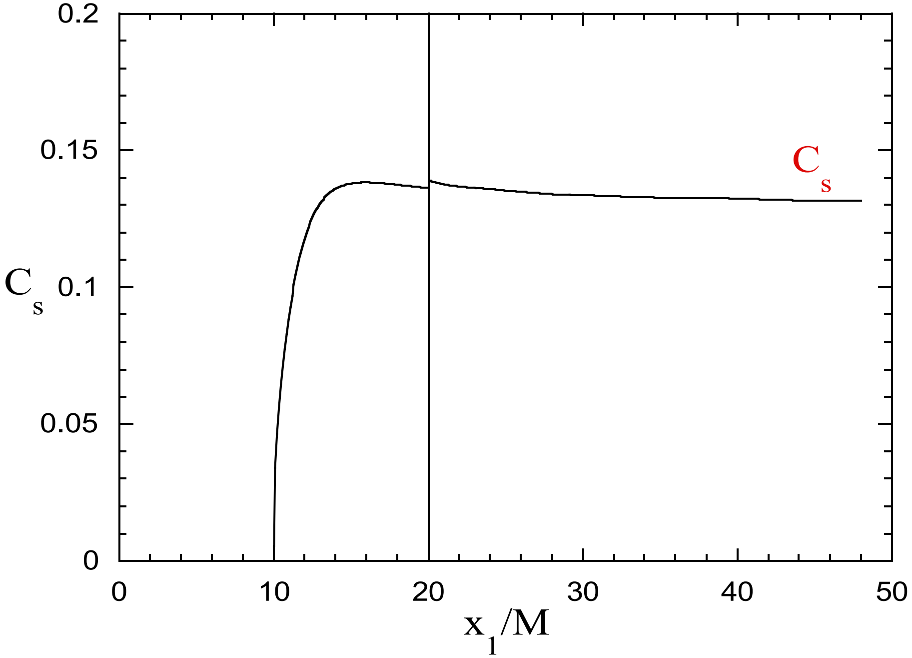
- Pseudo-spectral code: 128^3 nodes, carefully dealiased (3/2N)
- All parameters are equivalent to those of experiments.
- Initial energy distribution: 3-D energy spectrum at $x_1/M = 20$

- LES Models: standard Smagorinsky-Lilly model,
  dynamic Smagorinsky and
  dynamic mixed tensor eddy-visc. model

Results: Dynamic Model Coefficients

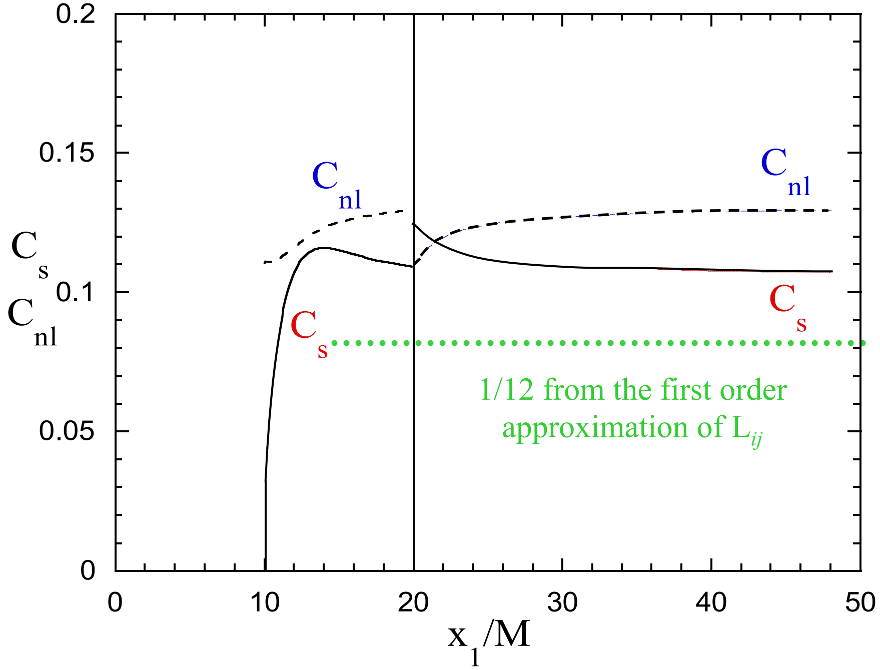
- **Dynamic Smagorinsky**

$$\tau_{ij}^{dyn-Smag} = -2\frac{\langle L_{ij}M_{ij}\rangle}{\langle M_{ij}M_{ij}\rangle}\Delta^2|\tilde{S}|\tilde{S}_{ij}$$



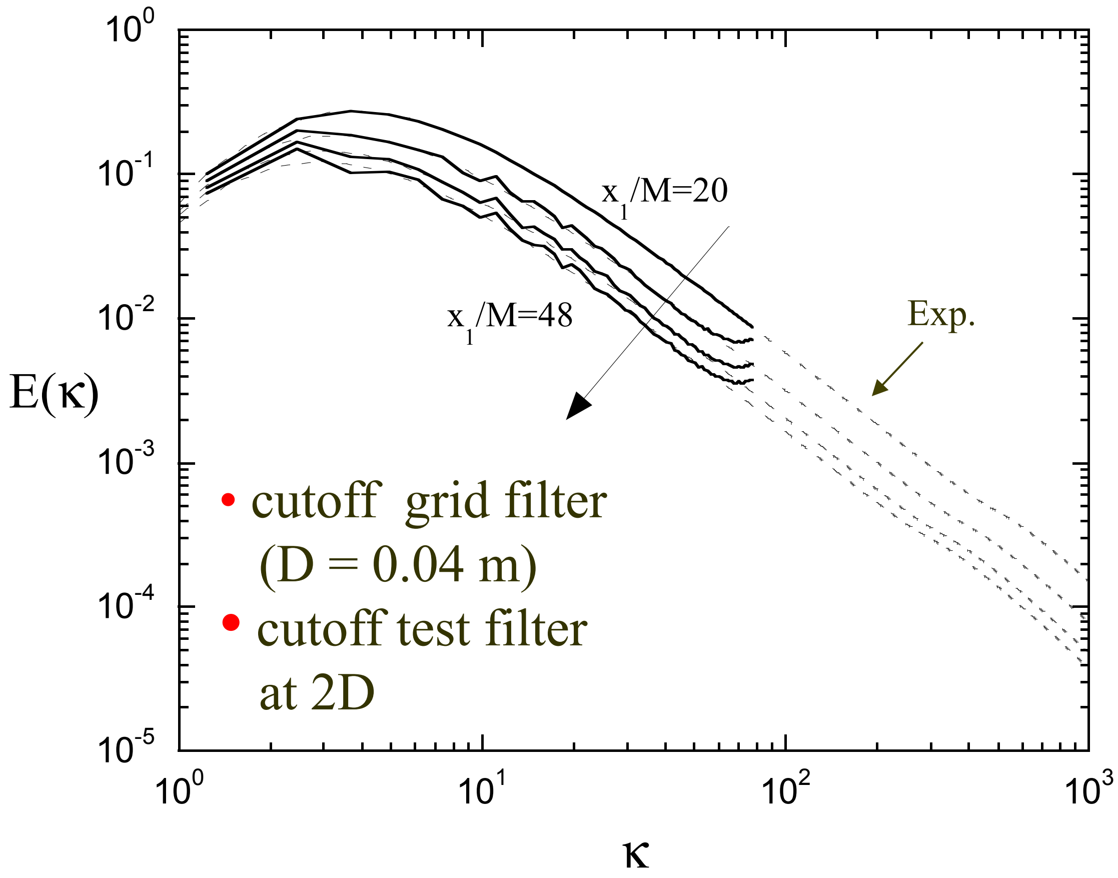
- **Dynamic Mixed tensor eddy visc:**

$$\tau_{ij}^{mnnl} = C_{nl}\Delta^2\frac{\partial\tilde{u}_i}{\partial x_k}\frac{\partial\tilde{u}_j}{\partial x_k} - 2(C_S\Delta)^2|\tilde{S}|\tilde{S}_{ij}$$



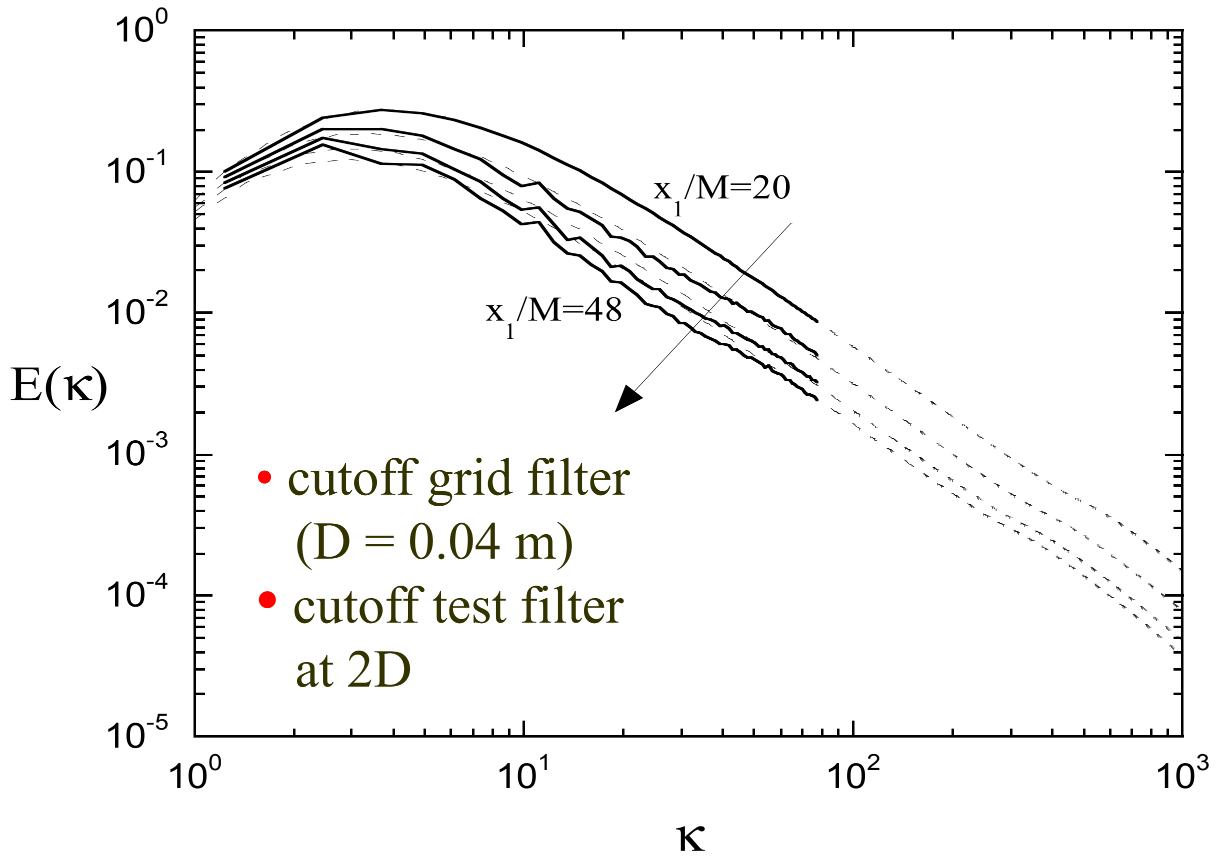
3-D Energy Spectra (LES vs experiment)

