Ciclo di seminari

Università degli studi di Roma "Tor Vergata"

May, 2013

LES of turbulent flows over multi-scale surfaces using a dynamic roughness model

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OVERVIEW:

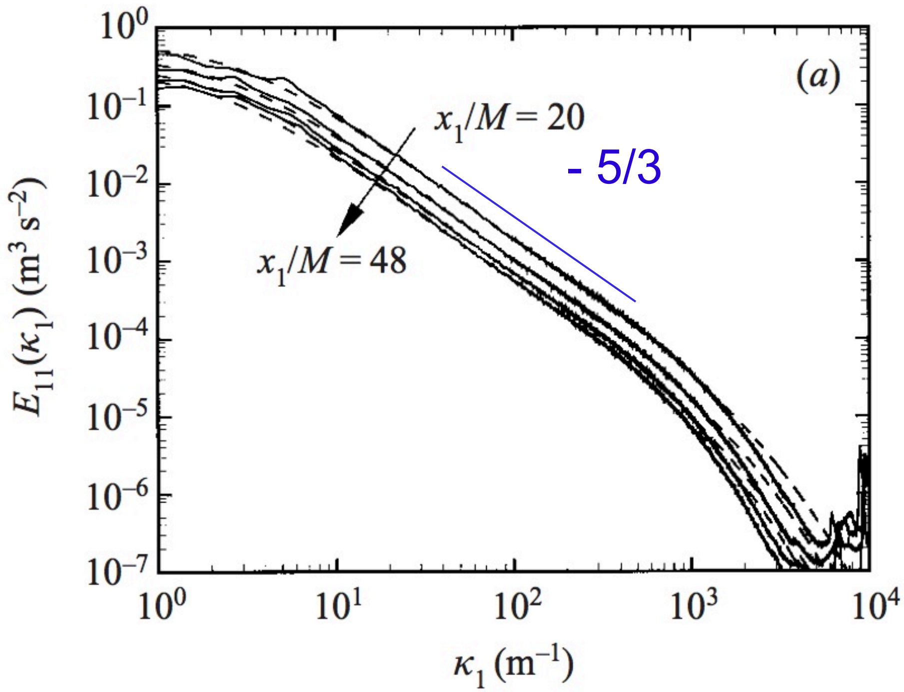
Mercoledì:

- Introduction to fundamentals of Turbulence
- Intro to Large Eddy Simulation (LES)
- Intro to Subgrid-scale (SGS) modeling
- The dynamic SGS model
- Some sample applications from our group

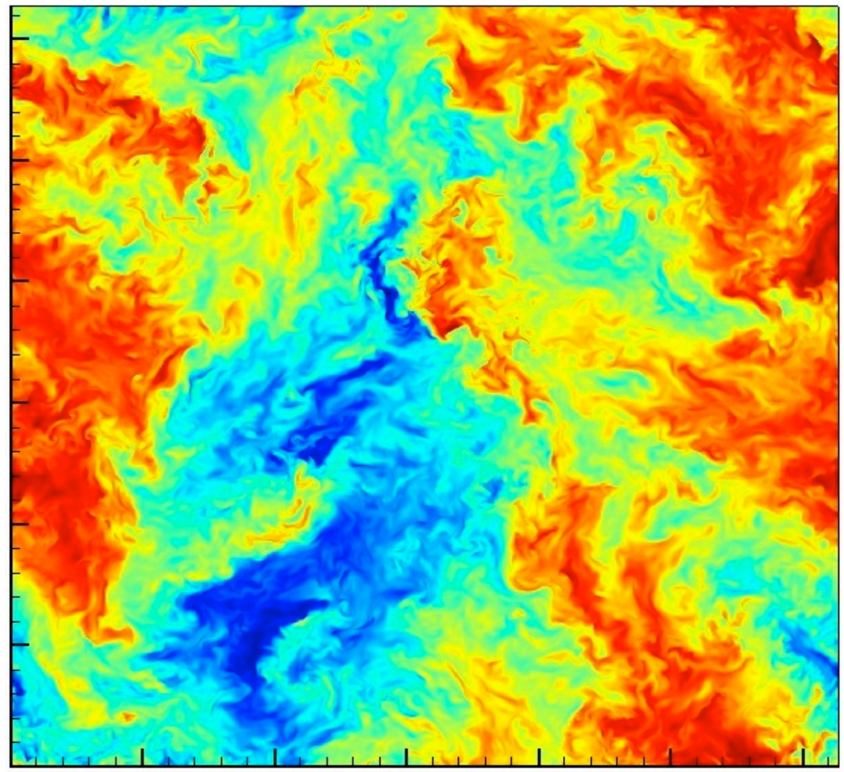
Venerdì:

- Dynamic model for LES over rough multiscale surfaces
- LES studies of large wind farms

Turbulent flow: multiscale

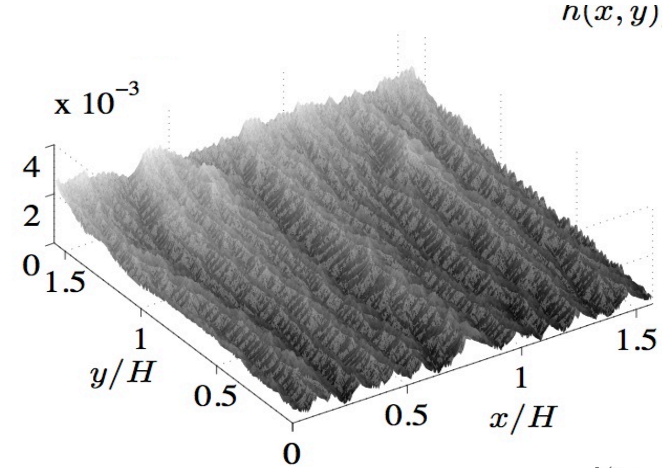


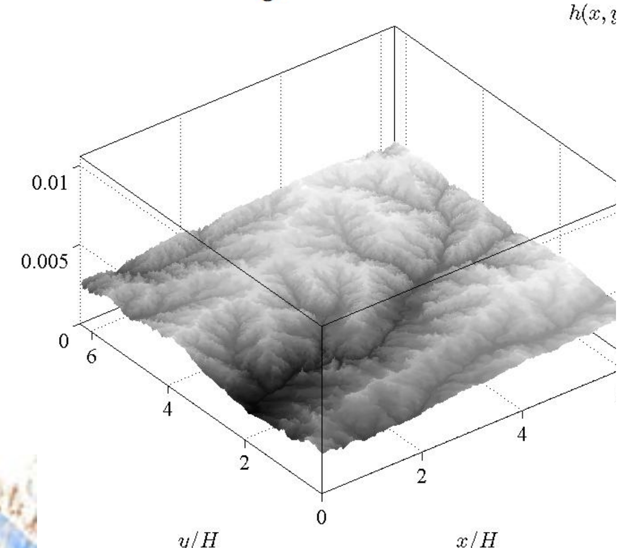
Spectra from wind-tunnel measurements (Kang et al. JFM 2003)



Velocity contours $u(x,y,z_0,t_0)$ extracted from JHU public database using Matlab (C. Verhulst) - turbulence.pha.jhu.edu (Li et al. J. Turbulence 2008)

Turbulence multi-scale by itself.
But often, turbulent flow interacts with multi-scale surfaces:

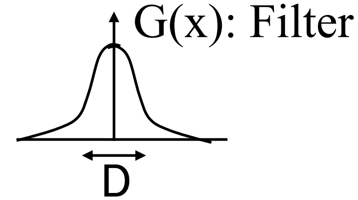




Context: Large-eddy-simulation (LES) and filtering:

N-S equations (incompress, single phase…):

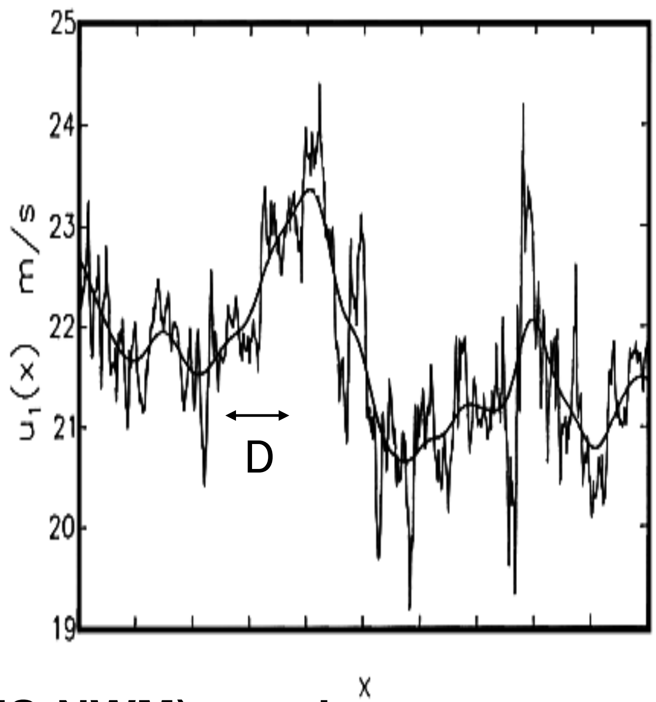
$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$



Filtered N-S equations:

$$\frac{\partial \tilde{u}_j}{\partial t} + \frac{\partial \widetilde{u_k u_j}}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j$$

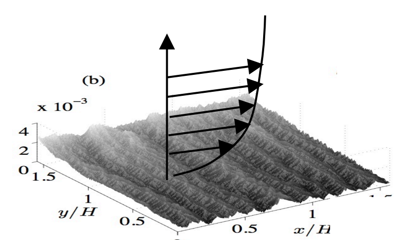
$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k}\tau_{jk}$$



where SGS stress tensor is (in bulk): and at wall (LES-NWM), need

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

$$\tau_w = \rho u_*^2$$



In bulk: eddy-viscosity, dynamic Smagorinsky model

$$\tau_{ij}^{d} = -2\left(c_s\Delta\right)^2 |\tilde{S}|\,\tilde{S}_{ij}$$

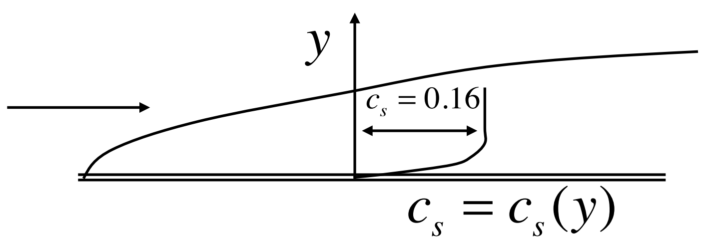
c_s: "Smagorinsky coefficient"

c_s=0.16 works well for isotropic,
high Reynolds number turbulence

In practice (complex flows), must adapt to local physics

Examples:

1. Transitional pipe flow: change from 0 to 0.16
2. Near wall damping for wall boundary layers (Piomelli et al 1989)



$$c_s = c_s(\mathbf{x}, t)$$

Recall: dynamic model

(Germano et al., Phys. Fluids 1991)

$$
\begin{aligned}
\overline{\widetilde{\Psi(\boldsymbol{q})}} &= \Psi(\bar{\tilde{\boldsymbol{q}}}) + \Psi_{\mathrm{mod}}(\bar{\tilde{\boldsymbol{q}}}, \alpha\Delta, C_1, C_2 \ldots) \\
&= \overline{\Psi(\tilde{\boldsymbol{q}})} + \overline{\Psi_{\mathrm{mod}}(\tilde{\boldsymbol{q}}, \Delta, C_1, C_2 ..)}
\end{aligned}
$$

Recall: dynamic model

(Germano et al., Phys. Fluids 1991)

$$\overline{\widetilde{\Psi(\boldsymbol{q})}} = \Psi(\bar{\tilde{\boldsymbol{q}}}) + \Psi_{\mathrm{mod}}(\bar{\tilde{\boldsymbol{q}}}, \alpha\Delta, C_1, C_2 \ldots)$$

$$= \overline{\Psi(\tilde{\boldsymbol{q}})} + \overline{\Psi_{\mathrm{mod}}(\tilde{\boldsymbol{q}}, \Delta, C_1, C_2 ..)}$$

$$\overline{\widetilde{u_i u_j}} = \bar{\tilde{u}}_i \bar{\tilde{u}}_j - 2\left(c_s 2\Delta\right)^2 |\bar{\tilde{S}}| \bar{\tilde{S}}_{ij}$$

$$= \overline{\tilde{u}_i \tilde{u}_j} - 2\left(c_s \Delta\right)^2 \overline{|\tilde{S}| \tilde{S}_{ij}}$$

FIRST PRINCIPLES CONSTRAINT:
Equality of momentum flux tensor

$$\Psi(\boldsymbol{u}) = u_i u_j$$

Recall: dynamic model

(Germano et al., Phys. Fluids 1991)

$$\overline{\widetilde{\Psi(\boldsymbol{q})}} = \Psi(\bar{\tilde{\boldsymbol{q}}}) + \Psi_{\text{mod}}(\bar{\tilde{\boldsymbol{q}}}, \alpha\Delta, C_1, C_2 \ldots)$$
$$= \overline{\Psi(\tilde{\boldsymbol{q}})} + \overline{\Psi_{\text{mod}}(\tilde{\boldsymbol{q}}, \Delta, C_1, C_2 ..)}$$

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$$= \overline{\tilde{u}_i \tilde{u}_j} - 2\left(c_s \Delta\right)^2 \overline{|\tilde{S}| \tilde{S}_{ij}}$$

$$0 = L_{ij} - c_s^2 M_{ij}$$

FIRST PRINCIPLES CONSTRAINT:
Equality of momentum flux tensor

$$\Psi(\boldsymbol{q}) = \widetilde{u_i u_j}$$

Recall: dynamic model

(Germano et al., Phys. Fluids 1991)

$$\overline{\widetilde{\Psi(\boldsymbol{q})}} = \Psi(\bar{\tilde{\boldsymbol{q}}}) + \Psi_{\mathrm{mod}}(\bar{\tilde{\boldsymbol{q}}}, \alpha\Delta, C_1, C_2 \ldots)$$
$$= \overline{\Psi(\tilde{\boldsymbol{q}})} + \overline{\Psi_{\mathrm{mod}}(\tilde{\boldsymbol{q}}, \Delta, C_1, C_2 ..)}$$

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$$= \overline{\tilde{u}_i \tilde{u}_j} - 2\left(c_s \Delta\right)^2 \overline{|\tilde{S}| \tilde{S}_{ij}}$$

FIRST PRINCIPLES CONSTRAINT:
Equality of momentum flux tensor

$$\Psi(\boldsymbol{q}) = \widetilde{u_i u_j}$$

$$0 = L_{ij} - c_s^2 M_{ij}$$

Least-square error minimization (Lilly 1992):

$$c_s^2 = \frac{\left\langle L_{ij} M_{ij} \right\rangle}{\left\langle M_{ij} M_{ij} \right\rangle}$$

Recall: dynamic model

(Germano et al., Phys. Fluids 1991)

Many refined and generalized versions

(there are many others):

Passive scalar & compressible LES (Moin et al. 1991)

k-equation (Wong 1992)

MHD (Carati et al. 2002)

…

Lagrangian dynamic model, scale-dependent

- CM, Lund & Cabot (JFM 1996)
- Anderson & CM (FTC 1999)
- Porté-Agel et al. (JFM 2000)
- Bou-Zeid et al. (Phys Fluids 2005)