- Dynamic Smagorinsky



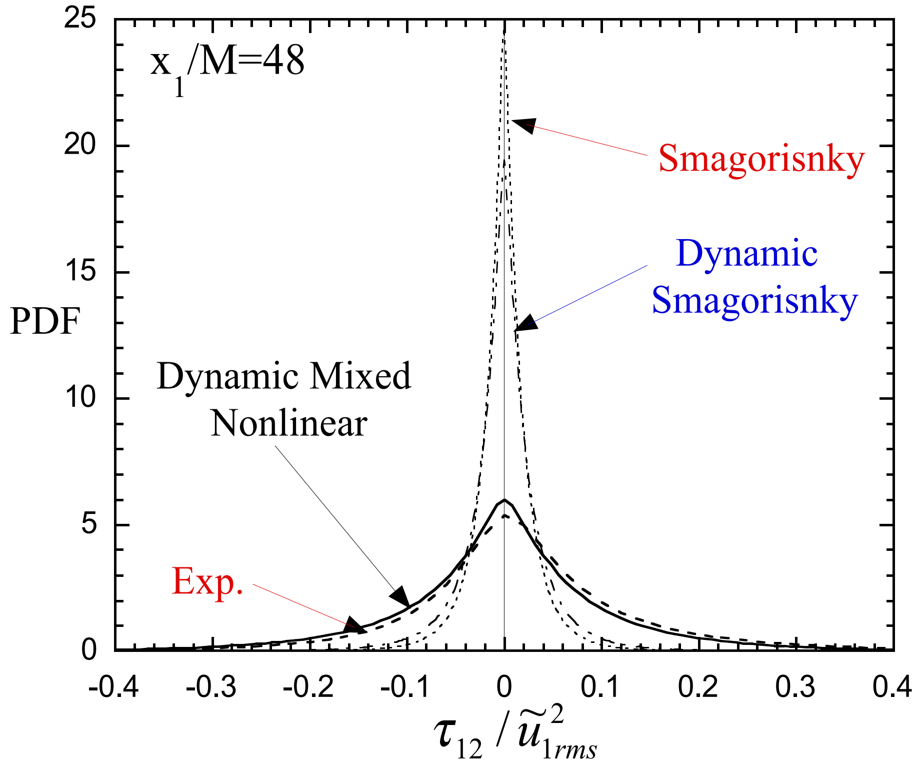
3-D Energy Spectra (LES vs experiment)

- Dynamic Mixed tensor eddy-visc. model



PDF of SGS Stress (LES vs experiment)

- SGS Stress $\tau_{12} \equiv \widetilde{u_1 u_2} - \tilde{u}_1 \tilde{u}_2$



Dynamic mixed tensor eddy-visc model predicts PDF of the SGS stress accurately.

Germano identity and dynamic model

(Germano et al. 1991):

Exact ("rare" in turbulence):

$$\underbrace{\overline{\widetilde{u_i u_j}} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j}_{T_{ij}} = \underbrace{\overline{\widetilde{u_i u_j}} \quad - \overline{\tilde{u}_i \tilde{u}_j}}_{\bar{\tau}_{ij}} + \underbrace{\overline{\tilde{u}_i \tilde{u}_j} \quad - \bar{\tilde{u}}_i \bar{\tilde{u}}_j}_{L_{ij}}$$

$$T_{ij} = \bar{\tau}_{ij} + L_{ij}$$

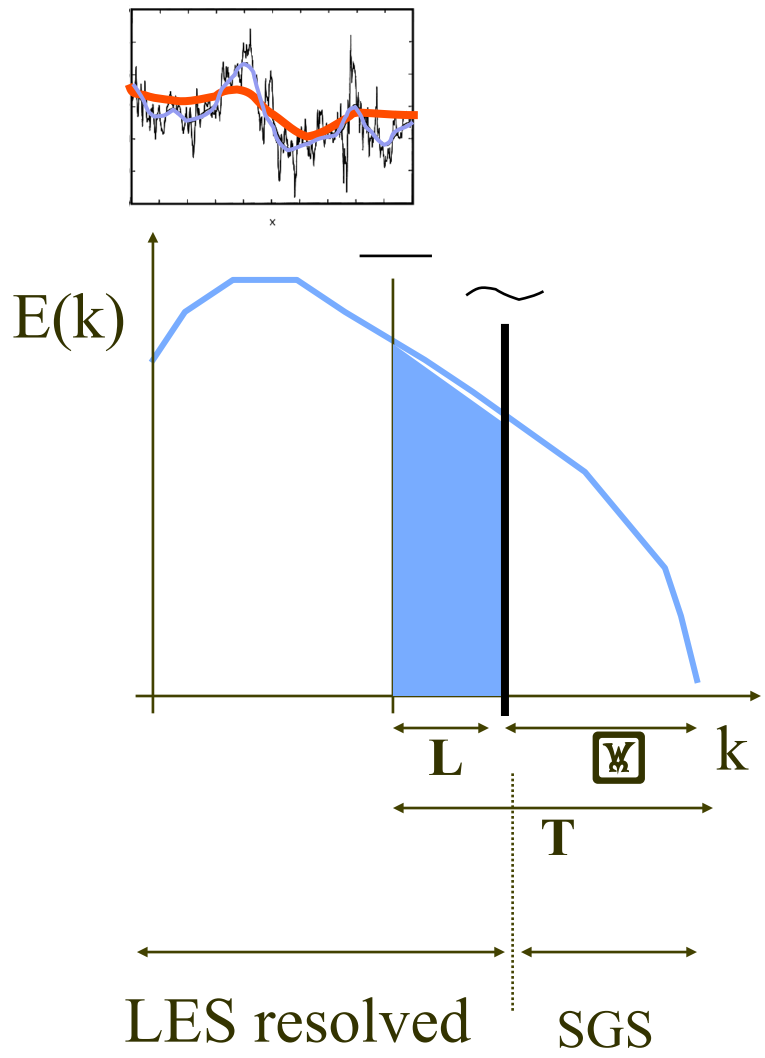
$$L_{ij} - (T_{ij} - \bar{\tau}_{ij}) = 0$$

$$-2(c_s 2\Delta)^2 \,|\bar{\tilde{S}}|\, \bar{\tilde{S}}_{ij} \qquad -2(c_s \Delta)^2 \,\overline{|\tilde{S}|\tilde{S}_{ij}}$$

Assumes scale-invariance:

$$L_{ij} - c_s^2 M_{ij} = 0$$

where $M_{ij} = 2\Delta^2 \left(\overline{|\tilde{S}|\tilde{S}_{ij}} - 4\,|\bar{\tilde{S}}|\,\bar{\tilde{S}}_{ij} \right)$



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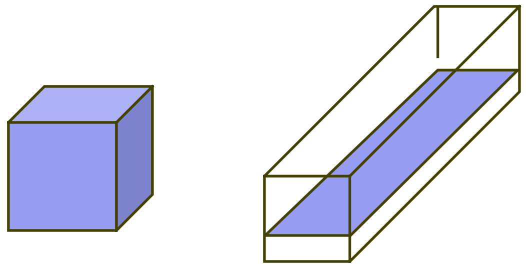
Averaging over regions of
statistical homogeneity
or fluid trajectories



Problem: what to do for non-homogeneous flows without directions over which to average ("learn", or "assimilate" larger-scale statistics?)

Lagrangian dynamic model (M, Lund & Cabot, JFM 1996):

Average in time, following fluid particles for Galilean invariance:

$$\langle E \rangle = \int_{-\infty}^{t} \left(L_{ij} - C_s^2 M_{ij} \right)^2 \; \frac{1}{T} e^{-\frac{(t-t')}{T}} \, dt'$$

Lagrangian dynamic model (M, Lund & Cabot, JFM 1996):

Average in time, following fluid particles for Galilean invariance:

$$\langle E \rangle = \int_{-\infty}^{t} \left(L_{ij} - C_s^2 M_{ij} \right)^2 \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

$$\delta \langle E \rangle = 0 \;\Rightarrow\; C_s^2 = \frac{\int_{-\infty}^{t} L_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'}{\int_{-\infty}^{t} M_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'}$$

$$\Im_{LM} = \int_{-\infty}^{t} L_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

$$\Im_{MM} = \int_{-\infty}^{t} M_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

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$$\delta \langle E \rangle = 0 \;\; \Rightarrow \;\; C_s^2 = \frac{\int_{-\infty}^{t} L_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'}{\int_{-\infty}^{t} M_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'}$$

$$\Im_{LM} = \int_{-\infty}^{t} L_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

$$\Im_{MM} = \int_{-\infty}^{t} M_{ij} M_{ij} \frac{1}{T} e^{-\frac{(t-t')}{T}} dt'$$

With exponential weight-function, equivalent to relaxation forward equations:

$$\frac{\partial \Im_{LM}}{\partial t} + \tilde{u}_k \frac{\partial \Im_{LM}}{\partial x_k} = \frac{1}{T} (L_{ij} M_{ij} - \Im_{LM})$$

$$\frac{\partial \Im_{MM}}{\partial t} + \tilde{u}_k \frac{\partial \Im_{MM}}{\partial x_k} = \frac{1}{T} (M_{ij} M_{ij} - \Im_{MM})$$

$$c_s^2 = \frac{\Im_{LM}(\mathbf{x}, t)}{\Im_{MM}(\mathbf{x}, t)}$$

Lagrangian dynamic model has allowed applying the Germano-identity to a number of complex-geometry engineering problems

LES of flows in internal combustion engines:

Haworth & Jansen (2000)

D.C. Haworth, K. Jansen / Computers & Fluids 29 (2000) 493–524 505

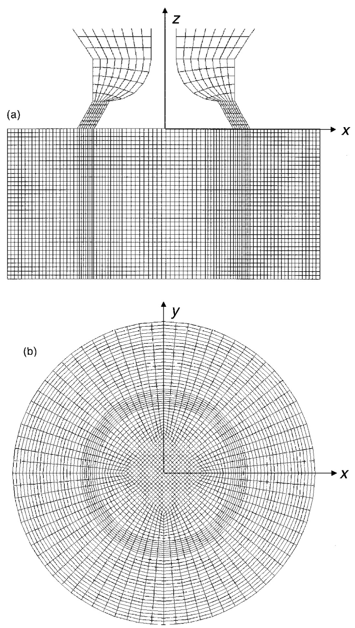
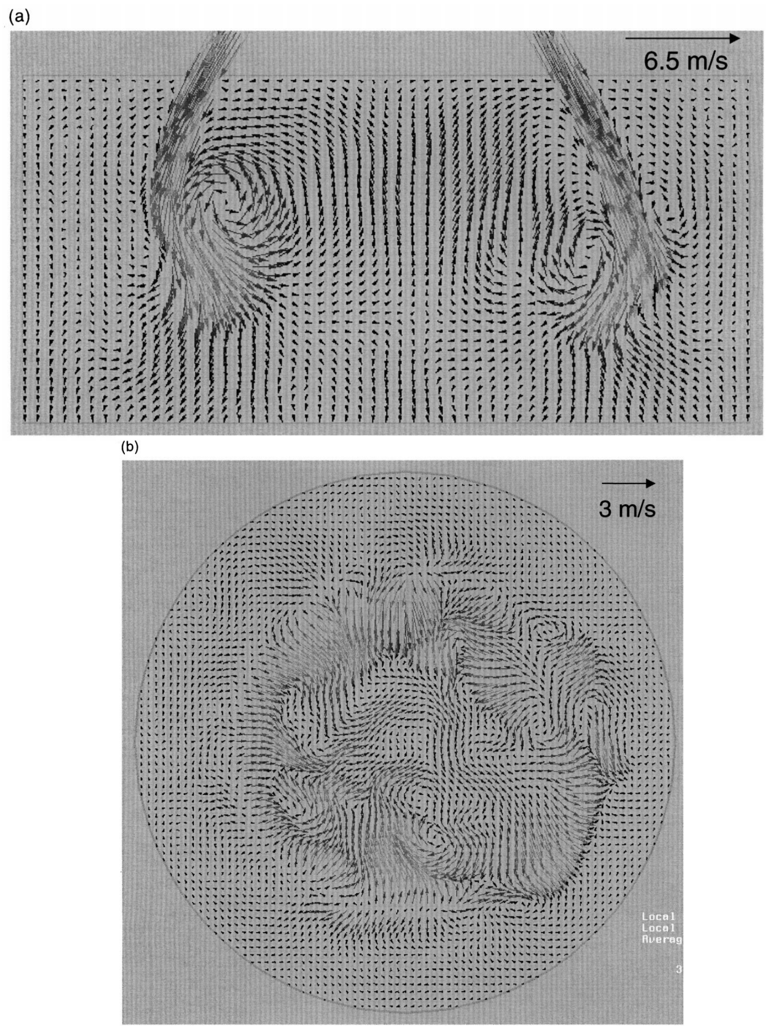


Fig. 2. Two-dimensional sections through computational mesh for the axisymmetric piston-cylinder assembly of Morse et al. [27], including coordinate system definition. (a) Cross section through the axis of symmetry ($y = 0$). (b) Cross section normal to the axis of symmetry (z = constant).



Examples:

LES of flow over wavy walls
Armenio & Piomelli (2000)
Flow, Turb. & Combustion.

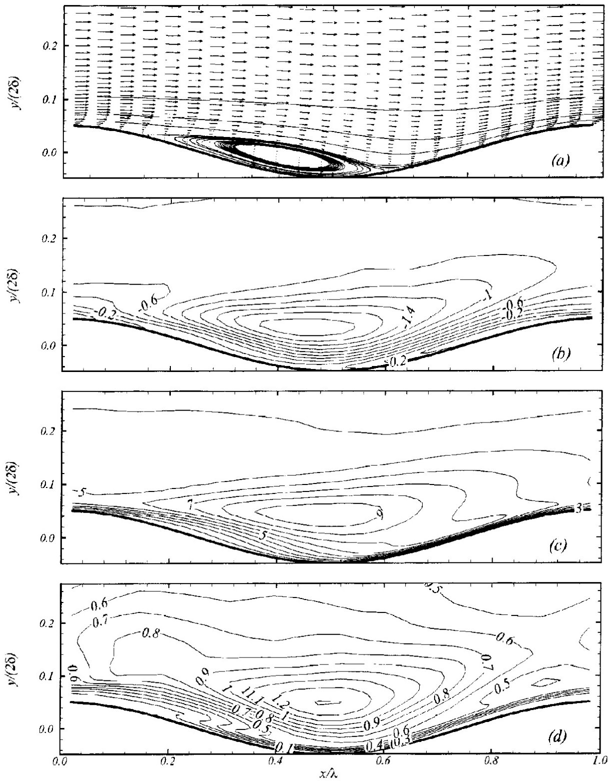
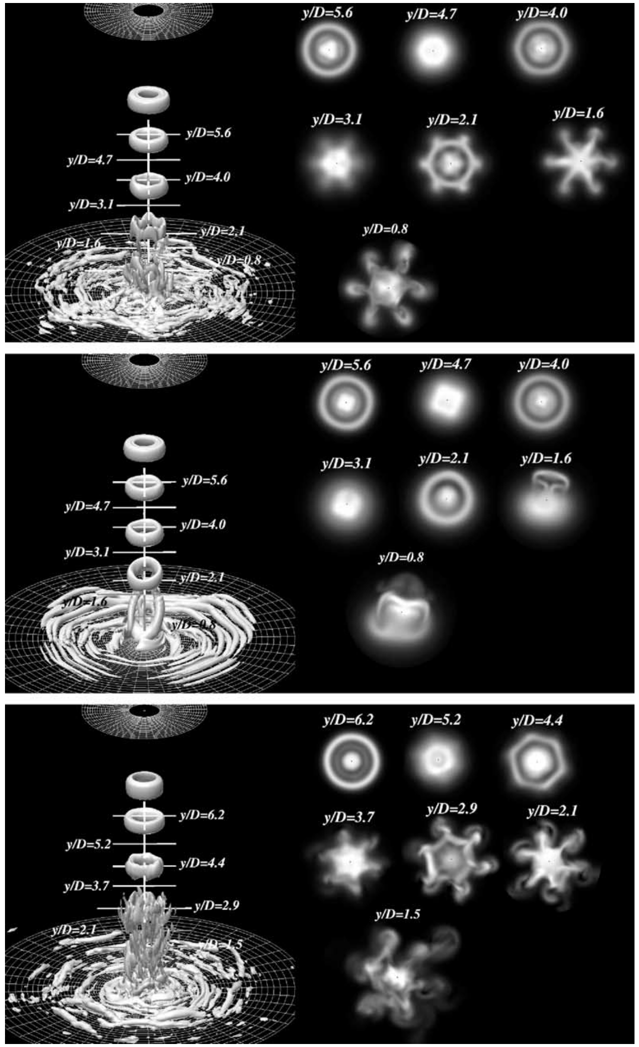


Figure 12. Wavy channel, $2a/\lambda = 0.1$; C1W simulation. (a) Mean velocity vectors and streamlines; (b) Reynolds stress $\langle u'v' \rangle$; (c) q^2; (d) normalized eddy viscosity.

4.2. LARGE-AMPLITUDE WAVE

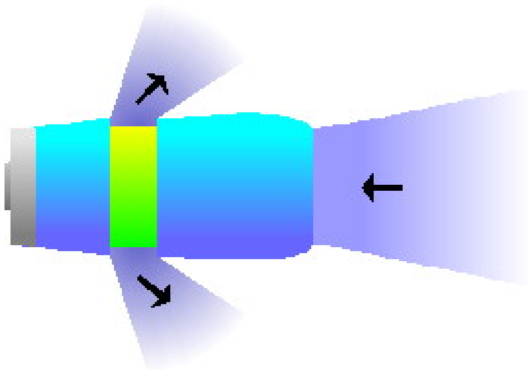
The grid and the flow parameters used for the simulation of the flow over a large-amplitude wavy wall were reported in Table III. As previously pointed out, the parameters have been chosen to fit the experiments of B93 and the LES of HS99.