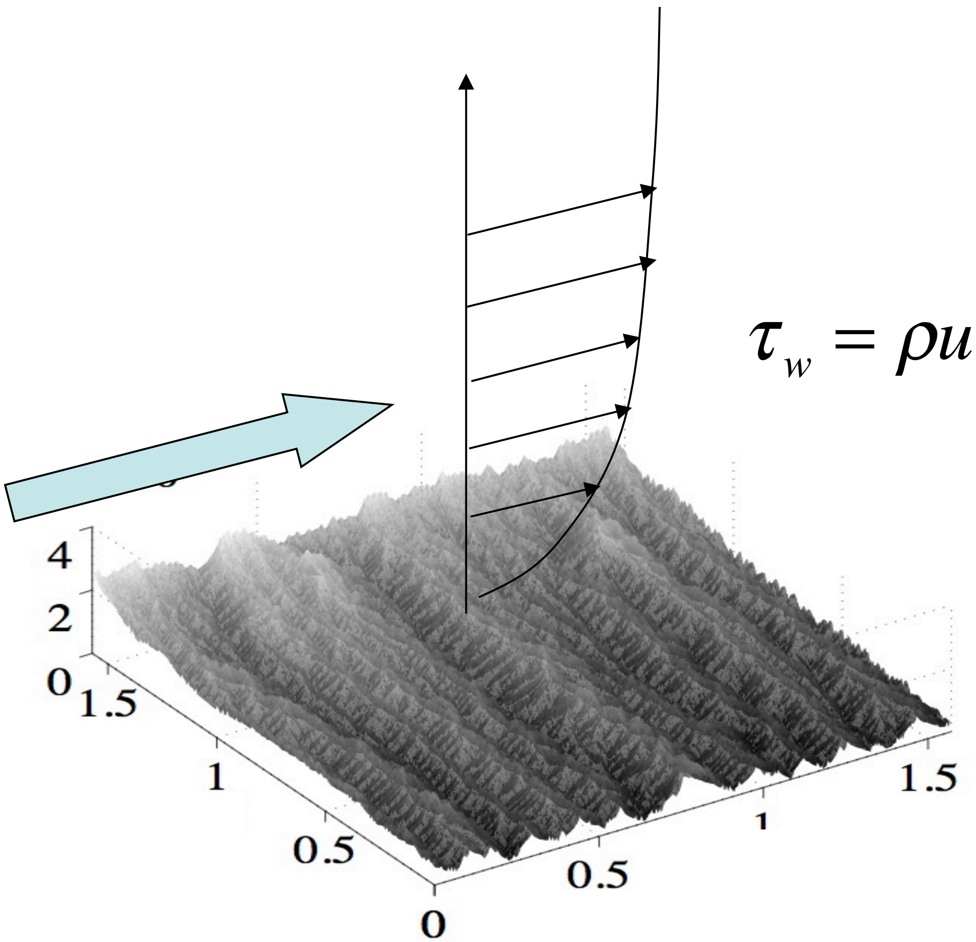
…

Constrained mixed model (Shi, Xiao & Chen, 2008)

Global dynamic Vreman model (You & Moin, 2006)

…

Back to main theme:
LES modeling of flow over multi-scale rough surfaces:

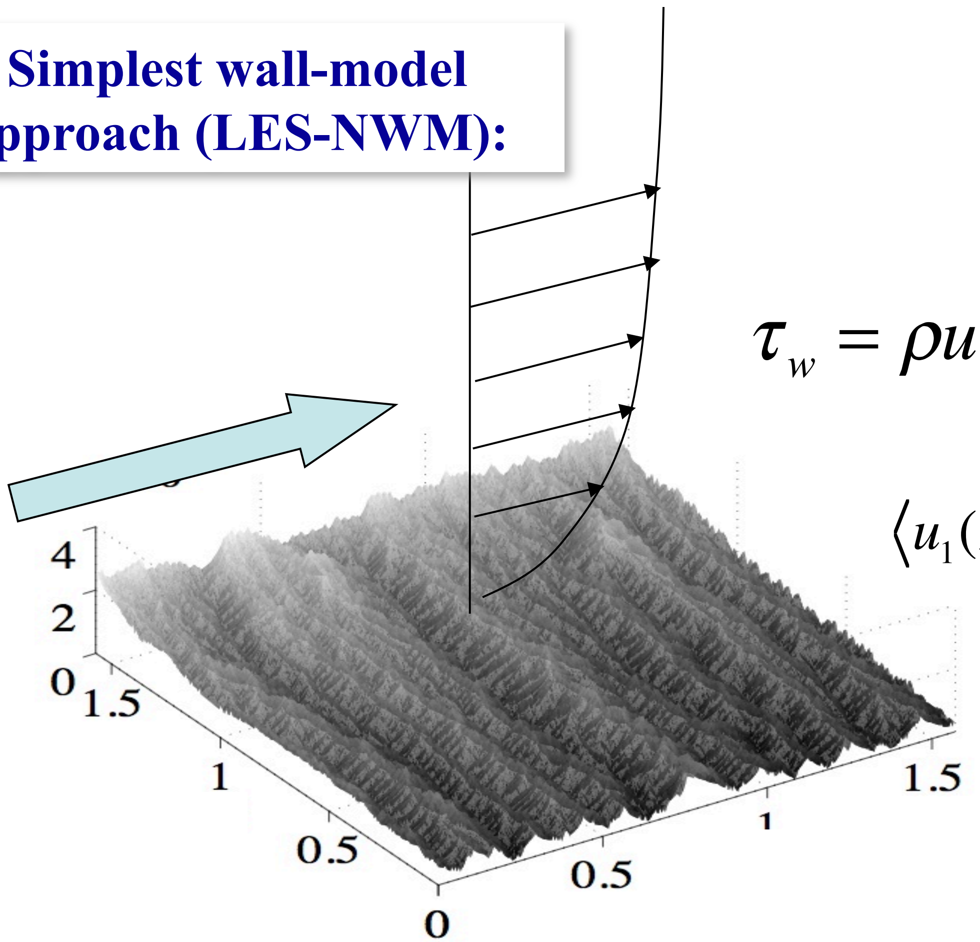


$$\tau_w = \rho u_*^2$$

In LES that does not resolve
wall-details, we need to specify t_w

Back to main theme:
LES modeling of flow over multi-scale rough surfaces:

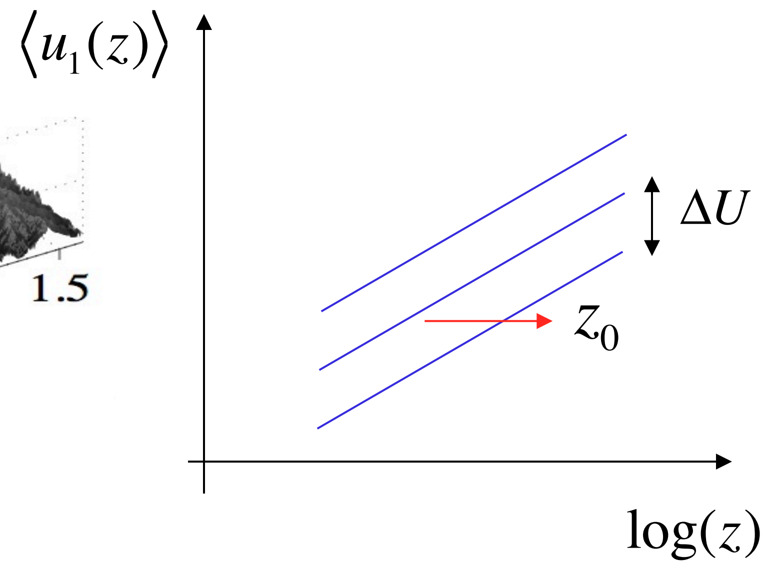
Simplest wall-model approach (LES-NWM):



$$\tau_w = \rho u_*^2$$

$$\frac{\langle u_1(z)\rangle}{u_*} = \frac{1}{\kappa}\log\left(\frac{z}{z_0}\right)$$

$z_0 = ?$



In LES that does not resolve wall-details, we need to specify t_w

Back to main theme:
LES modeling of flow over multi-scale rough surfaces:

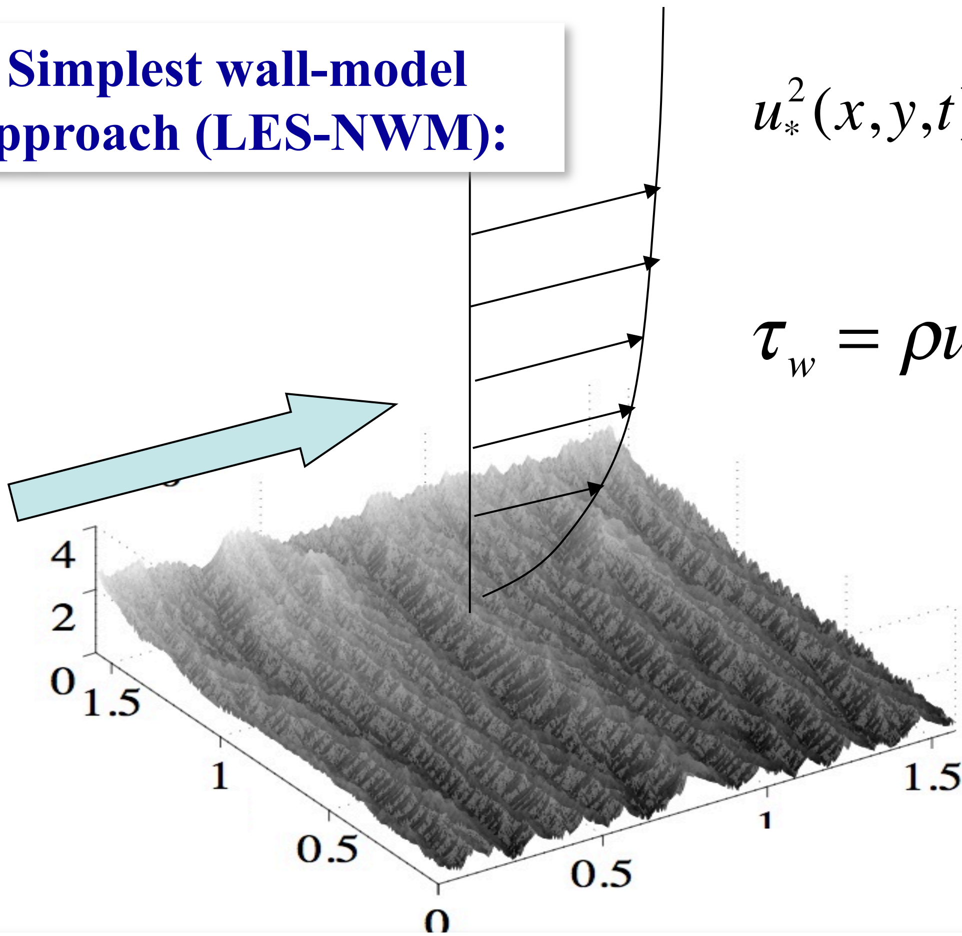
Simplest wall-model approach (LES-NWM):

$$u_*^2(x,y,t) = \left(\frac{\kappa}{\log(z/z_0)} \right)^2 |u|^2 (x,y,z,t)$$

$$\tau_w = \rho u_*^2$$

$z_0 = ?$

$$z_0^\Delta = \alpha \, \sigma_h^\Delta$$



"SGS hydrodynamic roughnes scale is proportional to the **SGS rms** of the height *h'(x,y)* variation"

α: hydrodynamic roughness parameter

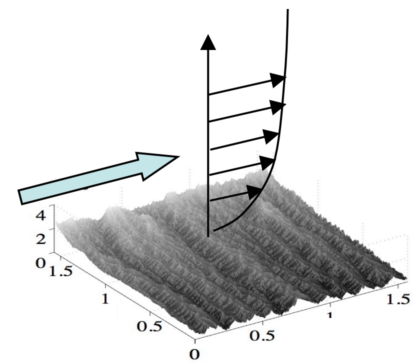
accept approximations

but how to select coefficient α ???

Dynamic surface roughness model:

Anderson & CM,
Journal of Fluid Mechanics, 2011

$$
\begin{aligned}
\overline{\widetilde{\Psi(\boldsymbol{q})}} &= \Psi(\bar{\tilde{\boldsymbol{q}}}) + \Psi_{\mathrm{mod}}(\bar{\tilde{\boldsymbol{q}}}, \alpha\Delta, C_1, C_2 ...) \\
&= \overline{\Psi(\tilde{\boldsymbol{q}})} + \overline{\Psi_{\mathrm{mod}}(\tilde{\boldsymbol{q}}, \Delta, C_1, C_2 ..)}
\end{aligned}
$$



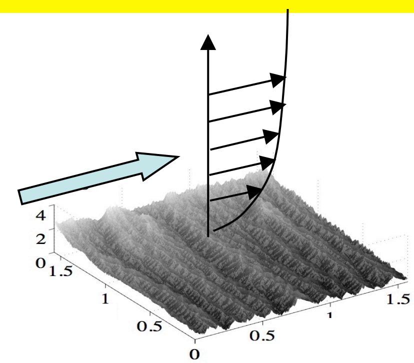
Dynamic surface roughness model:

Anderson & CM,
Journal of Fluid Mechanics, 2011

Let Phi be total force:

$$\overline{\widetilde{\Psi(\boldsymbol{q})}} = \Psi(\tilde{\bar{\boldsymbol{q}}}) + \Psi_{\text{mod}}(\tilde{\bar{\boldsymbol{q}}}, \alpha\Delta, C_1, C_2 \ldots)$$
$$= \overline{\Psi(\tilde{\boldsymbol{q}})} + \overline{\Psi_{\text{mod}}(\tilde{\boldsymbol{q}}, \Delta, C_1, C_2 ..)}$$

$$F_i = -\iint_S \tilde{p}^w \tilde{n}_i dS + \iint_S \tau_{ij}^{w,\Delta} \tilde{n}_j dS = -\iint_S \hat{\tilde{p}}^w \hat{\tilde{n}}_i dS + \iint_S \tau_{ij}^{w,2\Delta} \hat{\tilde{n}}_j dS$$



Resolved pressure field $\tilde{p}^w(x,y)$

Resolved test-filtered pressure field $\hat{\tilde{p}}^w(x,y)$

Wall stress at D:

$$\tau_{i3}^{w,\Delta} = -\left[\frac{\kappa U^{\Delta}}{ln\left(z/z_0^{\Delta}\right)}\right]^2 \frac{\tilde{u}_i}{U^{\Delta}}, \quad i = 1,2$$

Wall stress expressed at 2 D

$$\tau_{i3}^{w,2\Delta} = -\left[\frac{\kappa U^{2\Delta}}{ln\left(z/z_0^{2\Delta}\right)}\right]^2 \frac{\hat{\tilde{u}}_i}{U^{2\Delta}}, \quad i = 1,2$$