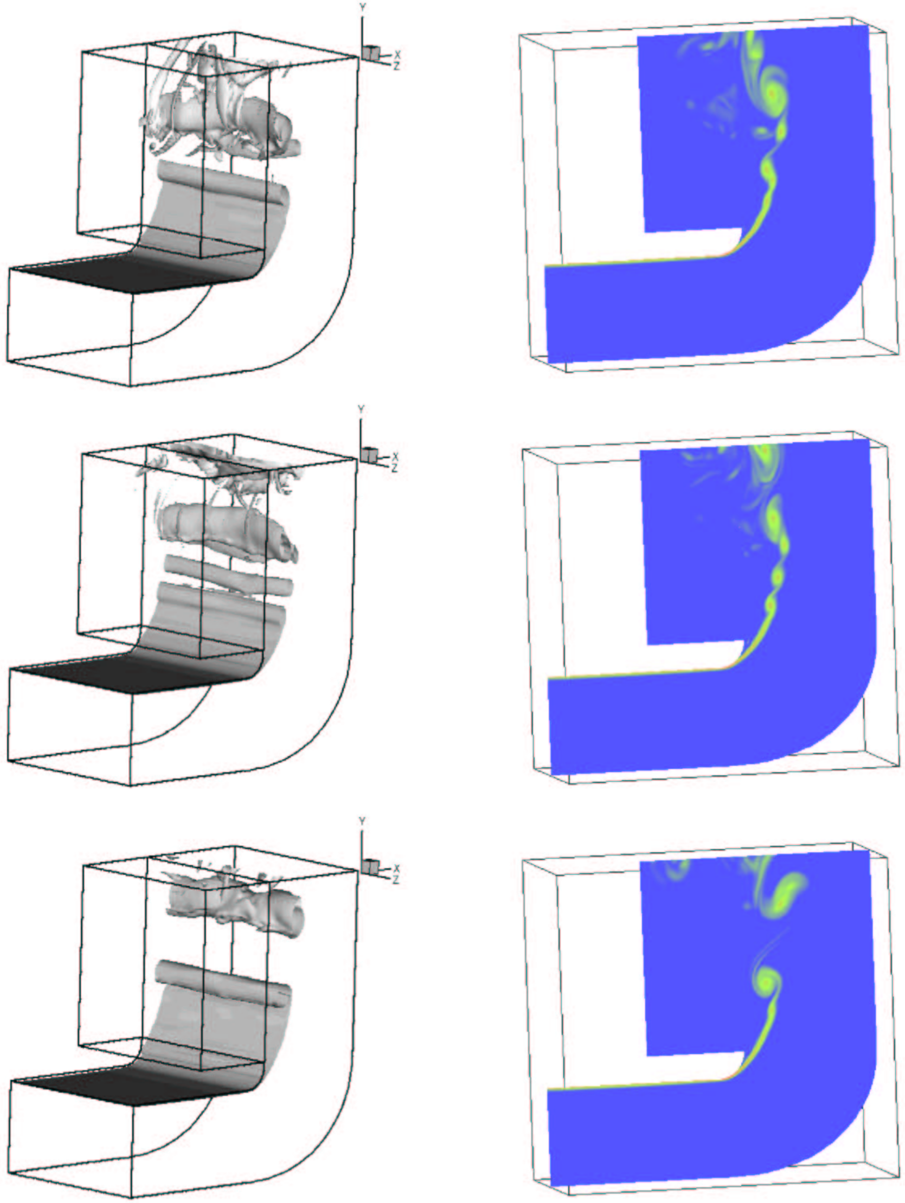
LES of structure of impinging jets:
Tsubokura et al. (2003)
Int Heat Fluid Flow **24**.



Examples:

LES of flow in thrust-reversers
Blin, Hadjadi & Vervisch (2002)
J. of Turbulence.

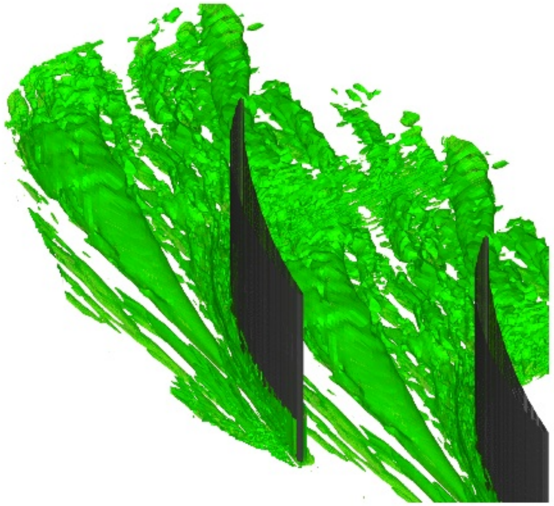


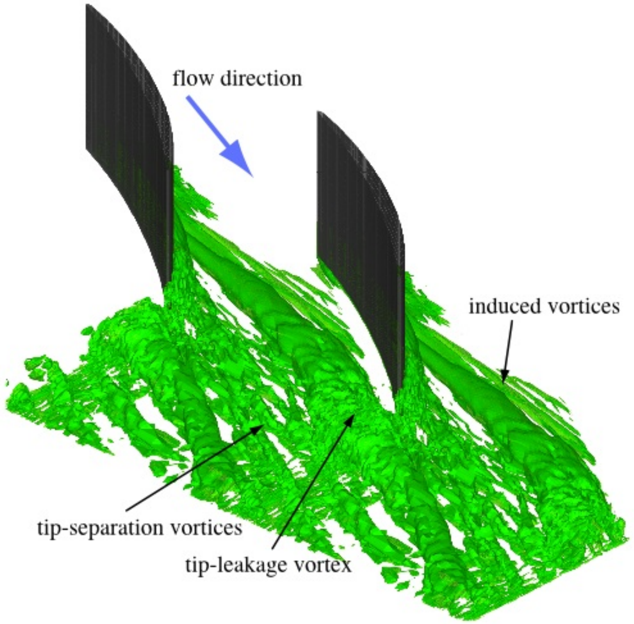


Examples:

LES of flow in turbomachinery
Zou, Wang, Moin, Mittal. (2007)
Journal of Fluid Mechanics.

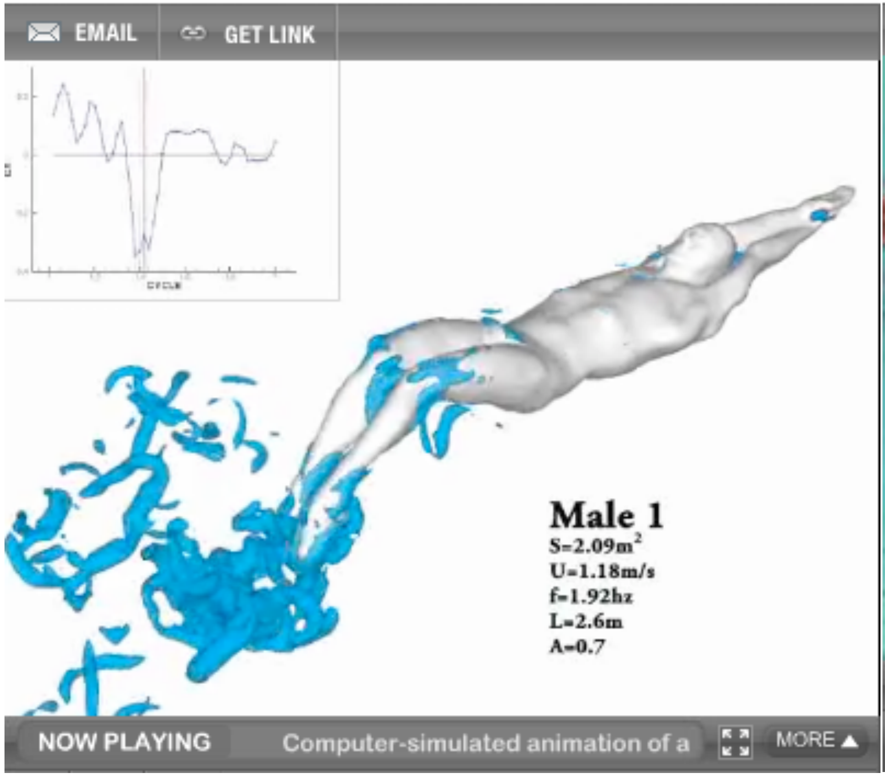
(a)

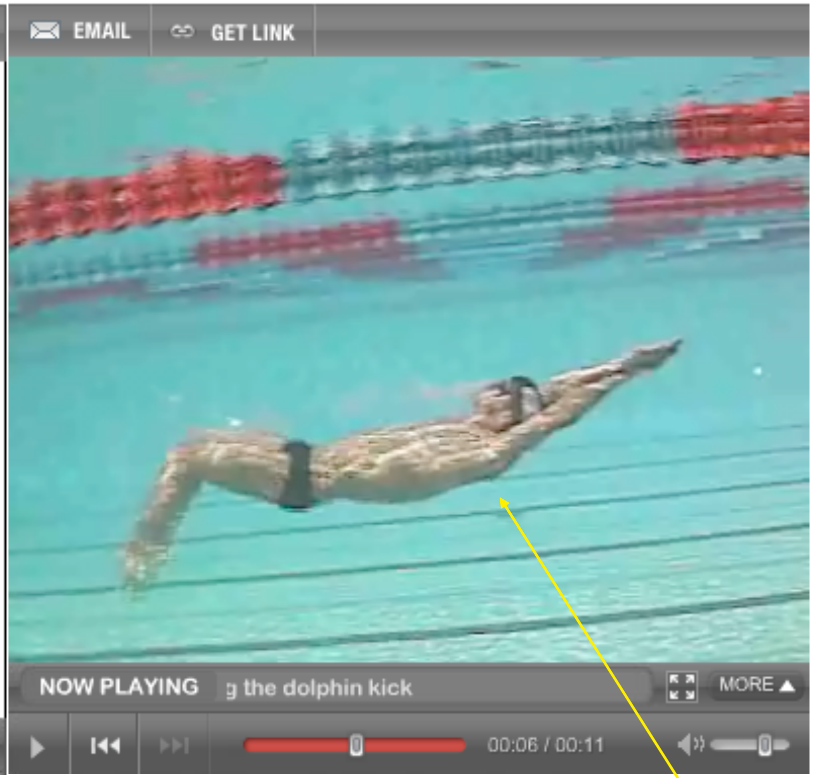




Examples:

Human swimming flow structures (Rajat Mittal, 2008)





Michael Phelps
local Baltimore boy

Ciclo di seminari

Universita degli studi di Roma, Tor Vergata

May, 2013

OVERVIEW:

Mercoledì:

- Introduction to fundamentals of Turbulence
- Intro to Large Eddy Simulation (LES)
- Intro to Subgrid-scale (SGS) modeling
- The dynamic SGS model
- Some sample applications from our group

Venerdì:

- Dynamic model for LES over rough multiscale surfaces
- LES studies of large wind farms

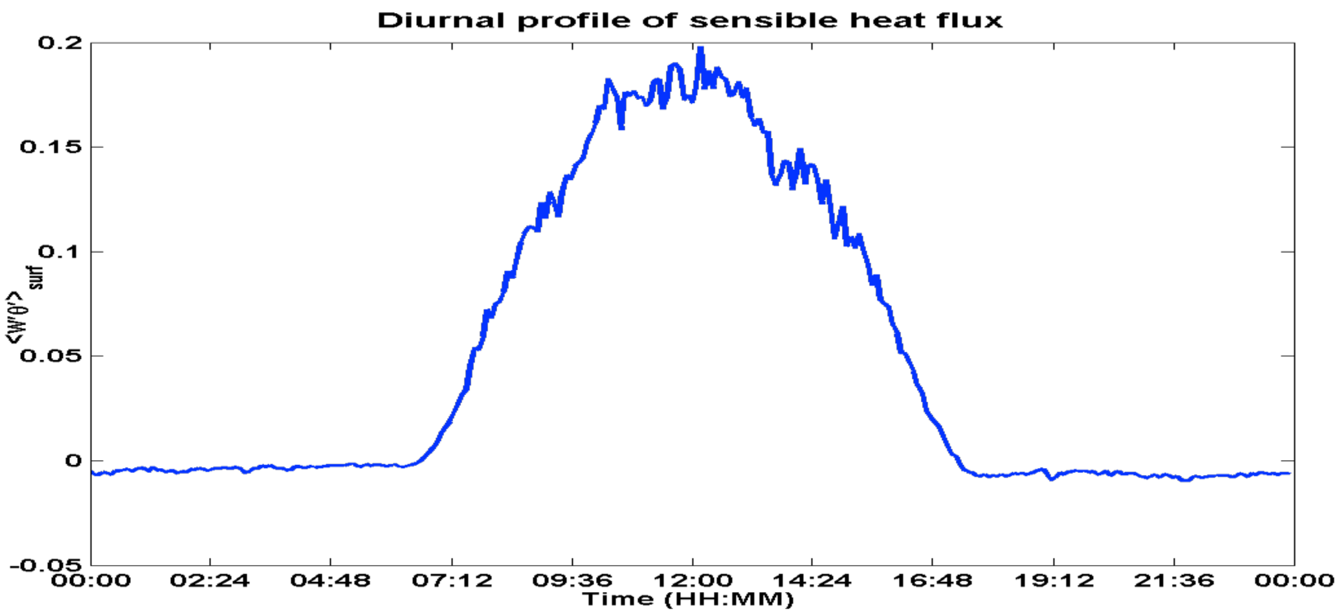
Examples:

LES of convective atmospheric boundary layer:

Kumar, M. & Parlange (Water Resources Research **42**, 2006)

- Transport equation for temperature
- Boussinesq approximation
- Coriolis forcing
- Lagrangian dynamic model with assumed $b=C_s(2\,D)/C_s(D)$
- Constant (non-dynamic) SGS Prandtl number $Pr_{sgs}=0.4$
- Imposed surface flux of sensible heat on ground
- Diurnal cycle: start stably stratified, then heating….

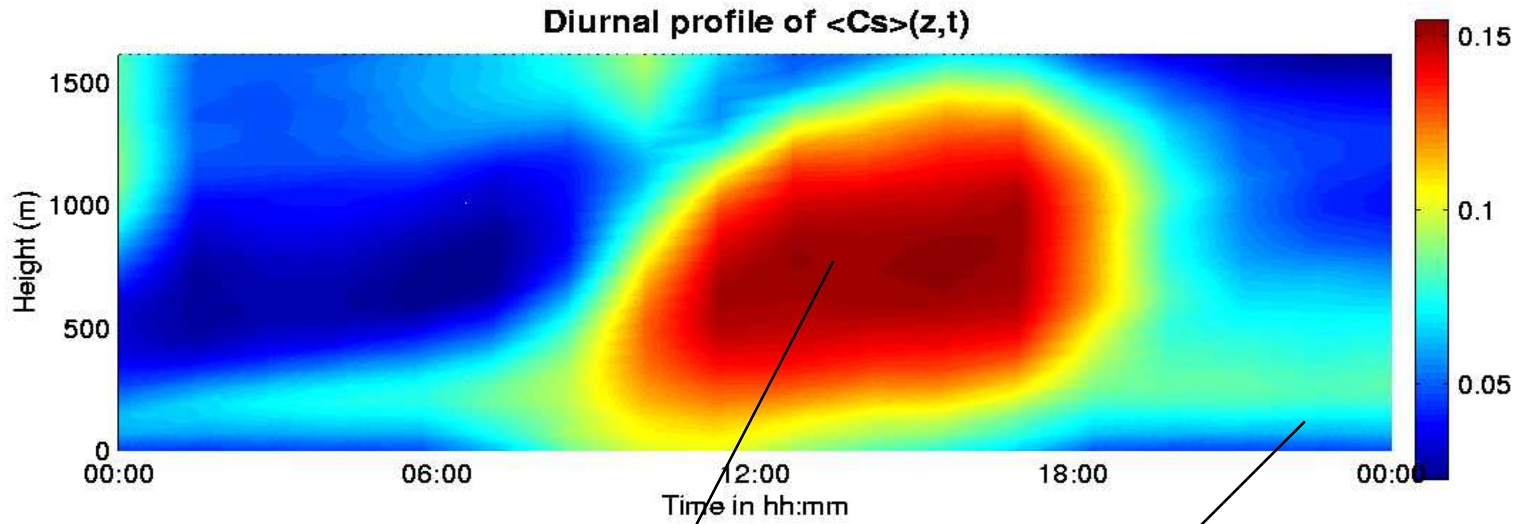
Imposed ground heat flux during day:



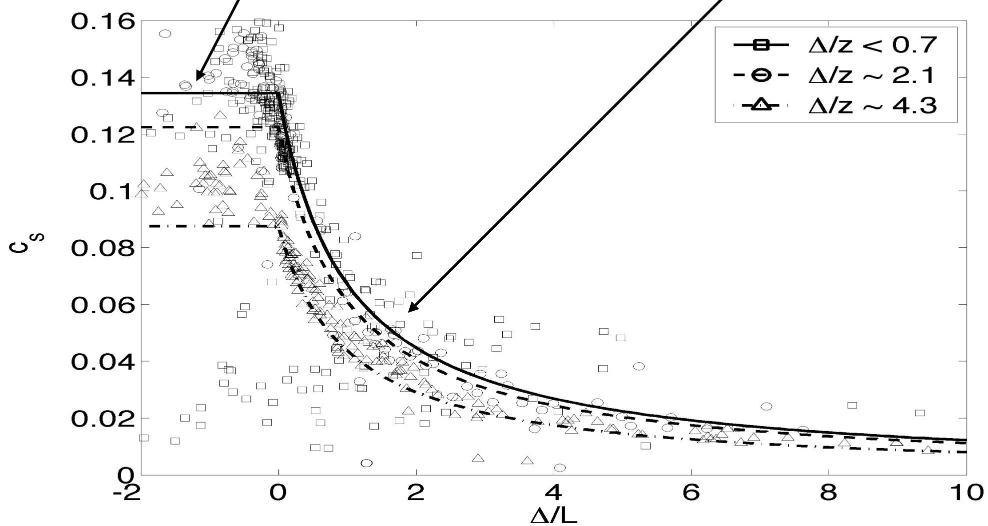
Examples:

- Diurnal cycle: start stably stratified, then heating….

Resulting dynamic coefficient (averaged):



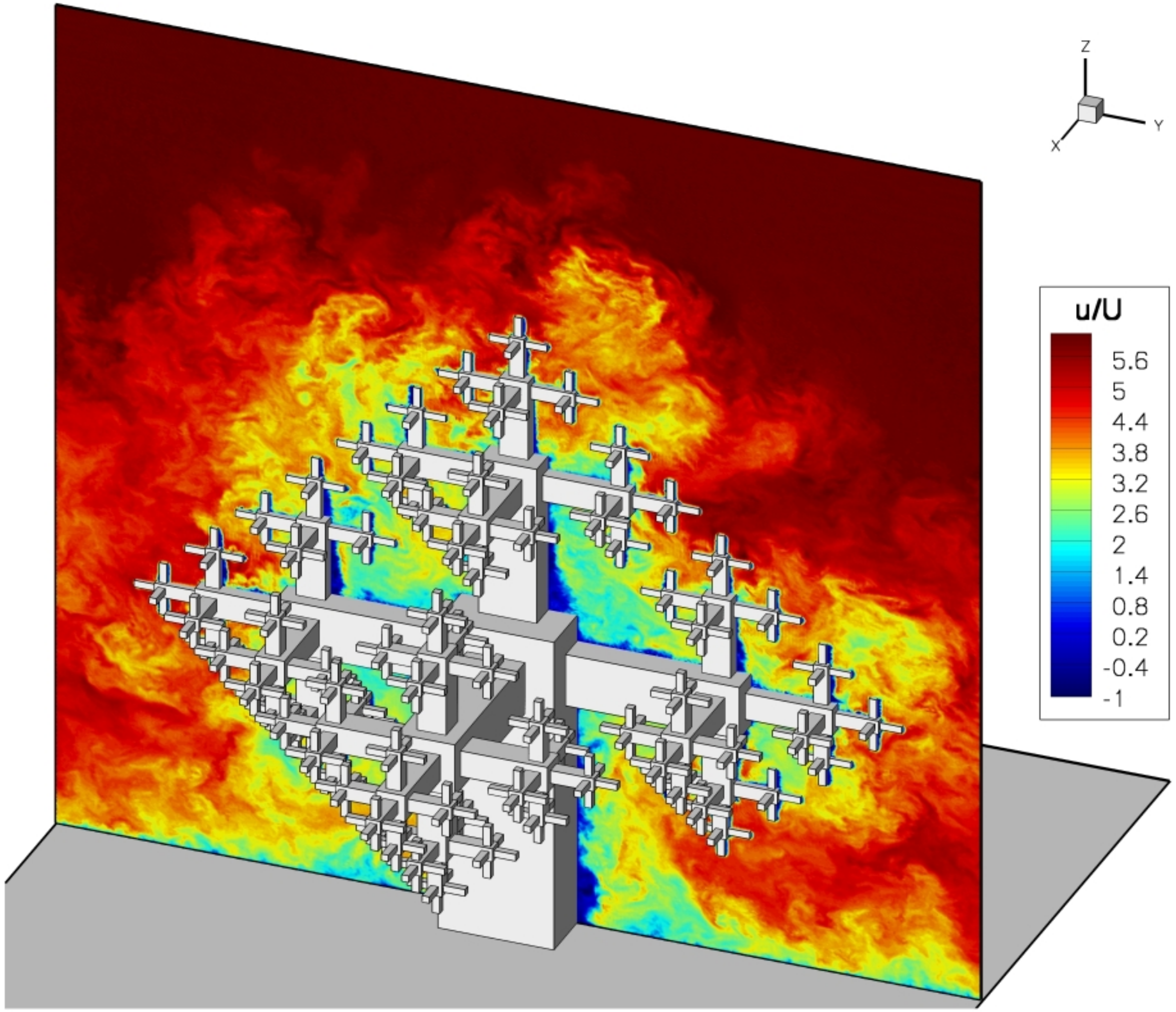
Consistent with HATS field measurements:

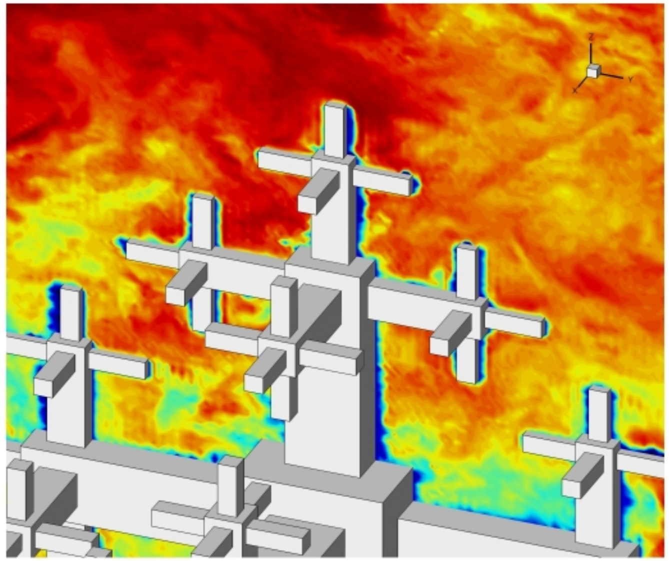


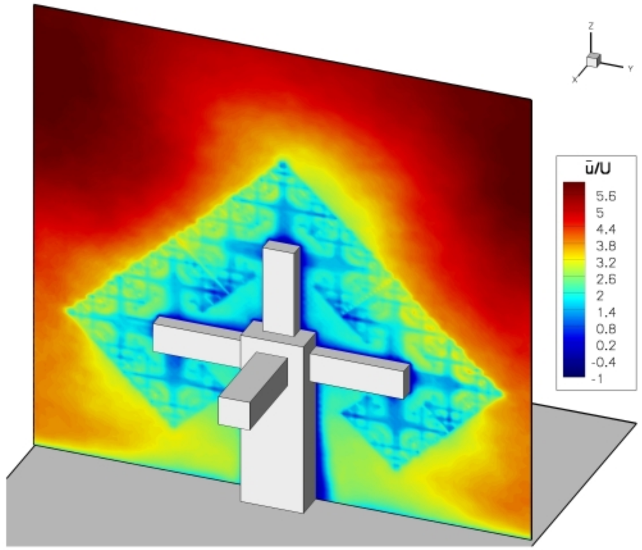
Large-eddy-Simulation of atmospheric flow over fractal trees:

Chester, M & Parlange

(J. Comp. Phys. **225**, 2007; J. Env. Fluid Mech. **7**, 2007)



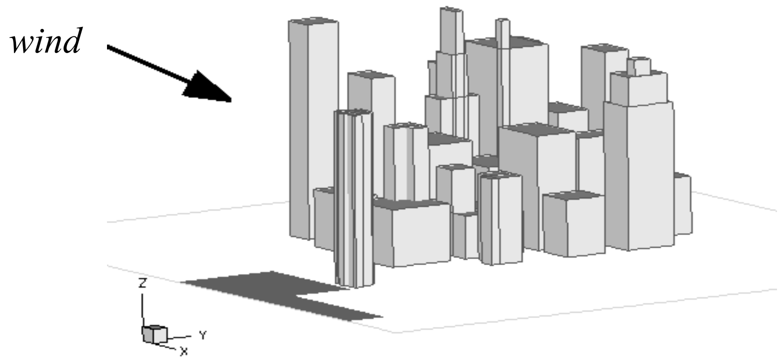




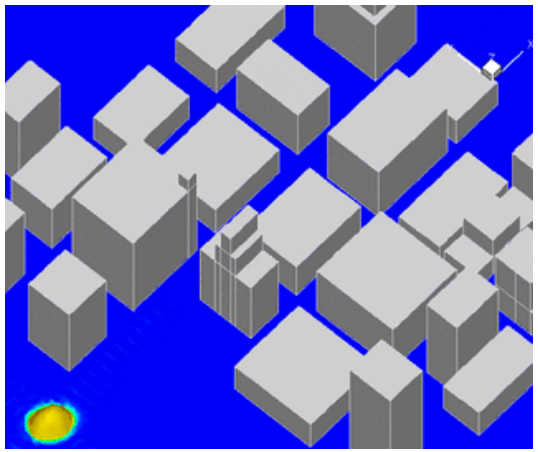
Urban contamination and Transport

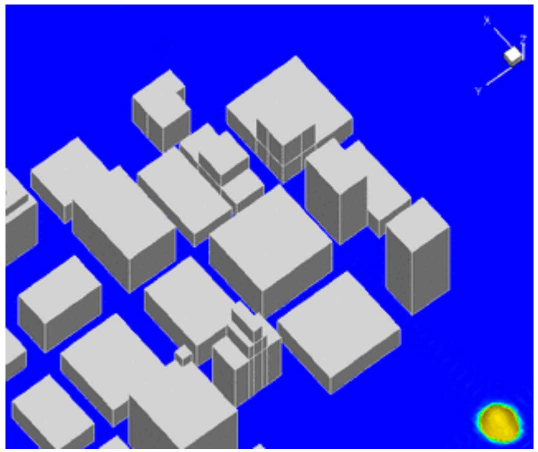
Tseng, M. & Parlange, Env. Sci & Tech. **40**, 2006

Downtown Baltimore:



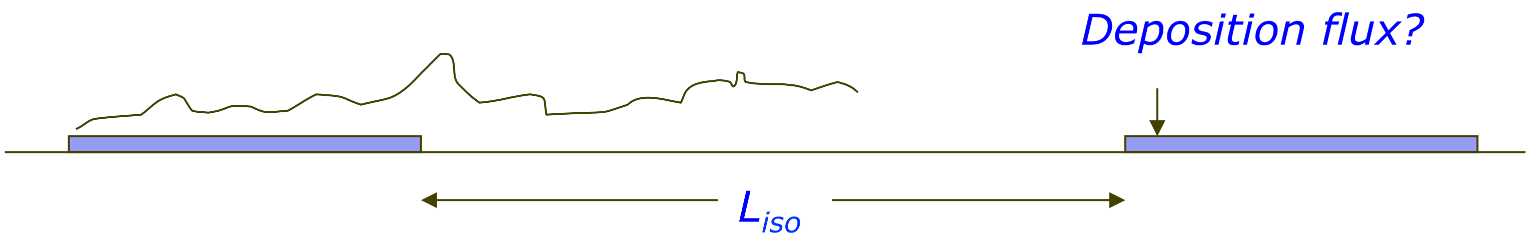
Momentum and scalar transport equations solved using LES and Lagrangian dynamic subgrid model. Buildings are simulated using immersed boundary method.





Agriculture: What is the isolation distance to avoid cross-polination?

Collaboration with **Marcelo Chamecki** (Penn State U)



LES:

Eulerian approach C(x,y,z,t)

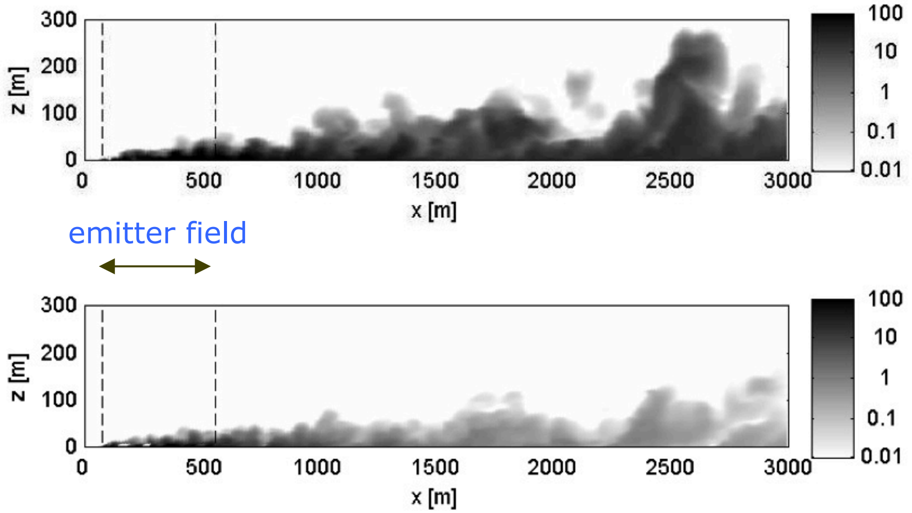
Vertical settling velocity w_s

Scale-dep dynamic eddy-viscosity eddy-diffusivity SGS

Log-law type boundary condition

For C, log-law, corrected by settling velocity

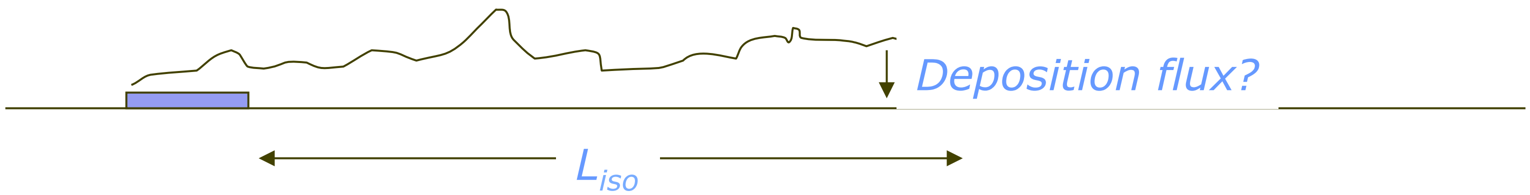
$$\frac{\partial \tilde{C}}{\partial t} + \left(\tilde{\mathbf{u}} - w_s \mathbf{e}_3\right) \bullet \nabla \tilde{C} = \nabla \bullet \left(\left(C_{s-dyn} \Delta \right)^2 Sc_{sgs}^{-1} \, |\tilde{\mathbf{S}}| \, \tilde{C} \right)$$



Low w_s

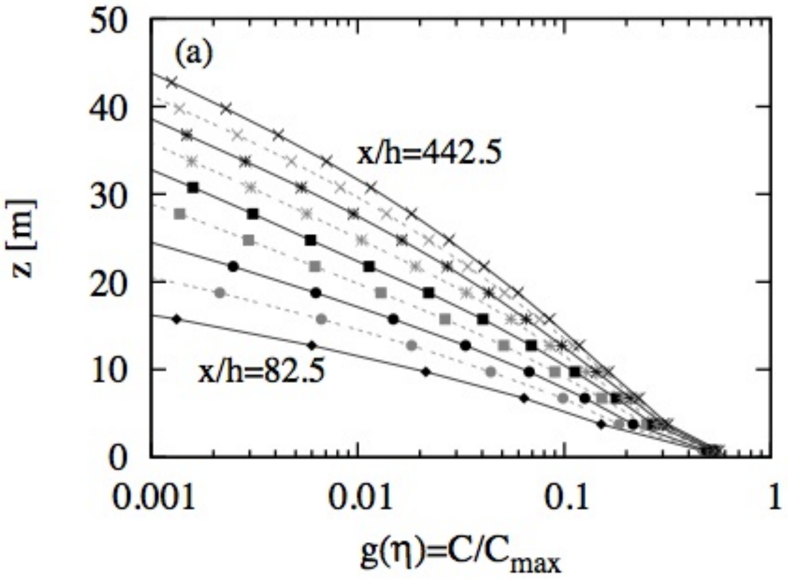
High w_s

FIGURE 4. Snapshots of iso-contours of resolved pollen concentration (on x-z plane) for (a) $\gamma = 0.125$ and (b) $\gamma = 0.625$. Dashed lines indicate the horizontal extent of the source field.