Dynamic surface roughness model:

$$\left\langle \tau_{13}^{\Delta}\Big|_{wall} \right\rangle + \left\langle \int_S \tilde{p}\tilde{n}_1 dS \right\rangle \frac{1}{S} = \left\langle \tau_{13}^{2\Delta}\Big|_{wall} \right\rangle + \left\langle \int_S \hat{\tilde{p}}\hat{\tilde{n}}_1 dS \right\rangle \frac{1}{S}$$

$$\left\langle \left[\frac{\kappa U^{\Delta}}{ln\left(\dfrac{\Delta_z/2 - h^{\Delta}}{\alpha\sigma_h^{\Delta}}\right)} \right]^2 \frac{\tilde{u}_1}{U^{\Delta}} \right\rangle + \left\langle \tilde{p}^{w} \frac{\partial \tilde{h}}{\partial x_1} \right\rangle = \left\langle \left[\frac{\kappa U^{2\Delta}}{ln\left(\dfrac{\Delta_z/2 - h^{2\Delta}}{\alpha\sigma_h^{2\Delta}}\right)} \right]^2 \frac{\hat{\tilde{u}}_1}{U^{2\Delta}} \right\rangle + \left\langle \hat{\tilde{p}}^{w} \frac{\partial \hat{\tilde{h}}}{\partial x_1} \right\rangle$$

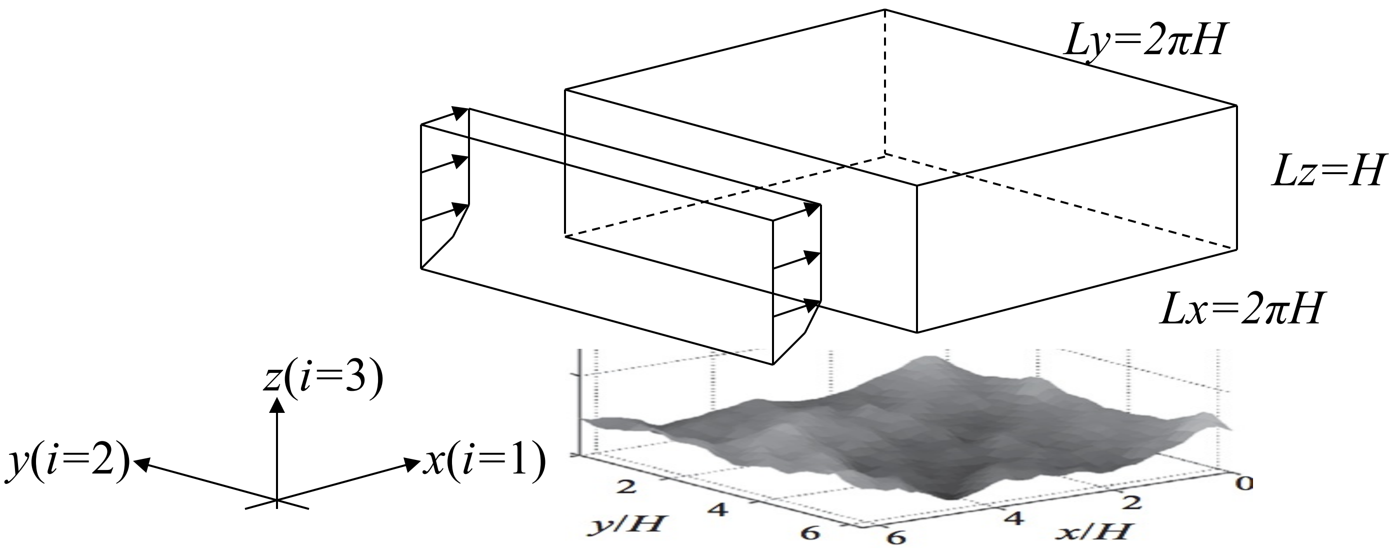
In LES: only unknown is α
(solve e.g. using bi-section method)

Large Eddy Simulation of boundary layers over rough surfaces:

Implement into ABL LES code (neutral)

- Pseudospectral in horizontal, 2nd-order finite difference in vertical (Moeng 1984-type, Albertson and Parlange, 1999: *Water Resour. Res., Porté-Agel, Parlange & Menevau, 2000*)

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right) = -\frac{1}{\rho} \left(\frac{\partial \tilde{p}^*}{\partial x_i} + \delta_{i1} \Pi \right) - \frac{\partial \tau_{ij}}{\partial x_j} + f_i$$



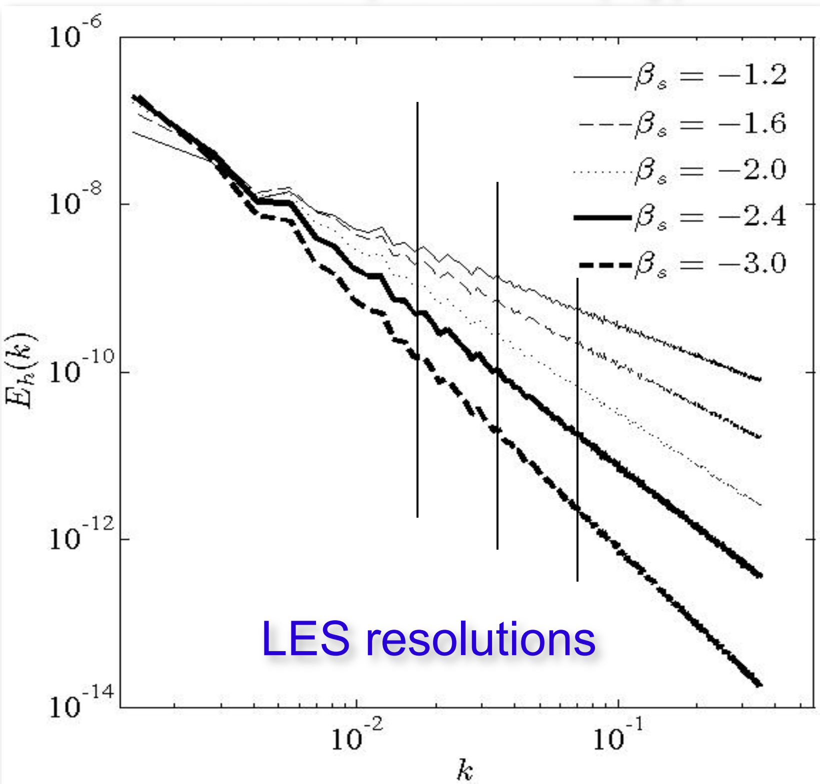
SGS closure: Lagrangian scale-dep. dynamic model (E. Bou-Zeid et al., 2005: *Phys. Fluids*)

Bottom BC(resolved): Surface gradient drag model (Anderson and CM, 2010: *Bound. Layer Met.*)

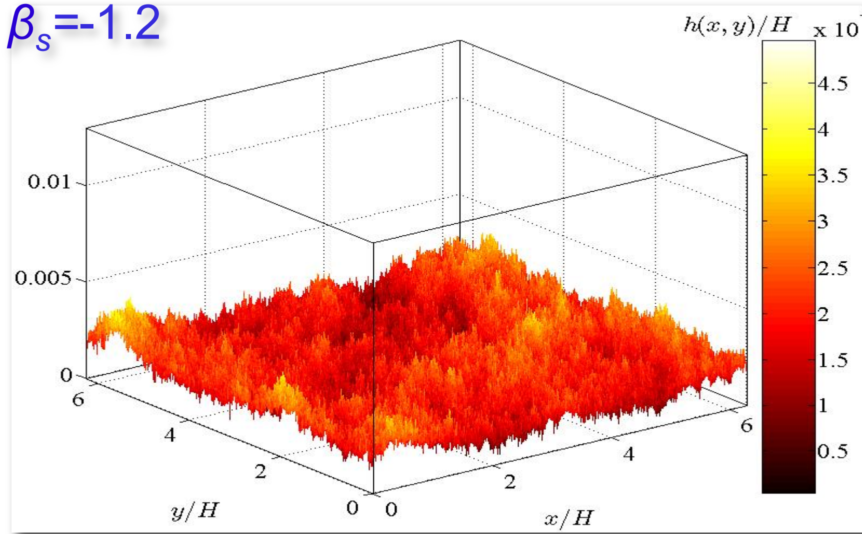
LES of atmospheric boundary layer flow over surfaces created using random-phase Fourier modes and power-law spectra:

$$h(x,y)=\sum_{\underline{k}} c k^{-\beta_s/2} e^{i(\underline{k}\bullet\underline{x}+\varphi)}$$

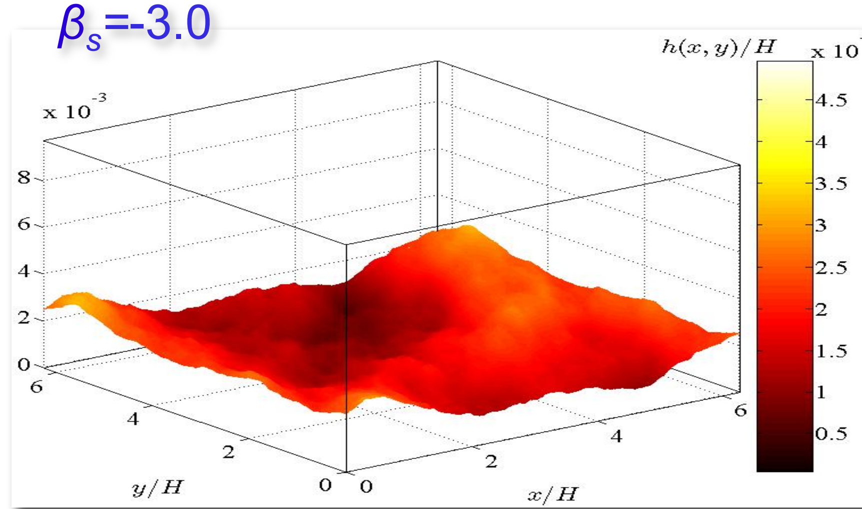
Radial Spectra of h(x,y)



β_s=-1.2

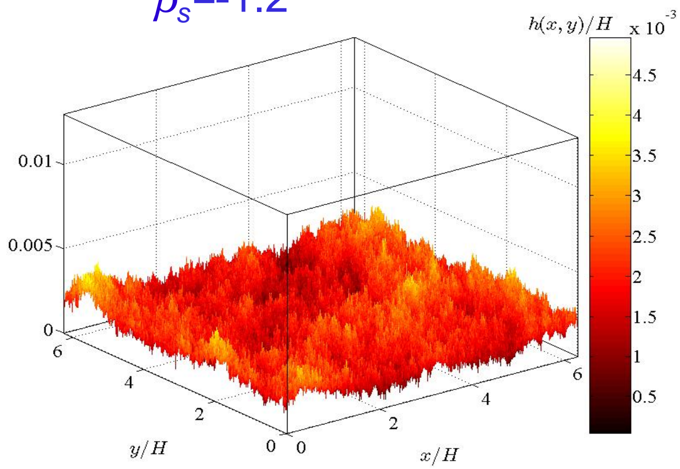


β_s=-3.0

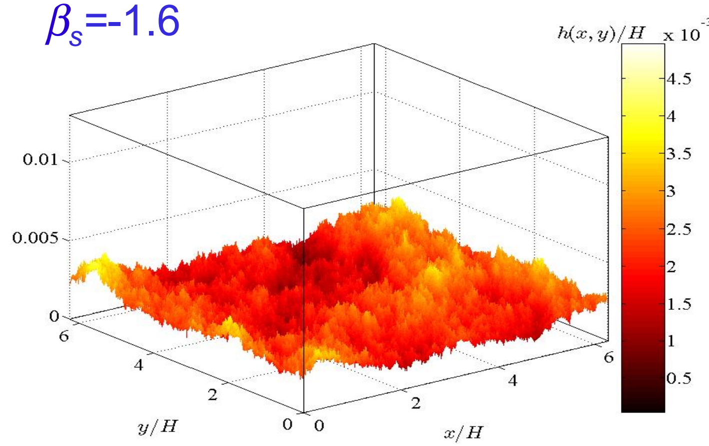


Suite of LES cases with bottom surface with different spectral exponents:

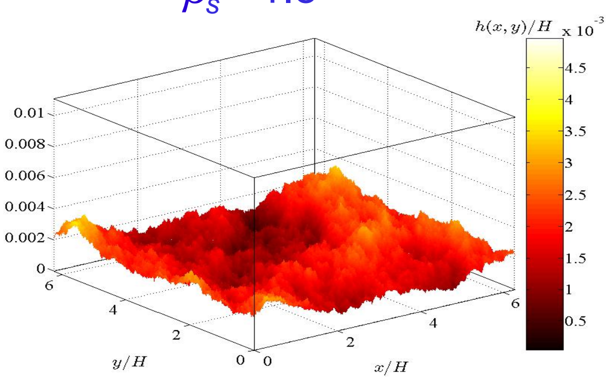
β_s=-1.2



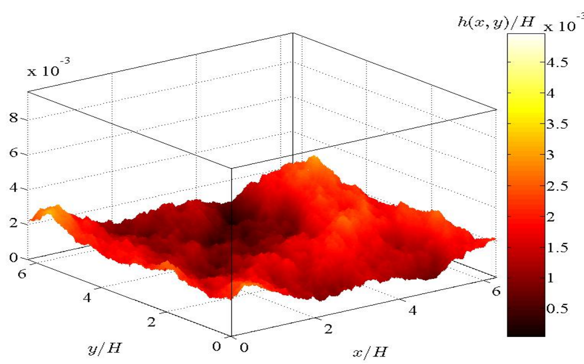
β_s=-1.6



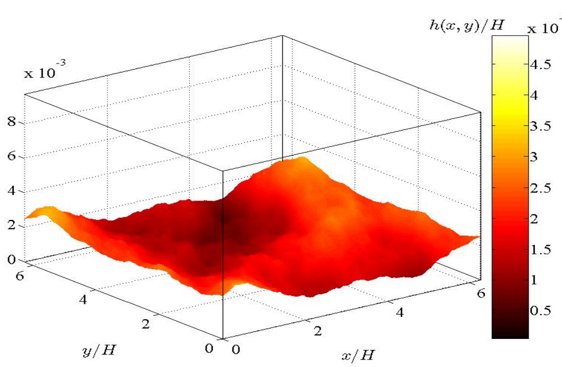
β_s=-1.8



β_s=-2.4

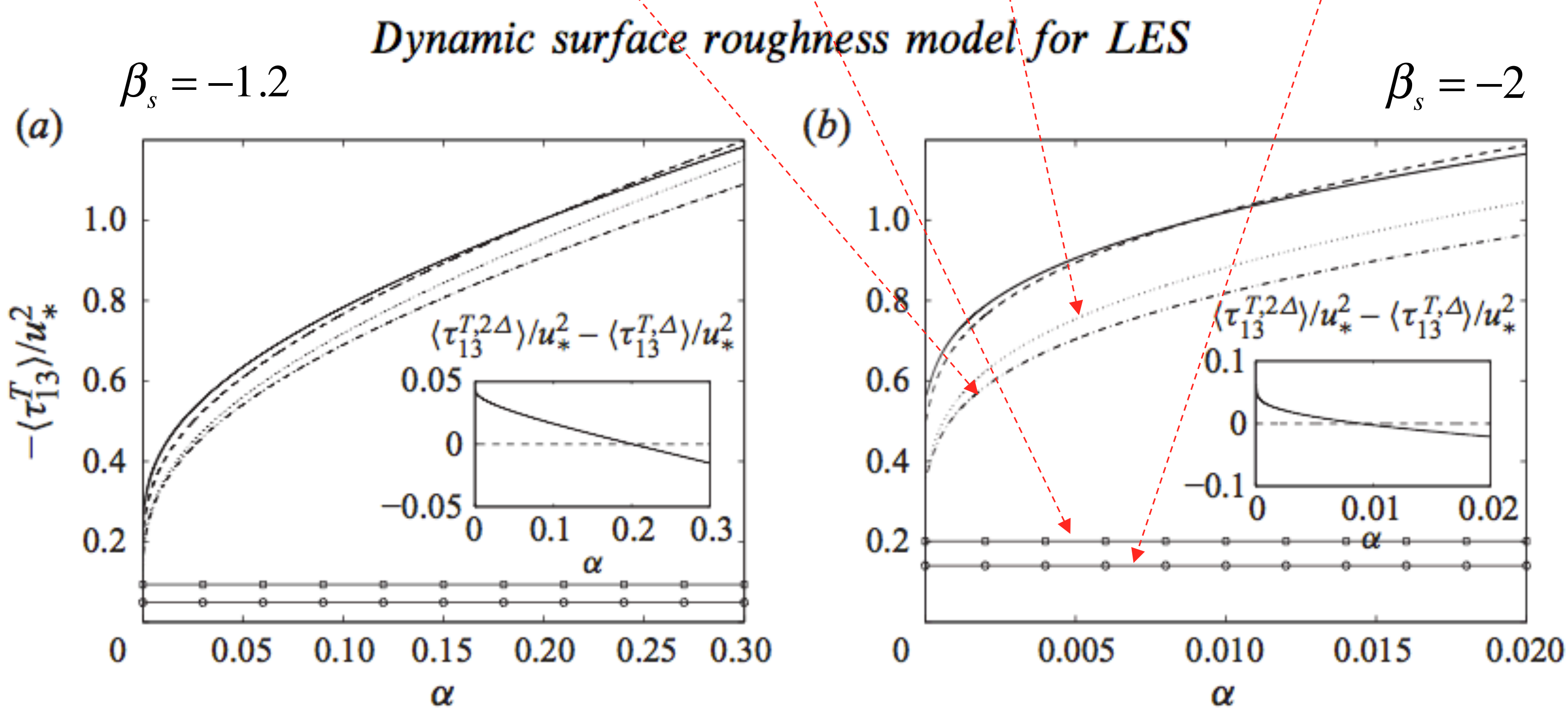


β_s=-3.0

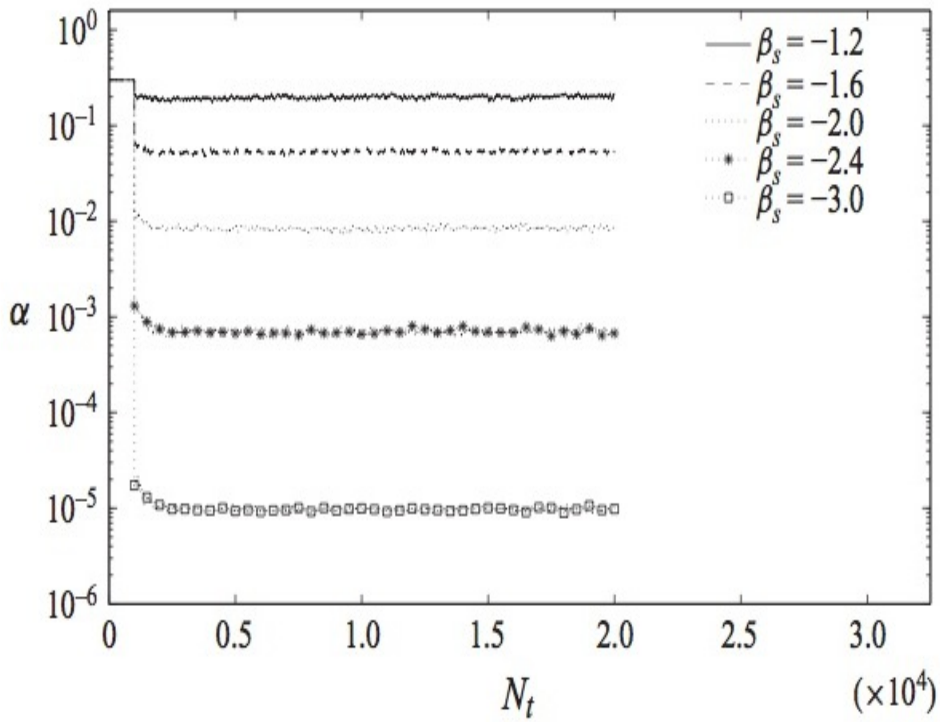


Bisection Method Solution: DSR Model (32^3 LES)

$$\left\langle \left[\frac{\kappa U^{\Delta}}{ln\left(\frac{\Delta_z/2 - h^{\Delta}}{\alpha\sigma_h^{\Delta}}\right)}\right]^2 \frac{\tilde{u}_1}{U^{\Delta}} \right\rangle + \left\langle \tilde{u}_1 R\left(\tilde{u}_k \frac{\partial \tilde{h}}{\partial x_k}\right)\right\rangle = \left\langle \left[\frac{\kappa U^{2\Delta}}{ln\left(\frac{\Delta_z/2 - h^{2\Delta}}{\alpha\sigma_h^{2\Delta}}\right)}\right]^2 \frac{\hat{\tilde{u}}_1}{U^{2\Delta}} \right\rangle + \left\langle \hat{\tilde{u}}_1 R\left(\hat{\tilde{u}}_k \frac{\partial \hat{\tilde{h}}}{\partial x_k}\right)\right\rangle$$



Results: time-evolution of a and dependence on surface spectral slope

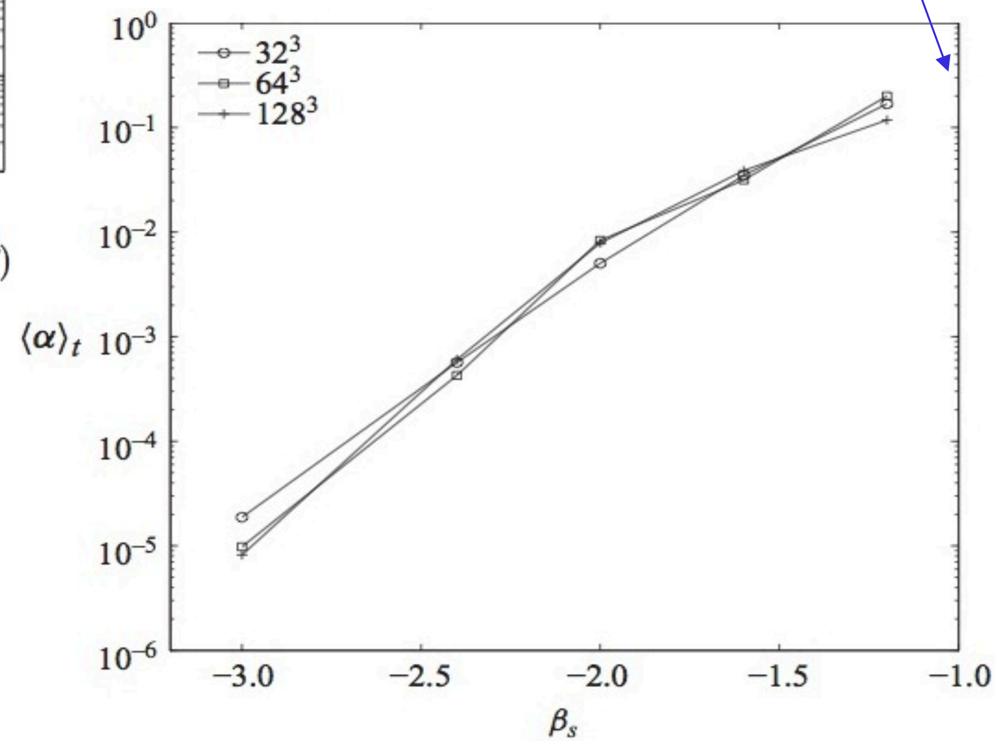


For β_S>-1, the total SGS variance diverges

$$z_0^\Delta = \alpha \sigma_h^\Delta$$

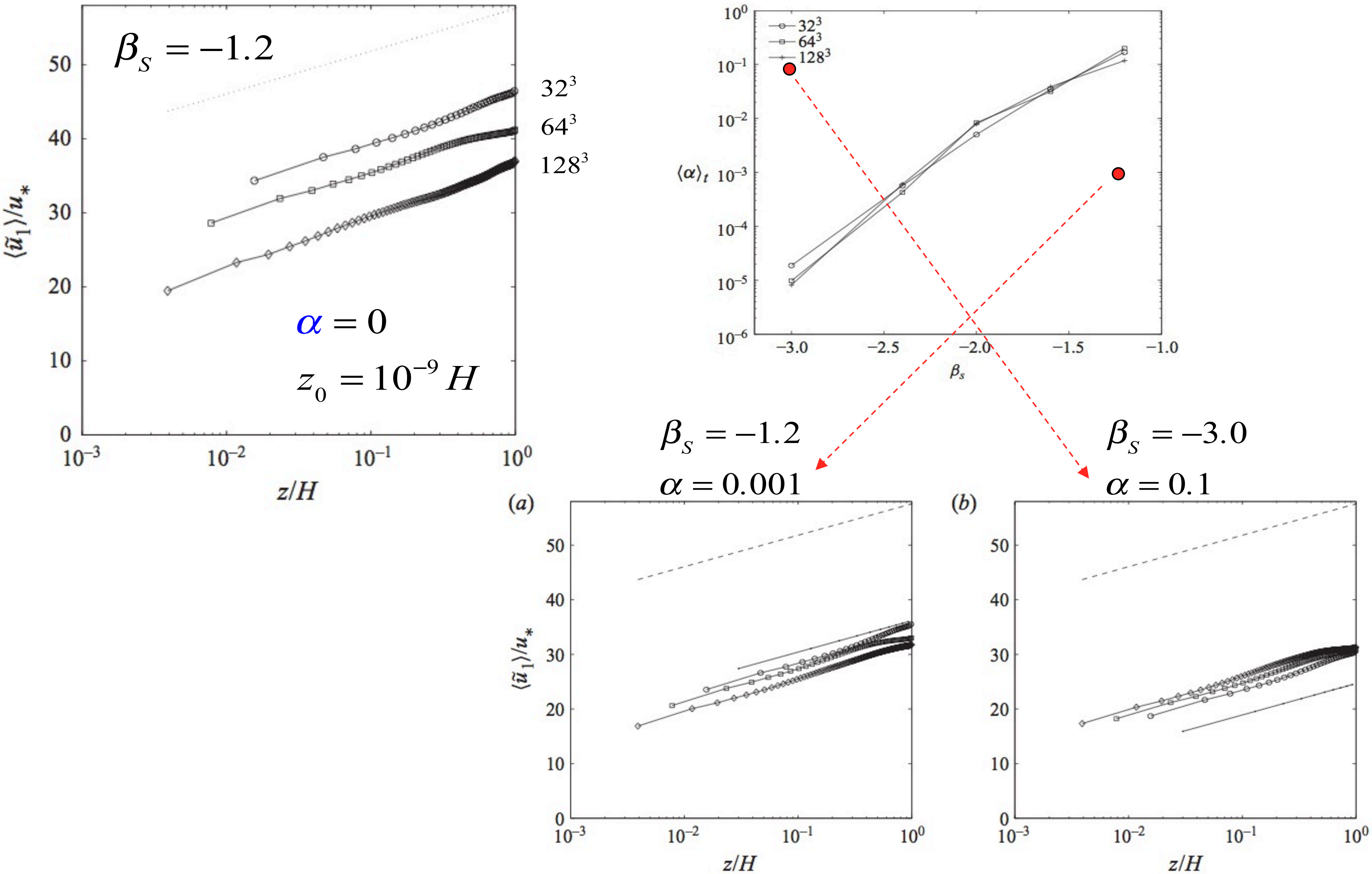
$$E_h(k) \sim k^{-\beta_s}$$

$$\left(\sigma_h^\Delta\right)^2 = \int_{\pi/\Delta}^{\infty} E_h(k)dk$$



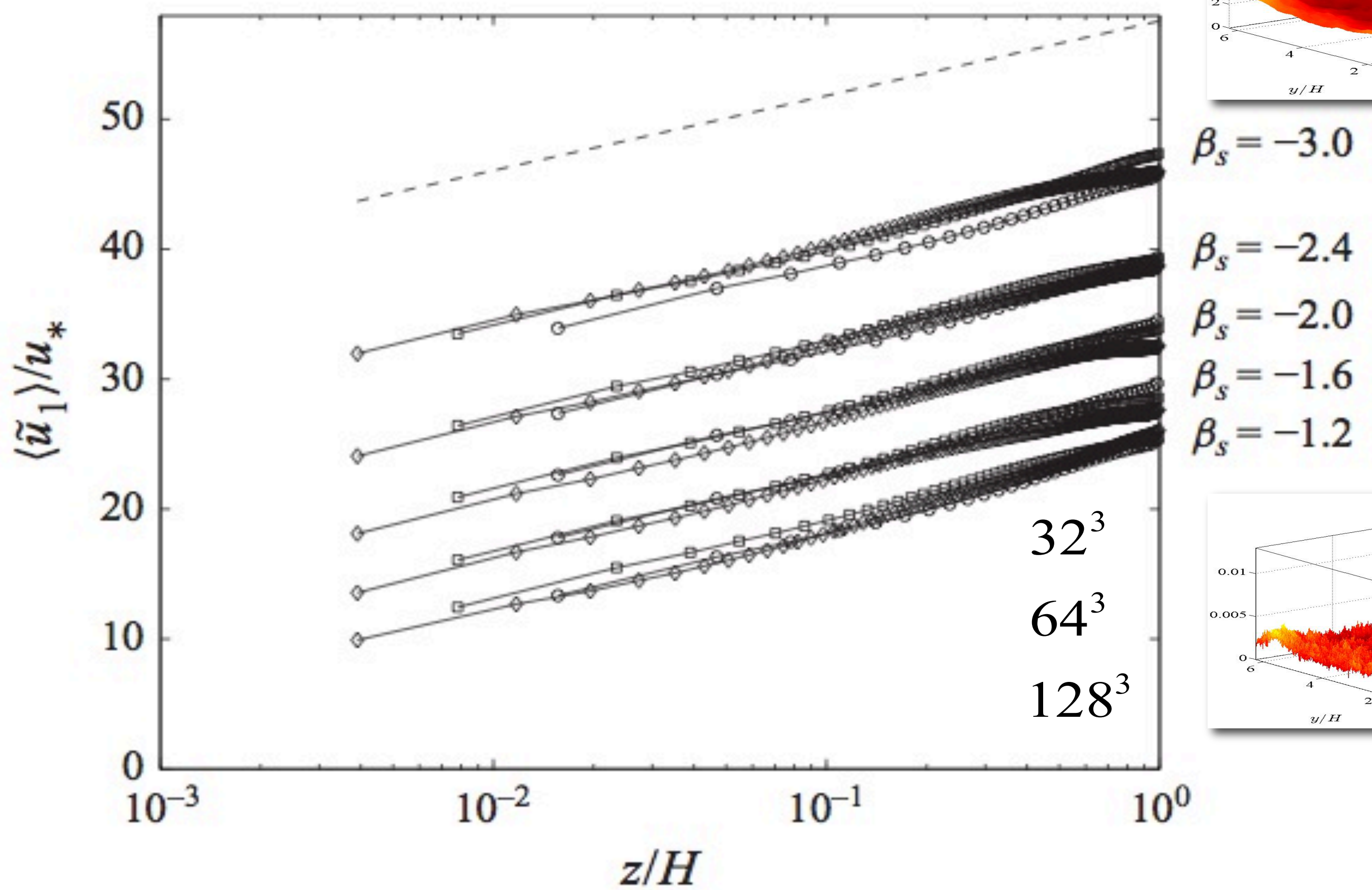
Mean velocity profiles using 32^3, 64^3, 128^3 :

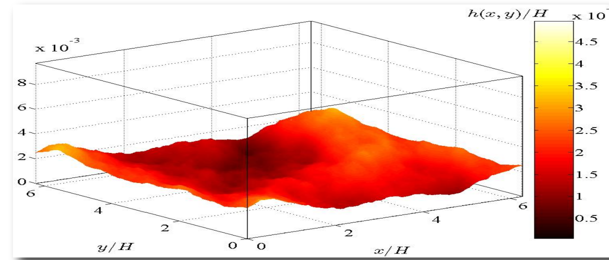
resolution-dependence if α=0 or α =“wrong values”

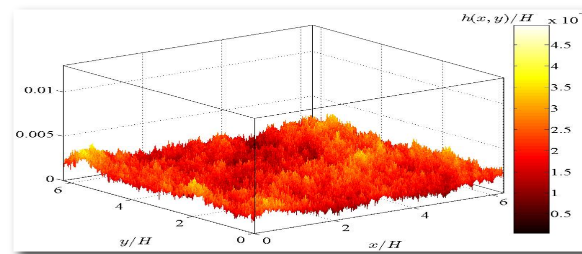


Mean velocity profiles:

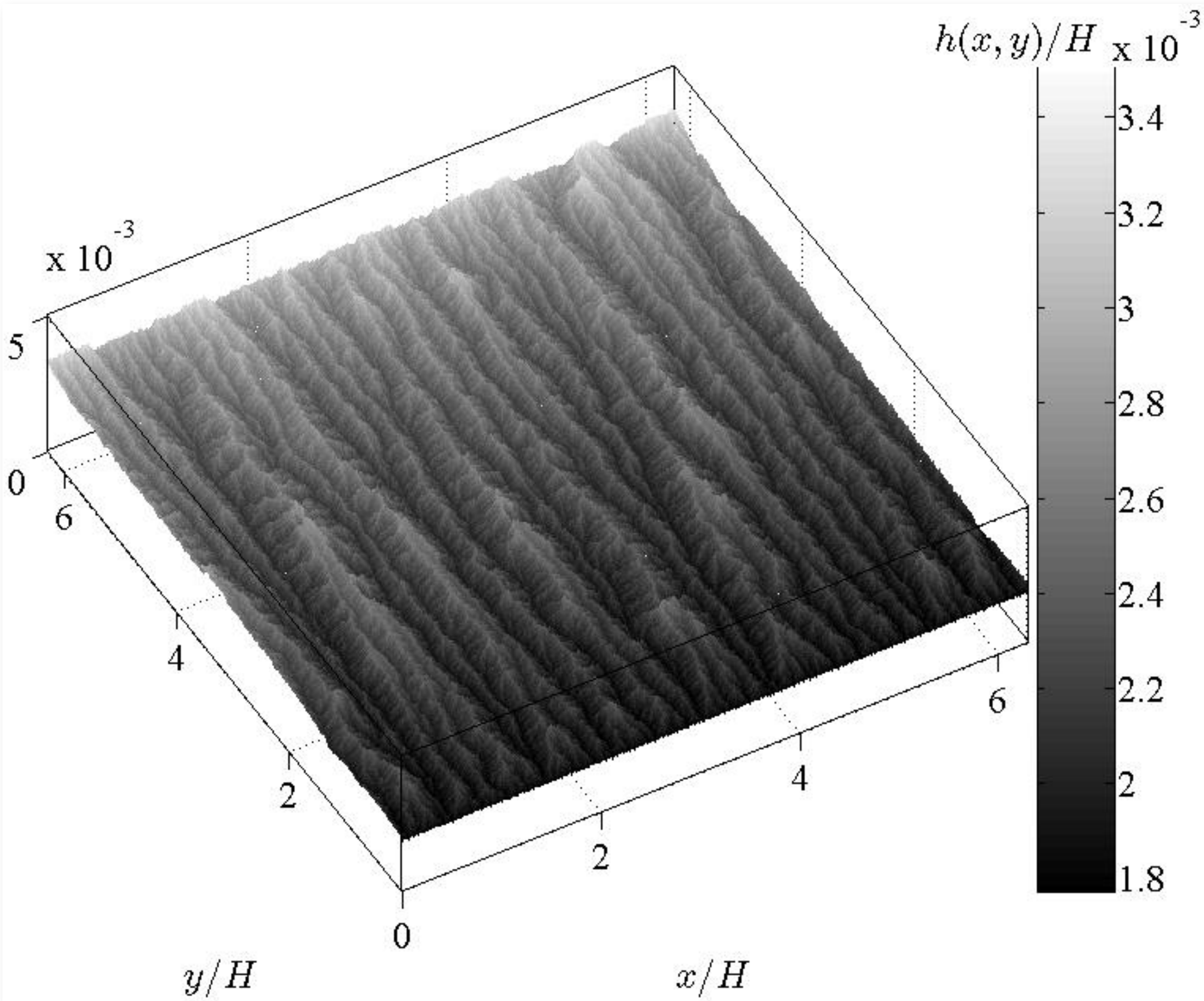
Nearly resolution-independent if **=**“dynamically determined”







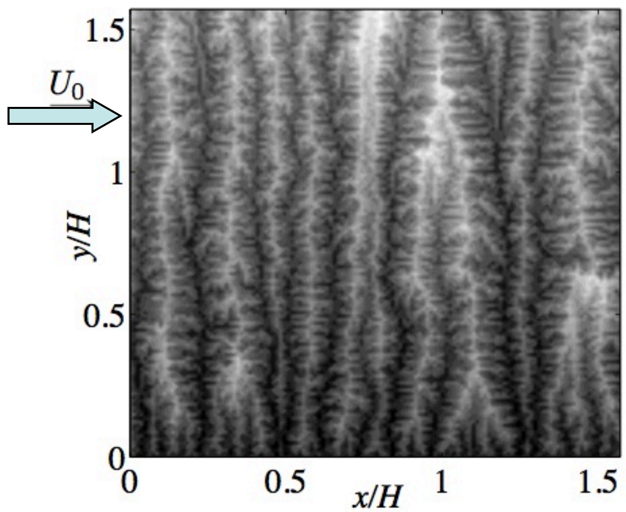
Numerically eroded (Kardar-Parisi-Zhang) surface:



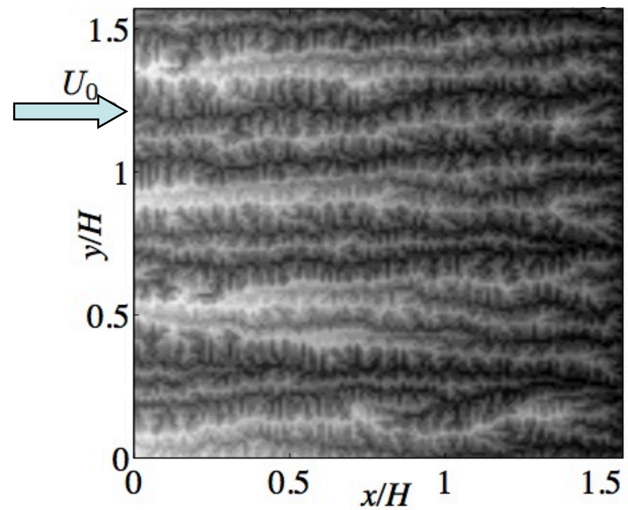
Numerical solution of KPZ equation for fluvial landscape evolution
(data by of P. Passalacqua and F. Porté-Agel)

Anisotropic surfaces: Applications to fluvial evolved landscapes (KPZ):

Flow across channels

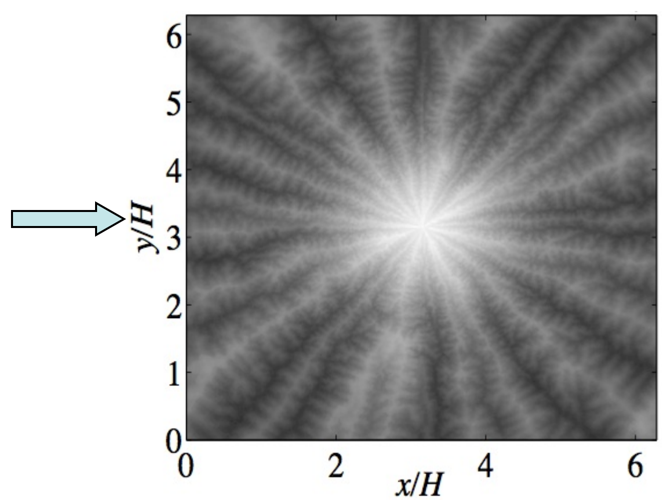


Flow along channels

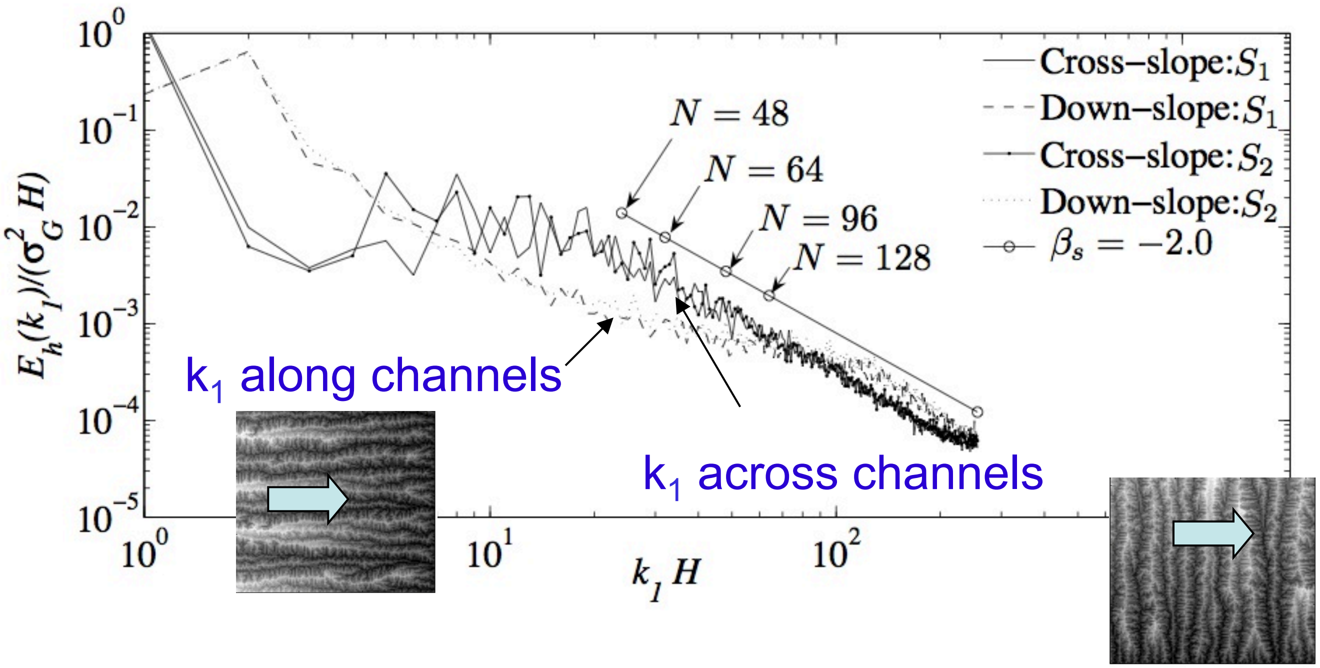


Anderson, Passalacqua, Porté-Agel & CM (BLM, 2012)

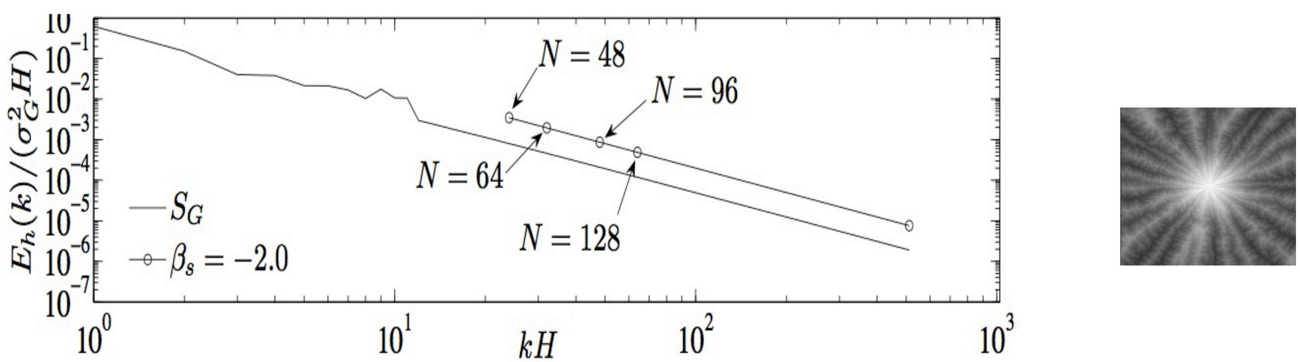
Flow over a Gaussian bump+evolved fluvial landscape (+spectral rescaling k^{-2})



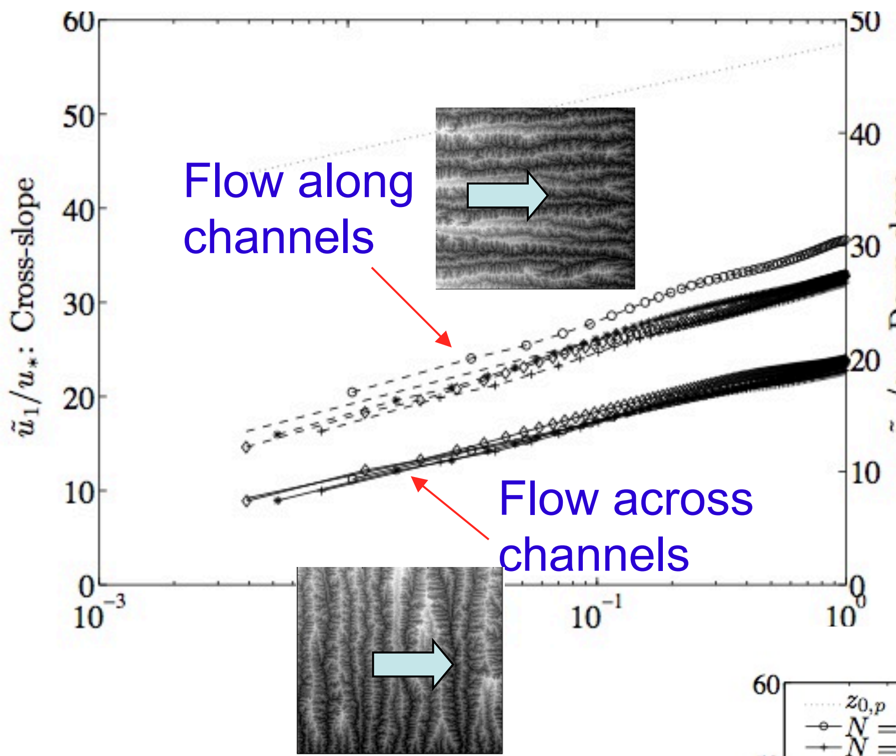
Streamwise spectra of fluvial evolved landscapes (KPZ):