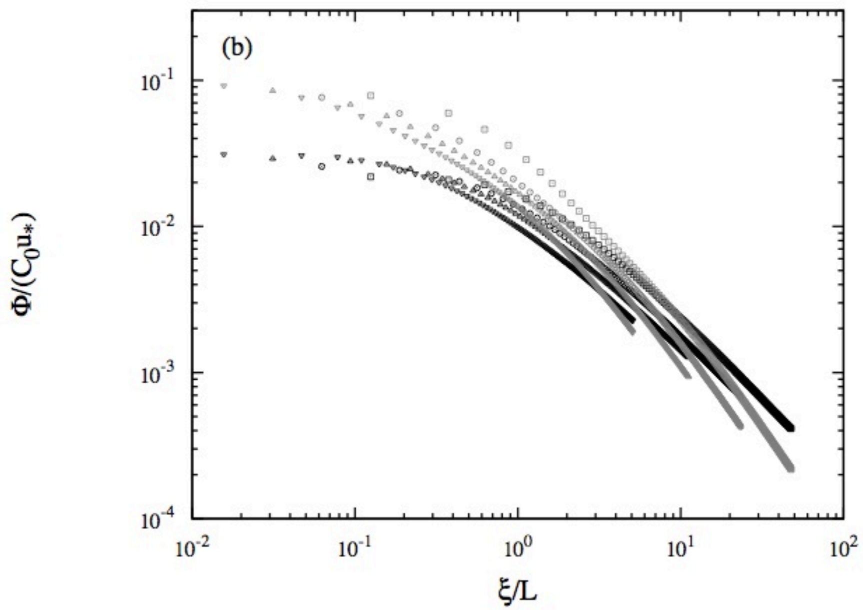


LES results:

Downstream evolution of concentration profiles



Downstream evolution of deposition flux



1-D “reduction” (back to early 1900s) - vertical profile

$$\overline{u}(z)\frac{\partial \overline{C}}{\partial x} - w_s\frac{\partial \overline{C}}{\partial z} = \frac{\partial}{\partial z}\left(K_c(z)\frac{\partial \overline{C}}{\partial z}\right),$$

$$\overline{C}(x=0,z)=0$$
$$\overline{C}(x,z\to\infty)=0$$
$$\overline{C}(x,z=z_{0,c})=\overline{C}_0,$$

$$\overline{u}(z) = u_* C_p\left(\frac{z}{z_o}\right)^m, \quad K_c(z) = \frac{\kappa u_* z}{\mathrm{Sc}},$$

$$z\frac{\partial^2 \overline{C}}{\partial z^2} + (1+\gamma)\frac{\partial \overline{C}}{\partial z} - \frac{\mathrm{Sc}\, C_p}{\kappa}\left(\frac{z}{z_o}\right)^m\frac{\partial \overline{C}}{\partial x} = 0,$$

$$\frac{d^2 g}{d\eta^2} + (1+\gamma)\frac{1}{\eta}\frac{dg}{d\eta} + \frac{\mathrm{Sc}\, C_p}{\kappa z_0^m}\left(\delta_c^m\frac{d\delta_c}{dx}\right)\eta^m\frac{dg}{d\eta} = 0.$$

$$\frac{\overline{C}(x,z)}{\overline{C}_{max}(x)} = g(\eta),$$

$$g'' + \left[\frac{(1+\gamma)}{\eta} + C_1(\gamma)\eta^m\right]g' = 0,$$

$$g(\eta) = -C_I\frac{1}{m+1}\left(\frac{C_1(\gamma)}{m+1}\right)^{\gamma/(m+1)}\int_{\frac{C_1(\gamma)}{m+1}\eta^{m+1}}^{\infty} t^{-[1+\gamma/(m+1)]}\exp[-t]\,dt$$

$$g(\eta) = \frac{\Gamma\left(-\frac{\gamma}{m+1}, \frac{C_1(\gamma)}{m+1}\eta^{m+1}\right)}{\Gamma\left(-\frac{\gamma}{m+1}, \frac{C_1(\gamma)}{m+1}\eta_0^{m+1}\right)},$$

Similarity solution
for any eta (Rouse #)

Deposition downstream of field:

$$\frac{d^2 f}{d\eta^2} + (1+\gamma)\frac{1}{\eta}\frac{df}{d\eta} + \frac{\mathrm{Sc}C_p}{\kappa z_o^m}\left(\delta_c^m \frac{d\delta_c}{d\xi}\right)\eta^m \frac{df}{d\eta} - \frac{\mathrm{Sc}C_p}{\kappa z_o^m}\left(\frac{\delta_c^{m+1}}{\overline{C}_{max}}\frac{d\overline{C}_{max}}{d\xi}\right)\eta^{m-1} f = 0. \quad (2.30)$$

Since both δ_c and $\overline{C}_{max}$ depend on ξ, constraint (2.14) still has to be satisfied. There is one additional requirement for the existence of a similarity solution, namely that

$$C_3(\gamma) = -\frac{\mathrm{Sc}C_p}{\kappa z_o^m}\left(\frac{\delta_c^{m+1}}{\overline{C}_{max}}\frac{d\overline{C}_{max}}{d\xi}\right) \quad (2.31)$$

is independent of downstream distance. The final ODE contains three terms, two of which are similar to (2.15), and is given by

$$f'' + \left[\frac{(1+\gamma)}{\eta} + C_2(\gamma)\eta^m\right] f' + C_3(\gamma)\eta^{m-1} f = 0 \quad (2.32)$$

where $C_2(\gamma)$ is used instead of $C_1(\gamma)$ to indicate that the function may actually be different from the one obtained in the previous section.

Equation (2.24) should still be valid if a non-zero initial boundary layer height $\delta_c(\xi = 0) = \delta_L$ is imposed

$$\delta_c(\xi) = \left[\delta_L^{m+1} + C_2(\gamma)\frac{\kappa z_o^m}{\mathrm{Sc}C_p}(m+1)\xi\right]^{\frac{1}{m+1}}. \quad (2.33)$$

Replacing this expression for $\delta_c(\xi)$ into equation (2.31) and solving for $\overline{C}_{max}(\xi)$ yields

$$\overline{C}_{max}(\xi) = \overline{C}_{ini}(\gamma)\left[1 + C_2(\gamma)\frac{\kappa(m+1)}{\mathrm{Sc}C_p}\left(\frac{z_o}{\delta_L}\right)^m \frac{\xi}{\delta_L}\right]^{-\frac{C_3(\gamma)}{(m+1)C_2(\gamma)}} \quad (2.34)$$

where the initial condition $\overline{C}_{max}(\xi = 0) = \overline{C}_{ini}(\gamma)$ was used. Equation (2.34) can be written as

$$\overline{C}_{max}(\xi) = \overline{C}_{ini}(\gamma)\left[1 + \frac{1}{b(\gamma)}\frac{\xi}{\delta_L}\right]^{-\beta(\gamma)} \quad (2.35)$$

where the following definitions were used

$$b(\gamma) = \left[C_2(\gamma)\frac{\kappa(m+1)}{\mathrm{Sc}C_p}\left(\frac{z_o}{\delta_L}\right)^m\right]^{-1} \quad (2.36)$$

$$\beta(\gamma) = \frac{C_3(\gamma)}{(m+1)C_2(\gamma)}. \quad (2.37)$$

$$\gamma = \frac{Sc}{\kappa}\frac{w_s}{u_*}$$

$$\Phi(\xi) = \left[w_s\overline{C} + K_c\frac{\partial\overline{C}}{\partial z}\right]_{z=z_{0,c}}$$

$$\frac{\Phi(\xi)}{\overline{C}_{max}(\xi)u_*} = \frac{w_s}{u_*}\left[f + \frac{\eta}{\gamma}\frac{df}{d\eta}\right]_{\eta=\eta_0}$$