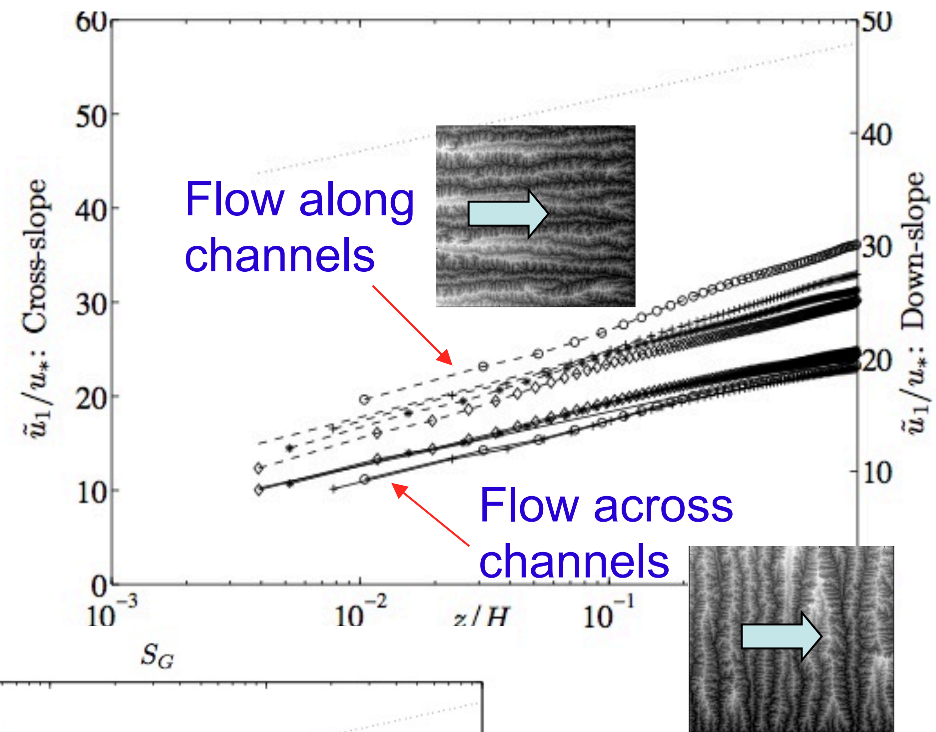
Gaussian bump+evolved fluvial landscape (radial spectrum)

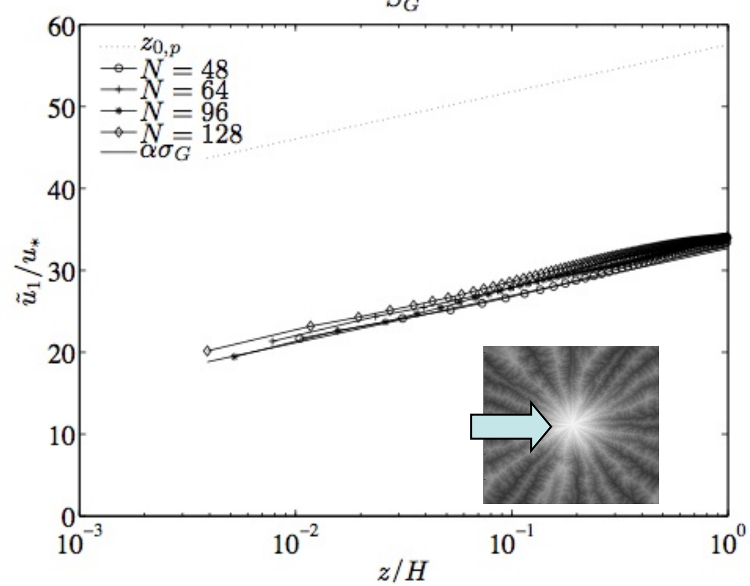


Results: Mean velocity profiles



Different realization:





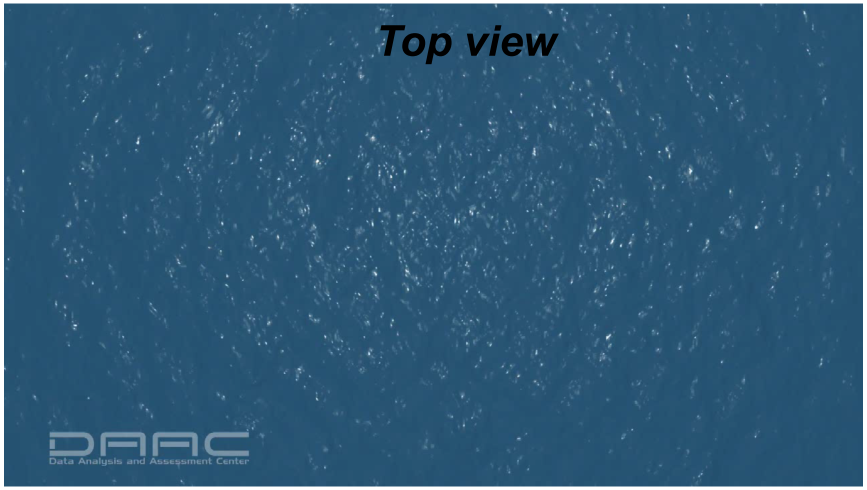
Conclusion:

Good results when surface dispays power-law spectrum in prevailing flow direction

Development of dynamic surface model for wind flow over ocean wave-field

(D. Yang, CM & L. Shen 2012, JFM, in press)

Nonlinear Ocean Wave-field



- *Ocean surface is covered by waves with a wide range of wavelength.*
- *Waves with different wavelength propagate at different speeds.*
- *Different waves also propagate in different directions, and interact with each other in a complex way.*

Strategy of Wind-Wave Coupled Simulation

Computation of wind turbulence using LES (boundary-fitted).

Dynamic modeling of sea-surface roughness for LES 11

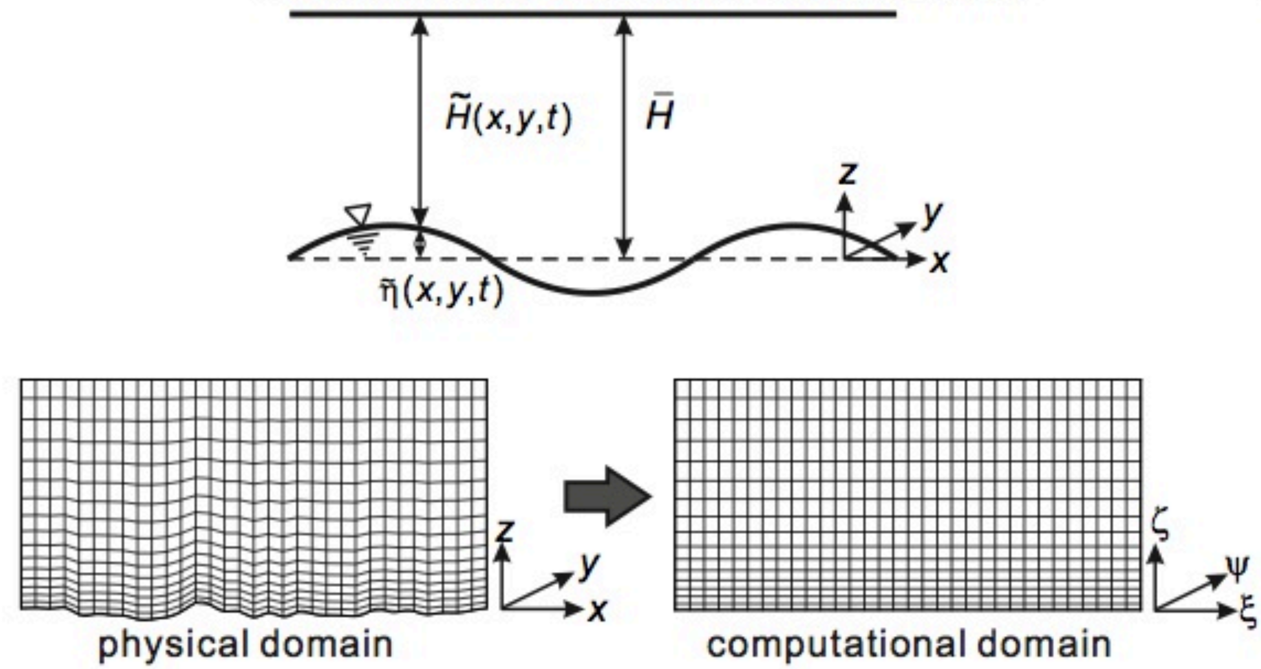


FIGURE 2. Illustration of coordinate transformation. The irregular wave surface-bounded domain in the physical space (x, y, z, t) is transformed to a right rectangular prism in the computational space (ξ, ψ, ζ, τ). Only a vertical cross-section in the three dimensional space is plotted here.

Strategy of Wind-Wave Coupled Simulation

- *Phase-resolving computation of nonlinear ocean wavefield based on potential flow theory, described by surface elevation h(x,y,t) and surface potential F^s(x,y,h,t).*
- *Wind and wave simulations can be dynamically coupled using a two-way feedback approach.*
- *Here: 1-way coupling: p_a from air effect on wave evolution neglected (for cases considered, effect was negligible)*

perturbation series of Φ with respect to wave steepness to order M,

$$\Phi(x,y,z,t)=\sum_{m=1}^{M}\Phi^{(m)}(x,y,z,t)\,; \tag{3.5}$$

(ii) express Φ^s using Taylor series expansion about $z=0$ to the order corresponding to (i),

$$\Phi^s(x,y,t)=\sum_{m=1}^{M}\sum_{\ell=0}^{M-m}\frac{\eta^\ell}{\ell!}\frac{\partial^\ell}{\partial z^\ell}\Phi^{(m)}(x,y,z,t)\bigg|_{z=0}\,; \tag{3.6}$$

and (iii) represent $\Phi^{(m)}$ using an eigenfunction expansion with N modes,

$$\Phi^{(m)}(x,y,z,t)=\sum_{n=1}^{N}\Phi_n^{(m)}(t)\Psi_n(x,y,z)\,. \tag{3.7}$$

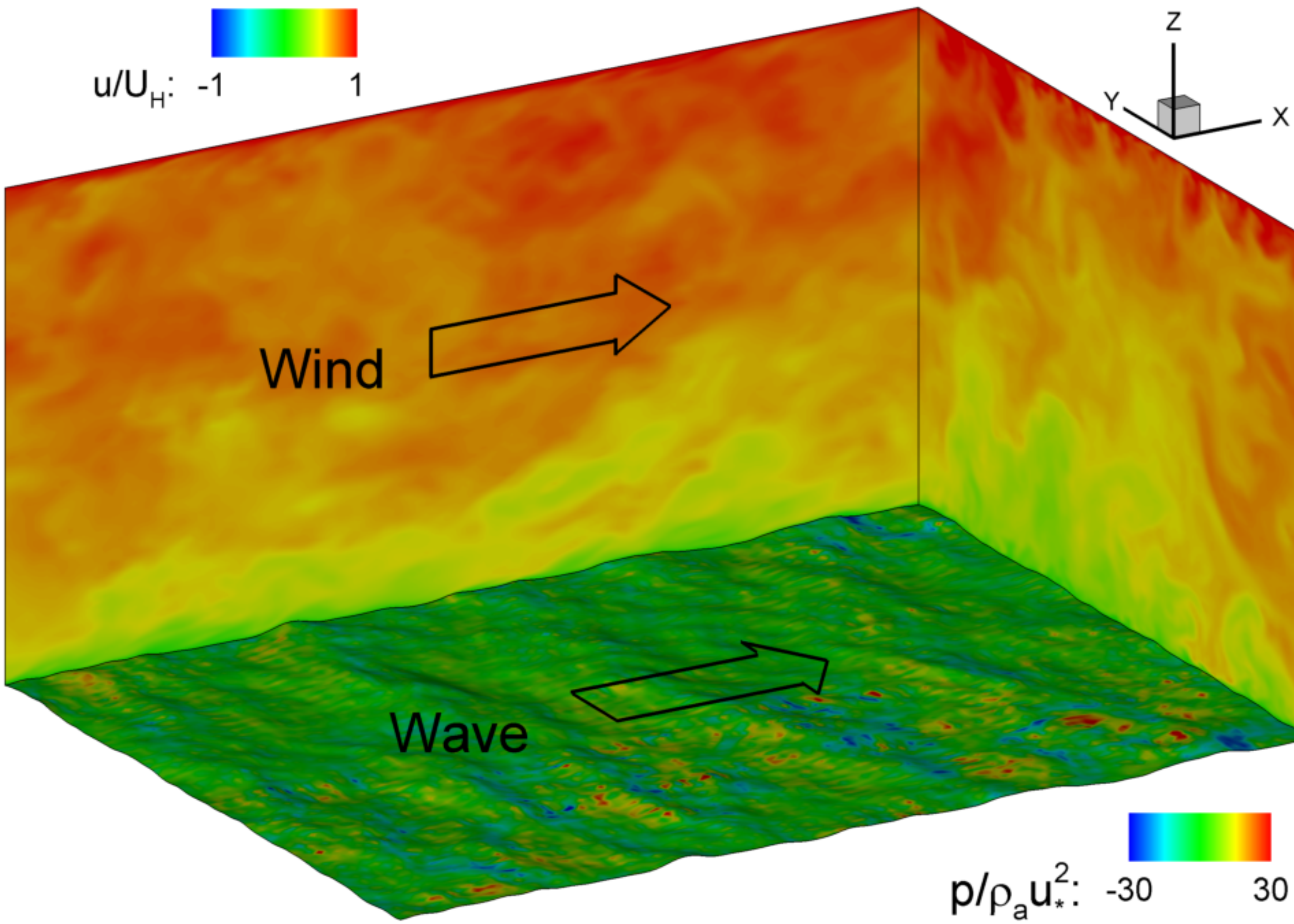
The evolution equations for η and Φ^s are obtained as (Dommermuth & Yue 1987)

$$\frac{\partial\eta}{\partial t}=-\nabla_h\eta\cdot\nabla_h\Phi^s+(1+\nabla_h\eta\cdot\nabla_h\eta)\times\left[\sum_{m=1}^{M}\sum_{\ell=0}^{M-m}\frac{\eta^\ell}{\ell!}\sum_{n=1}^{N}\Phi_n^{(m)}(t)\,\frac{\partial^{\ell+1}\Psi_n(x,y,z)}{\partial z^{\ell+1}}\bigg|_{z=0}\right], \tag{3.8}$$

$$\frac{\partial\Phi^s}{\partial t}=-g\eta-\frac{1}{2}\nabla_h\Phi^s\cdot\nabla_h\Phi^s+\frac{1}{2}(1+\nabla_h\eta\cdot\nabla_h\eta)\times\left[\sum_{m=1}^{M}\sum_{\ell=0}^{M-m}\frac{\eta^\ell}{\ell!}\sum_{n=1}^{N}\Phi_n^{(m)}(t)\,\frac{\partial^{\ell+1}\Psi_n(x,y,z)}{\partial z^{\ell+1}}\bigg|_{z=0}\right]^2. \tag{3.9}$$

Here, $\nabla_h=(\partial/\partial x,\partial/\partial y)$ is the horizontal gradient. For the deep-water waves considered in this study, the eigenfunction in the above equations is $\Psi_n(x,y,z)=\exp(|\boldsymbol{k}|z+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x})$, where $\boldsymbol{k}=(k_x,k_y)$ is the wavenumber vector and $\mathrm{i}=\sqrt{-1}$. The relation between the scalar wavenumber k (see §2) and the wavenumber vector is $k=|\boldsymbol{k}|=\sqrt{k_x^2+k_y^2}$. A pseudo-spectral method with Fourier series is used for spatial discretization. A fourth-

Strategy of Wind-Wave Coupled Simulation



Modeling of Wave-Induced Sea-Surface Roughness

➤ *Total surface drag acting on wind:*

$$f_x = \underbrace{-\frac{1}{S}\iint_S \tilde{p}_a \frac{\partial \tilde{\eta}}{\partial x} dS}_{\textit{grid-scale (GS)}} \underbrace{-\frac{1}{S}\iint_S \rho_a \tau_{13}^{\Delta} dS}_{\textit{subgrid-scae (SGS)}}$$

$$\tau_{13}^{\Delta} = \frac{\kappa^2 \left|\tilde{\bar{u}}_{z=\Delta z/2}\right|}{\left[\log\left(\frac{\Delta z/2}{z_{0,\Delta}}\right)\right]^2} \tilde{u}_{z=\Delta z/2}$$

SGS surface roughness

How to model $z_{0,\Delta}$?

➤ *Empirical formula for wave surface roughness:*

$$z_0 = A\left(\frac{c_p}{u_*}\right)^B \frac{u_*^2}{g}$$

	A	B
Geernaert *et al.* (1983)	0.015	-0.74
Nordeng (1991)	0.11	-0.75
Smith *et al.* (1992)	0.48	-1.00
Johnson & Vested (1992)	0.06	-0.52
Martin (1998)	0.68	-1.24
Johnson *et al.* (1998)	1.89	-1.24

➤ *Dynamic model for wave surface roughness:*

$$z_{0,\Delta} = \alpha \; \varepsilon_{\Delta}^{SGS}$$

SGS roughness length scale

roughness index

Dynamic Model of Wave Surface Roughness

$$z_{0,\Delta} = \alpha \ \varepsilon_{\Delta}^{SGS}$$

SGS roughness length scale = ?

roughness index = ?

original surface η GS surface $\tilde{\eta}$

test filtered surface $\hat{\tilde{\eta}}$

total drag at grid scale Δ

$$f_x = -\frac{1}{S}\iint_S \tilde{p}_a \frac{\partial \tilde{\eta}}{\partial x} dS - \frac{1}{S}\iint_S \rho_a \frac{\kappa^2 \left| \tilde{\vec{u}}_{z=\Delta z/2} \right|}{\left[\log\left(\frac{\Delta z/2}{\alpha \varepsilon_{\Delta}^{SGS}} \right) \right]^2} \tilde{u}_{z=\Delta z/2}\, dS$$

=

total drag at test filter scale $\hat{\Delta}$

$$f_x = -\frac{1}{S}\iint_S \hat{\tilde{p}}_a \frac{\partial \hat{\tilde{\eta}}}{\partial x} dS - \frac{1}{S}\iint_S \rho_a \frac{\kappa^2 \left| \hat{\tilde{\vec{u}}}_{z=\Delta z/2} \right|}{\left[\log\left(\frac{\Delta z/2}{\alpha \varepsilon_{\hat{\Delta}}^{SGS}} \right) \right]^2} \hat{\tilde{u}}_{z=\Delta z/2}\, dS$$

$\Downarrow$

α

Still need model for $\varepsilon_{\Delta}^{SGS}$

Dynamic Model of Wave Surface Roughness

total drag at grid scale Δ

$$f_x = -\frac{1}{S}\iint_S \tilde{p}_a \frac{\partial \tilde{\eta}}{\partial x} dS - \frac{1}{S}\iint_S \rho_a \frac{\kappa^2 \left|\tilde{\vec{u}}_{z=\Delta z/2}\right|}{\left[\log\left(\frac{\Delta z/2}{\alpha \varepsilon_{\Delta}^{SGS}}\right)\right]^2} \tilde{u}_{z=\Delta z/2}\, dS$$

=

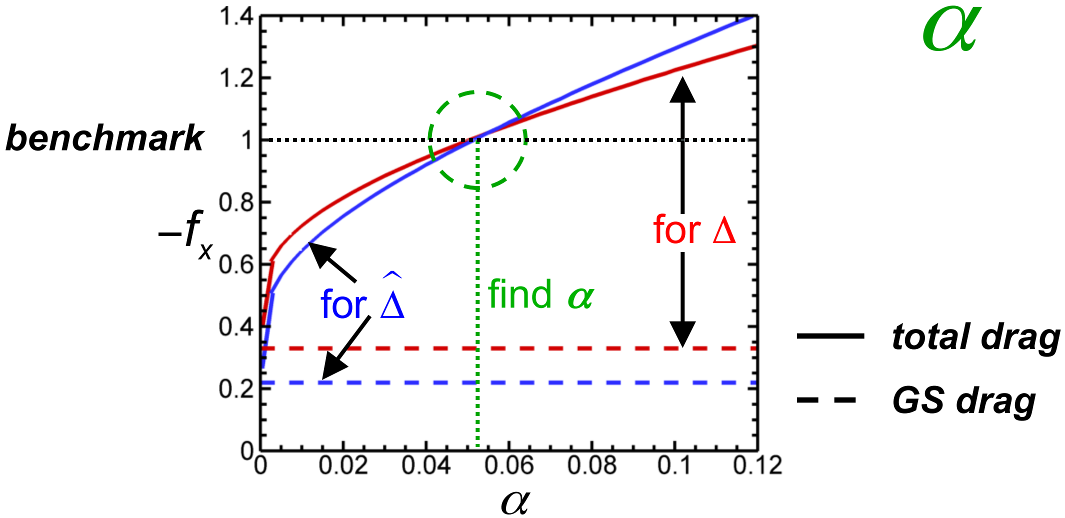
total drag at test filter scale $\hat{\Delta}$

$$f_x = -\frac{1}{S}\iint_S \hat{\tilde{p}}_a \frac{\partial \hat{\tilde{\eta}}}{\partial x} dS - \frac{1}{S}\iint_S \rho_a \frac{\kappa^2 \left|\hat{\tilde{\vec{u}}}_{z=\Delta z/2}\right|}{\left[\log\left(\frac{\Delta z/2}{\alpha \varepsilon_{\hat{\Delta}}^{SGS}}\right)\right]^2} \hat{\tilde{u}}_{z=\Delta z/2}\, dS$$

$\Downarrow$

α

Total surface drag as function of roughness index α



$\boxed{\textbf{Still need model for } \varepsilon_{\Delta}^{SGS}}$

- ***For land:***

$$\varepsilon_{\Delta}^{SGS} = \left[\int_{\pi/\Delta}^{\infty} \underbrace{E_{\eta}(k)}_{\text{surface elevation spectrum}} dk\right]^{1/2} = \underbrace{\eta_{RMS}}_{\text{RMS of surface elevation}}$$

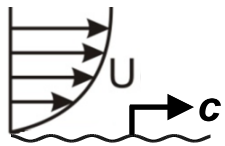
Anderson & Meneveau (2011)

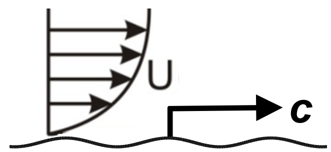
SGS Roughness Model for Waves

A-priori test
(filter DNS)

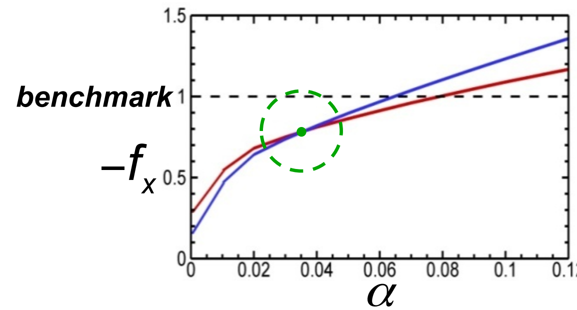
- **RMS-height model:**

$$\varepsilon_\Delta^{SGS} = \left[\int_{\pi/\Delta}^{\infty} E_\eta(k)dk\right]^{1/2}$$





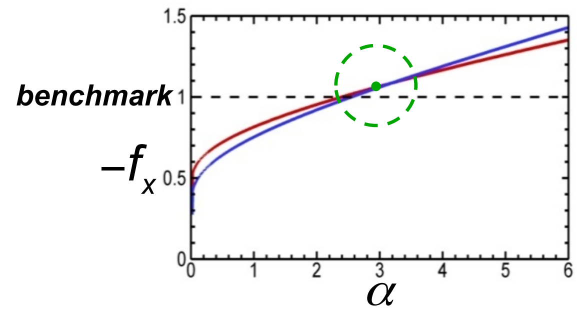
simply put equal weights on different wave components



- **Steepness-dependent Charnock model:**

$$\varepsilon_\Delta^{SGS} = \int_{\pi/\Delta}^{\infty} \overbrace{k^2 E_\eta(k)}^{\text{wave slope spectrum}} dk \frac{u_*^2}{g}$$

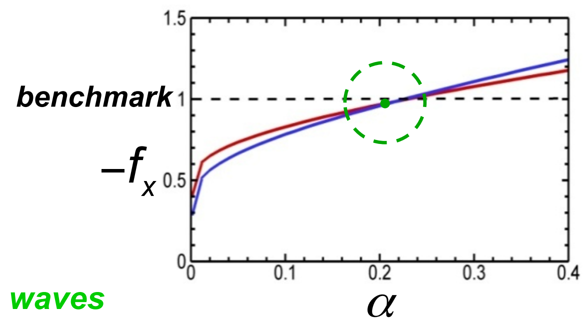
increase weights for short waves



- **Wave-kinematics-dependent model:**

$$\varepsilon_\Delta^{SGS} = \left[\int_{\pi/\Delta}^{\infty} E_\eta(k)\exp\left(-2\frac{\kappa}{u_*}\overbrace{\sqrt{\frac{g}{k}}}^{\text{wave phase speed}}\right)dk\right]^{1/2}$$

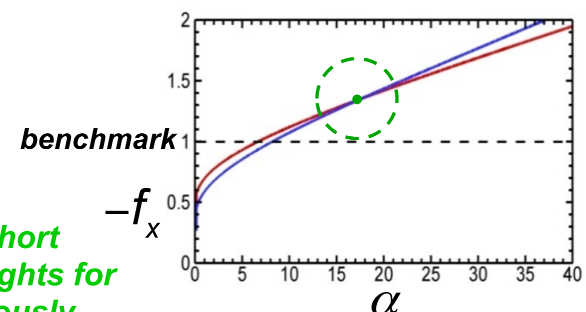
reduce weights for long waves



- **Combined-kinematics-steepness model:**

$$\varepsilon_\Delta^{SGS} = \int_{\pi/\Delta}^{\infty} k^2 E_\eta(k)\exp\left(-2\frac{\kappa}{u_*}\sqrt{\frac{g}{k}}\right)dk \frac{u_*^2}{g}$$

increase weights for short waves and reduce weights for long waves simultaneously



Analysis of Wave-Kinematics-Dependent Model

- **Correction by the wave age for each wave component:**

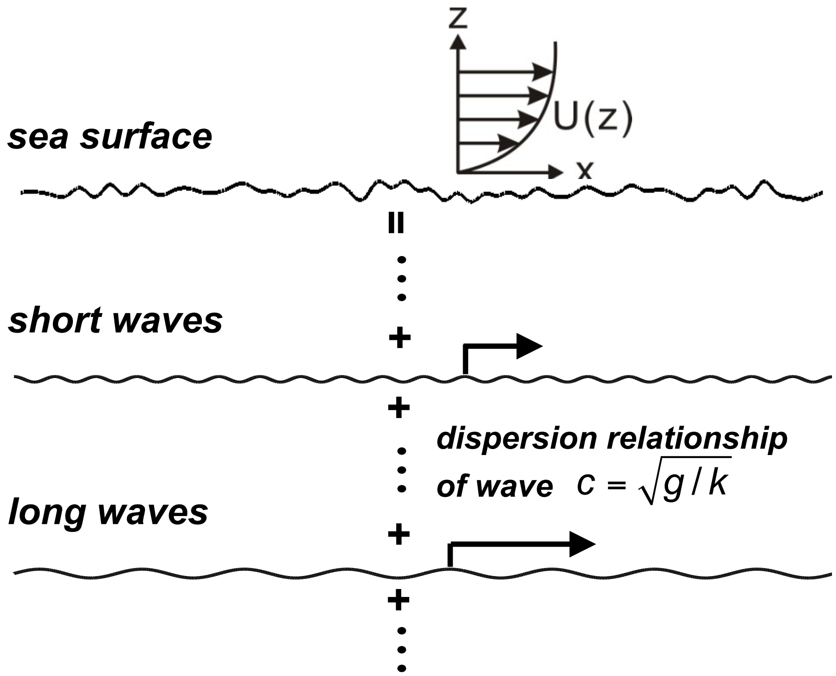
$$\frac{U(z)-c_k}{u_*} = \frac{1}{\kappa}\log\frac{z}{\alpha a_k}$$

$$U(z) = \frac{u_*}{\kappa}\log\frac{z}{\alpha a_k \exp(-\kappa c_k / u_*)}$$

$$c_k = \sqrt{\frac{g}{k}}$$

$$\varepsilon_{\Delta,k}^{SGS} = a_k \exp\left(-\frac{\kappa}{u_*}\sqrt{\frac{g}{k}}\right)$$

$$\varepsilon_{\Delta}^{SGS} = \left[\int E_\eta(k)\exp\left(-2\frac{\kappa}{u_*}\sqrt{\frac{g}{k}}\right)dk\right]^{1/2}$$



A Priori Test of Dynamic Sea Roughness model - Filtering scales and wave spectra

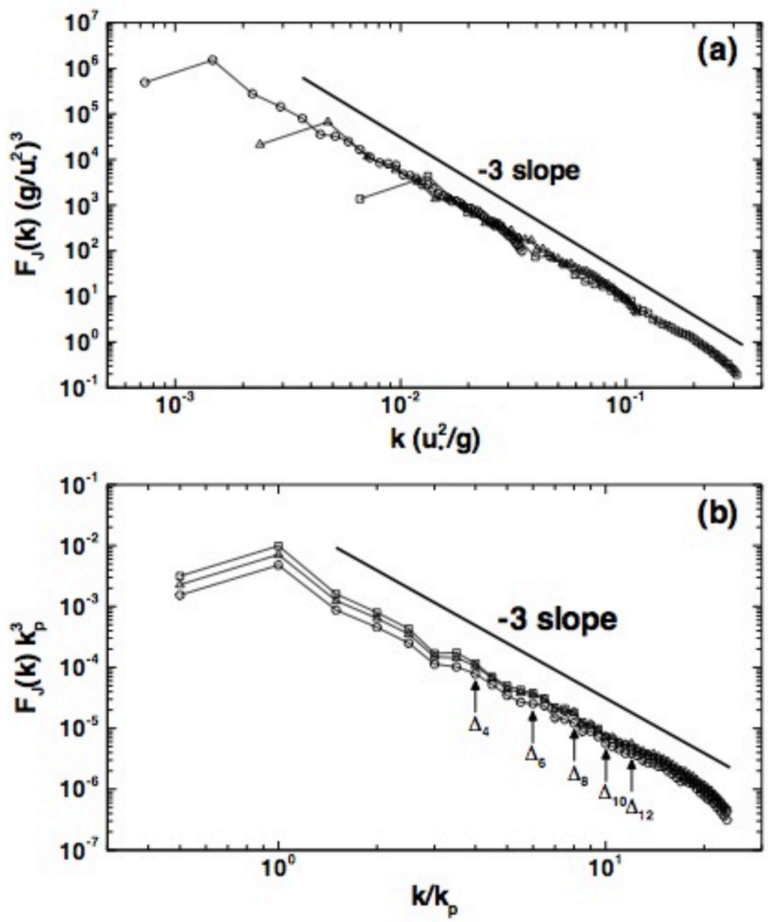


FIGURE 4. One-dimensional wavenumber spectra of surface elevation for the JONSWAP wave-fields simulated by HOSM: □, case CU6; △, case CU10; and ◯, case CU18. Variables in (*a*) are normalized by the wind friction velocity u_* and gravitational acceleration g; variables in (*b*) are normalized by the peak wavenumber k_p. The corresponding location of filters Δ_4, Δ_6, Δ_8, Δ_{10}, and Δ_{12} used for *a priori* tests are also indicated in (*b*).