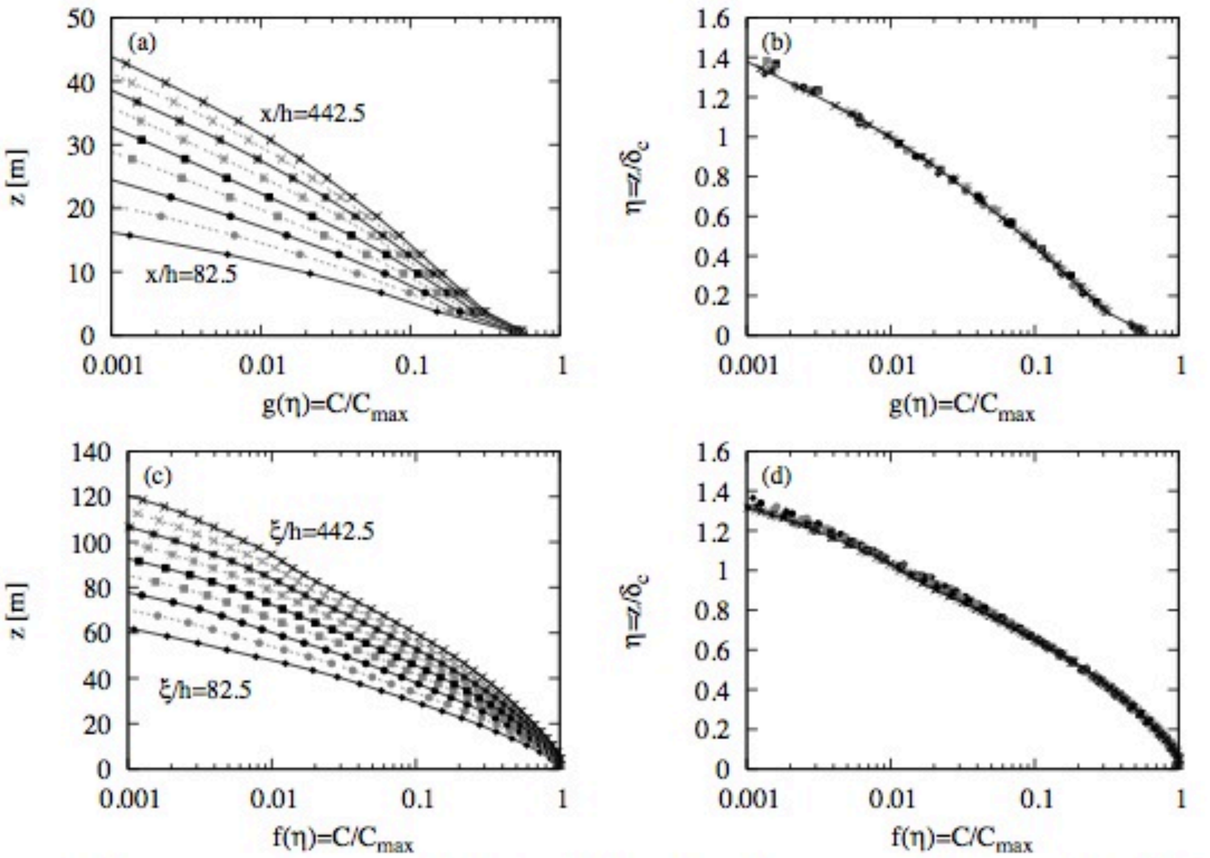
$$\Phi(\xi) = a(\gamma)\left[1 + \frac{1}{b(\gamma)}\frac{\xi}{\delta_L}\right]^{-\beta(\gamma)}$$

$delta_L$ is proper length-scale for deposition flux, not *L*

Comparison with LES:

$$\gamma = \frac{Sc}{\kappa}\frac{w_s}{u_*} = 0.125$$

14 *M. Chamecki and C. Meneveau*

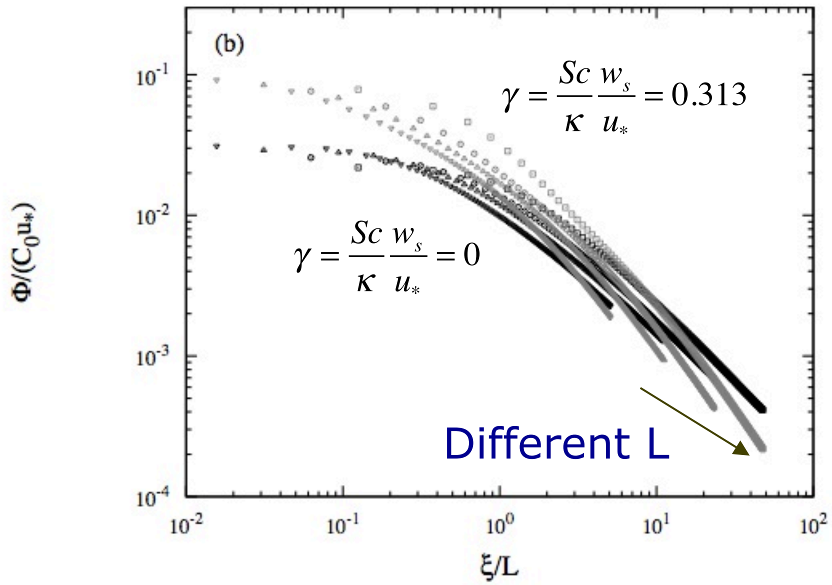


Over field

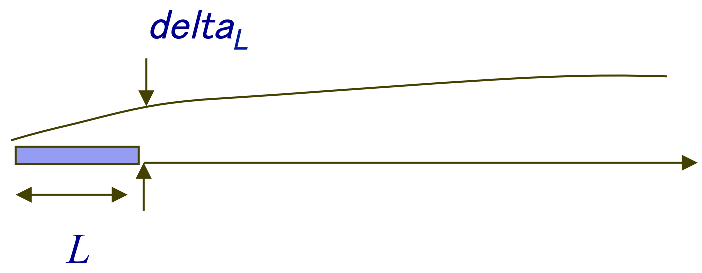
Downstream

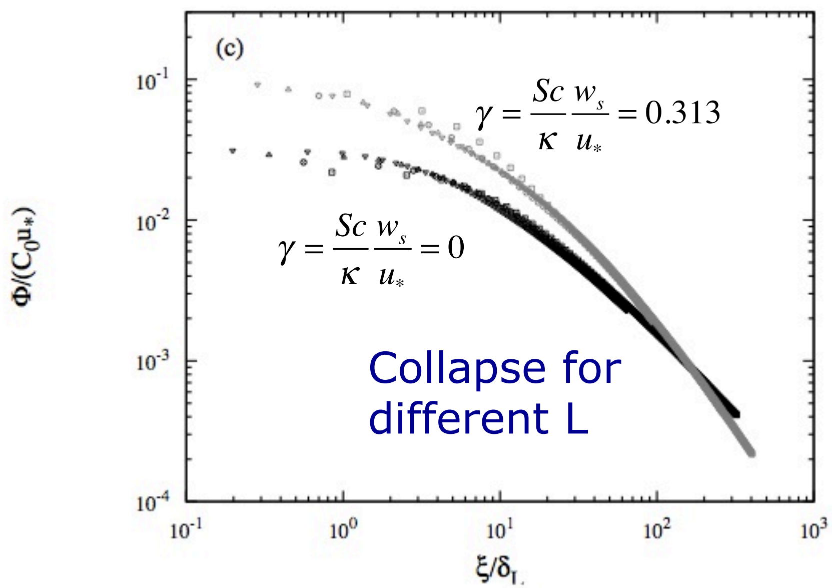
FIGURE 5. Average pollen concentration profiles above the source field normalized by local maximum concentration at different distances from the leading edge x/h (from $x/h = 82.5$ to $x/h = 442.5$ in increments of 50) for $\gamma = 0.125$. (a) Plotted against height above the ground and (b) against dimensionless height η illustrating collapse consistent with self-preservation. Panels (c) and (d) are similar for concentration profiles downstream from the source at different distances from the trailing edge ξ/h.

Scaling of deposition flux: field edge *delta* instead of *L*



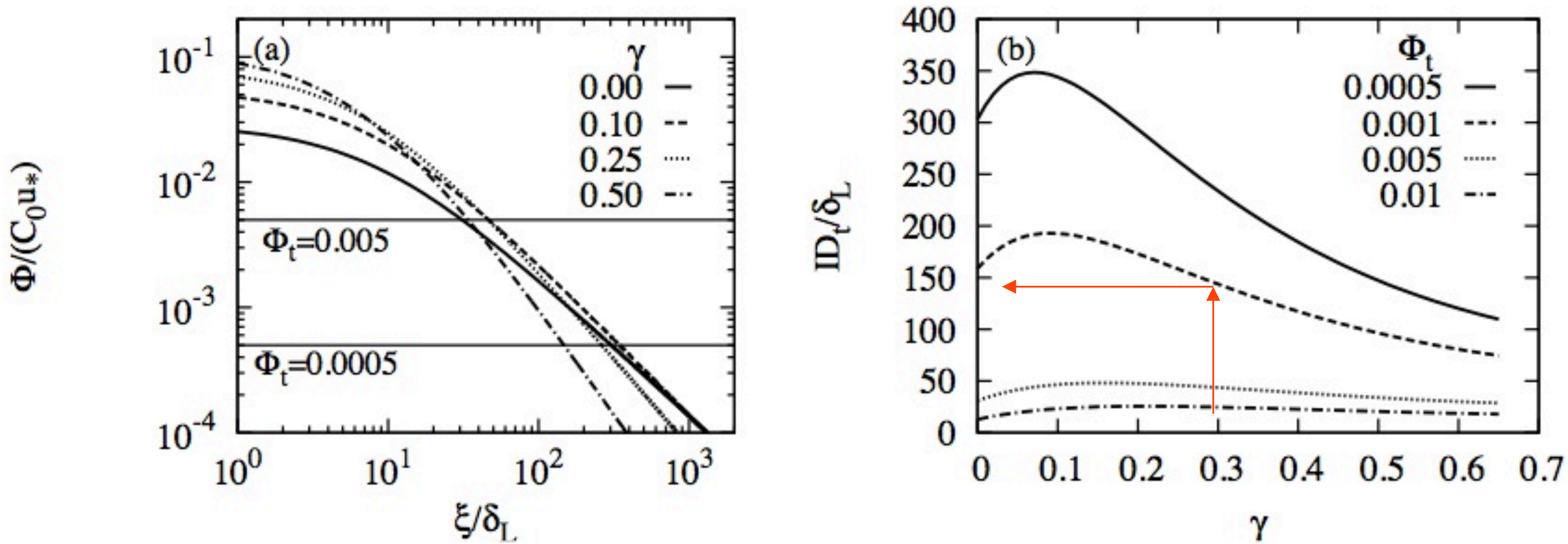
Traditional approach
(current rules)





Scaling with *delta$_L$*

Isolation distance as function of flux threshold:



E.g., w_s=0.06 m/s (d=40 micro-m), u*=0.5 m/s, Sc=1, k=0.4, $\gamma = \frac{Sc}{\kappa}\frac{w_s}{u_*} = 0.3$

L=500m -> $delta_L$=30m, E.g., Phi=10^{-3} -> ID/$delta_L$ ~150

ID = 4.5km !!

Useful references on LES and SGS modeling:

- P. Sagaut: "Large Eddy Simulation of Incompressible Flow" (Springer, 3rd ed., 2006)

- U. Piomelli, Progr. Aerospace Sci., 1999

- C. Meneveau & J. Katz, Annu Rev. Fluid Mech., 2000