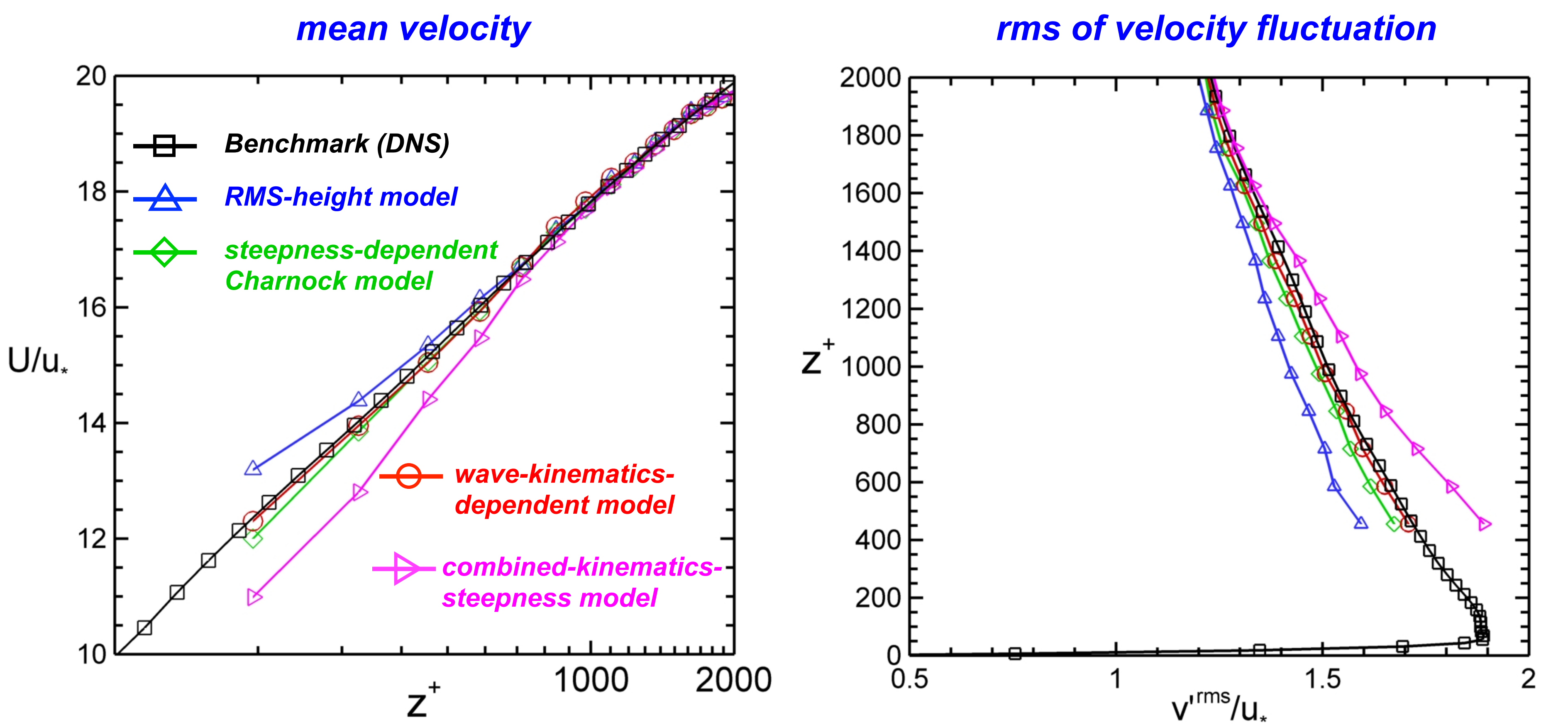
A Priori Test of Dynamic Sea Roughness model
- Invariance of Roughness Index

Filter size	Roughness index for various models			
	RMS-height model	steepness-dependent Charnock model	wave-kinematics-dependent model	combined-kinematics-steepness model
$\lambda_p/4$	0.0858	3.75	0.321	16.31
$\lambda_p/5$	0.0982	3.73	0.331	15.60
$\lambda_p/6$	0.1024	3.54	0.323	14.37
$\lambda_p/7$	0.1060	3.38	0.317	13.38
$\lambda_p/8$	0.1096	3.26	0.314	12.62
$\lambda_p/9$	0.1138	3.19	0.315	12.06
$\lambda_p/10$	0.1207	3.17	0.322	11.67
$\lambda_p/11$	0.1289	3.20	0.334	11.53
$\lambda_p/12$	0.1350	3.21	0.342	11.37
Norm. standard deviation	0.138	0.069	**0.029**	0.139

- ***Roughness index α will be a weak function of scale if the quantification of SGS roughness length scale works.***
- ***The wave-kinematics-dependent model gives the best result.***

$$z_{0,\Delta} = \alpha \ \varepsilon_{\Delta}^{SGS}$$

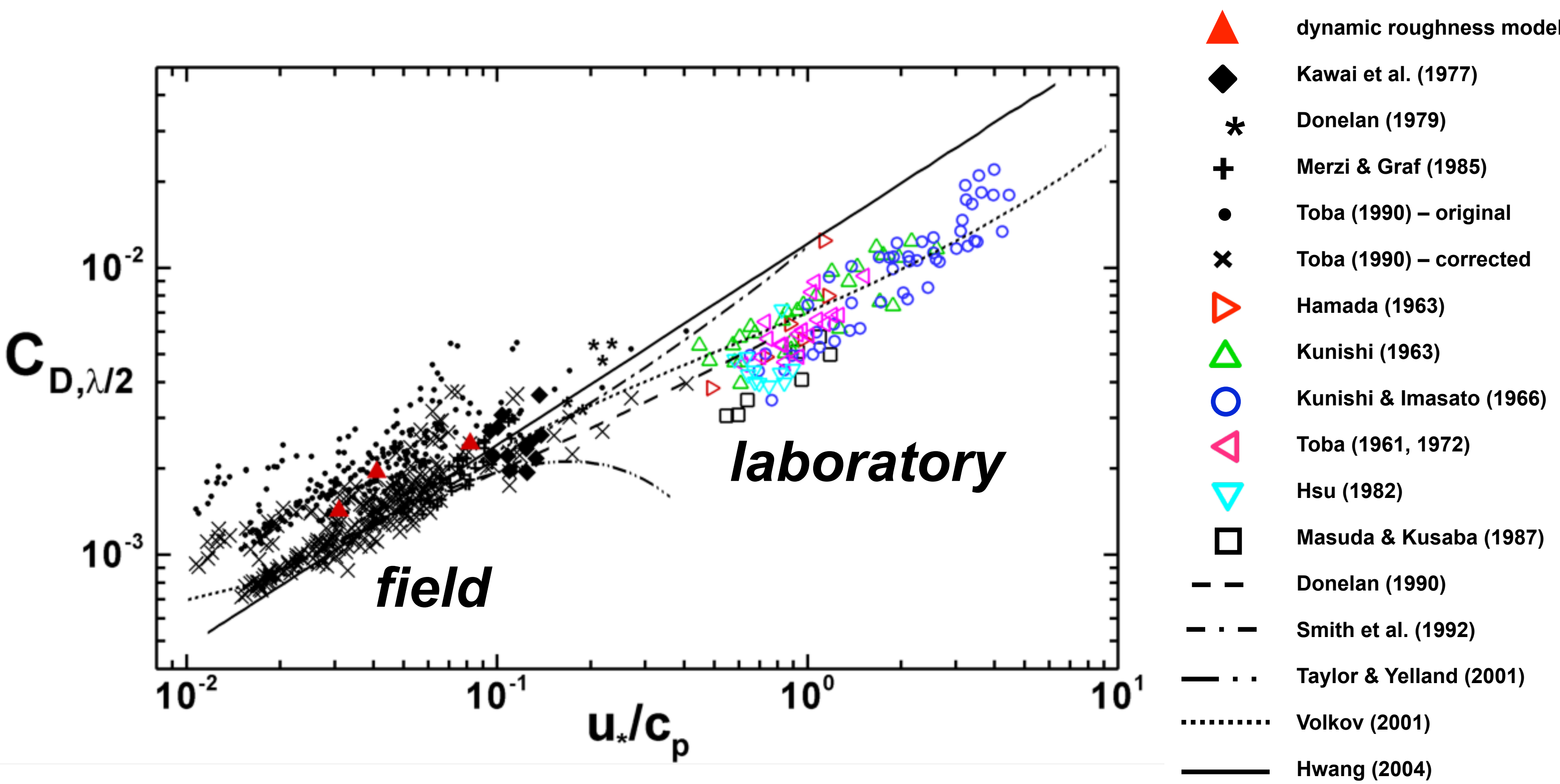
A Posteriori Test of Dynamic Sea Roughness Model



- *Wave-kinematics-dependent model gives the best result among different models.*
- *Steepness-dependent Charnock model also gives good result.*
- *RMS-height model is not good.* ← *lack of wave mechanics*
- *Combined-kinematics-steepness model performs poorly.* ← *double counting of wave effect*

A-posteriori tests:
Simulation with Dynamic Sea Surface Roughness Model

Spectrum	U_{10} (m/s)	u_* (m/s)	F (m)	c_p (m/s)	u_*/c_p	λ_p (m)
JONSWAP	10.0	0.37	10000	4.51	0.08	13.07
JONSWAP	10.0	0.37	80000	9.03	0.04	52.28
P-M	10.0	0.37	N/A	12.11	0.03	93.97



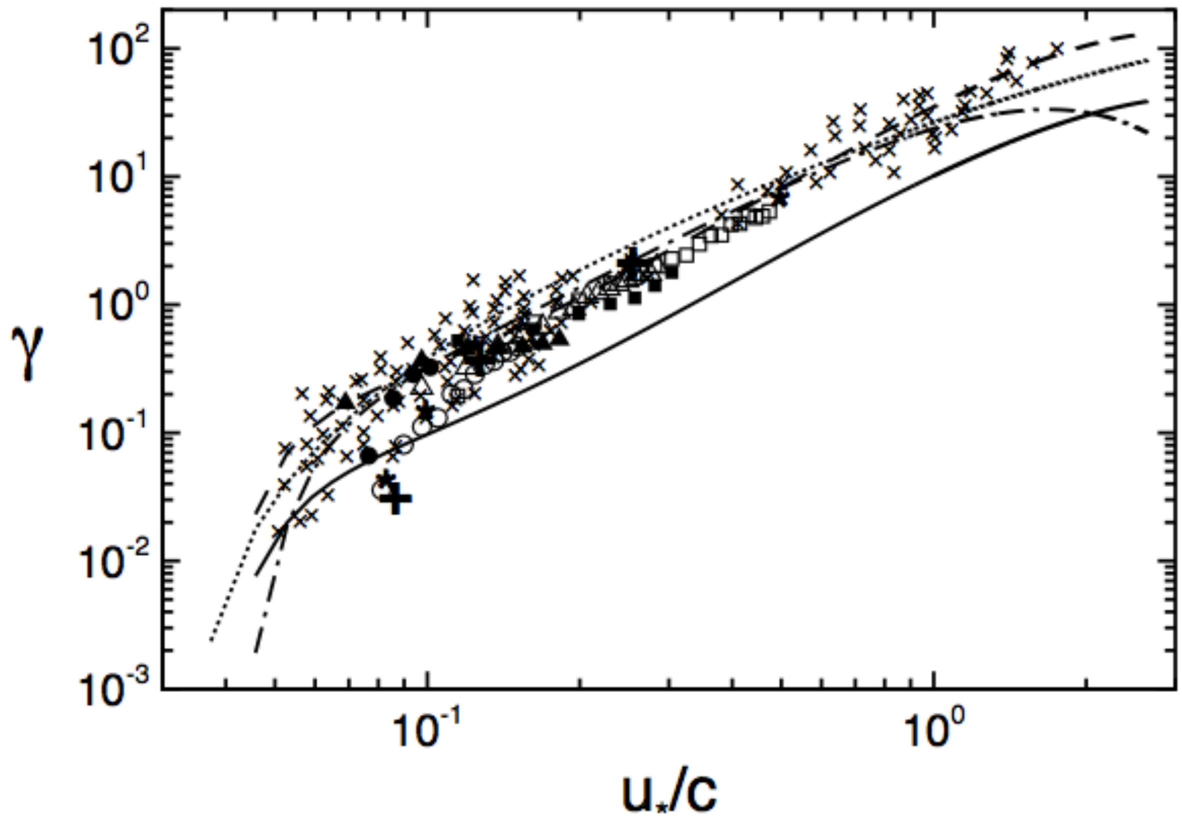


FIGURE 7. Dependence of wave temporal growth rate γ on reversed wave age u_*/c and comparison of the current LESns-R with previous experiments and simulations. Experimental data compiled by Plant (1982) are indicated by $\times$. Predictions by various wind-wave theories are indicated by lines: ——, Miles (1957); – –, Janssen (1991); and – · –, Miles (1993). Parameterization by Donelan *et al.* (2006) is indicated by $\cdots$. DNS results from Sullivan *et al.* (2000) are marked by +. DNS results from Kihara *et al.* (2007) are marked by $\star$. Values obtained by the current LESns-R are indicated by open symbols: □, case CU6; △, case CU10; and ○, case CU18. Values obtained by the current dynamic surface-modeled approach (LESns-M) with the wave-kinematics-dependent model for σ_{η}^{Δ} are indicated by solid symbols: ■, case CU6; ▲, case CU10; and ●, case CU18.

Conclusion: LES can be still much improved using "first principles constraints" on model parameters

$$
\begin{aligned}
\overline{\widetilde{\Psi(q)}} &= \Psi(\bar{\tilde{q}}) + \Psi_{\mathrm{mod}}(\bar{\tilde{q}}, \alpha\Delta, C_1, C_2 \ldots) \\
&= \overline{\Psi(\tilde{q})} + \overline{\Psi_{\mathrm{mod}}(\tilde{q}, \Delta, C_1, C_2 ..)}
\end{aligned}
$$

$$
\begin{aligned}
F_i = &-\iint_S \bar{\tilde{p}}^{w} \bar{\tilde{n}}_i \, dS + \iint_S \tau_{ij}^{w,2\Delta} \bar{\tilde{n}}_j \, dS \\
&- \iint_S \tilde{p}^{w} \tilde{n}_i \, dS + \iint_S \tau_{ij}^{w,\Delta} \tilde{n}_j \, dS
\end{aligned}
$$