Ciclo di seminari

Università degli studi di Roma "Tor Vergata"

May, 2013

Turbulence and the flow of kinetic energy in wind-turbine array boundary layers: LES studies

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WINDINSPIRE

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OVERVIEW:

Mercoledì:

- Introduction to fundamentals of Turbulence
- Intro to Large Eddy Simulation (LES)
- Intro to Subgrid-scale (SGS) modeling
- The dynamic SGS model
- Some sample applications from our group

Venerdì:

- Dynamic model for LES over rough multiscale surfaces
- LES studies of large wind farms

Acknowledgements: collaborations and discussions

- Marc Calaf (JHU & EPFL) - LES
- Prof. Johan Meyers (Univ. Leuven) - LES
- Marc B. Parlange (EPFL) – LES
- Dr. Richard Stevens (JHU & Twente) - LES
- Claire VerHulst (JHU) - LES

Also, related wind tunnel experiments:
- Raúl B. Cal (now at Portland State Univ.),
- Luciano Castillo (RPI, now Texas Tech),
- José Lebrón-Torres (RPI)

Funding: NSF CBET-0730922 (Energy for Sustainability)
NSF AGS-1045189 (Dynamic Meteorology)

Simulations: NCAR allocation (NSF)

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Renewables: low energy density

- solar, wind, wave energy
- need to cover "very, very big" areas
- wind: large wind-farms - on-land & off shore

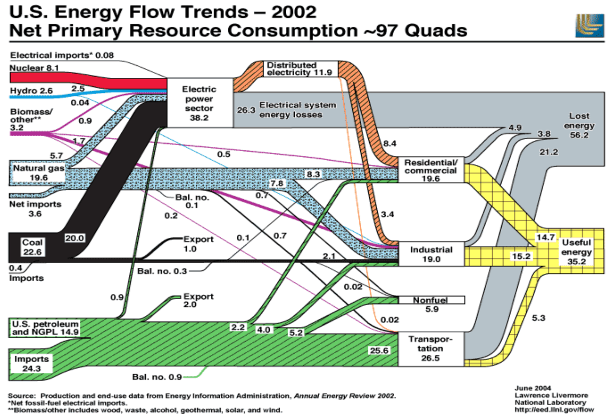
Land-based HAWT

Shell's Rock River windfarm in Carbon County, Wyoming, USA
Source: http://www.the-eic.com/News/Archive/2005/May/Article503.htm

Horns Rev HAWT
Copyright ELSAM/AS

Some thoughts on how much we (USA) consume:

(i.e. "a few solar collectors or little wind-mills simply won't do")



- Consider 3 TW US power consumption

- 3×10^{12} / 300×10^{6} = 10 kW per person in US

  (about 5-6 kW in Europe)

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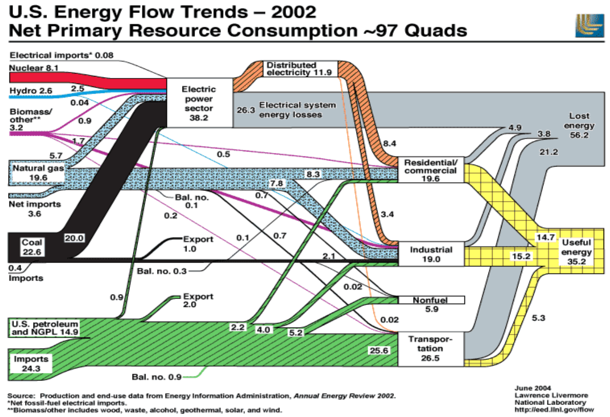
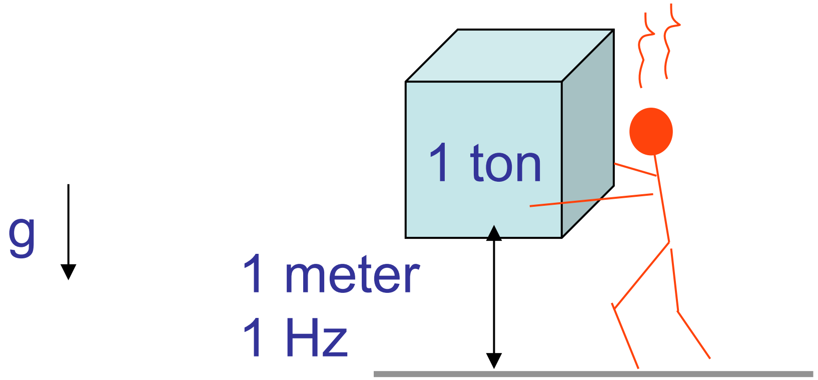


- Consider 3 TW US power consumption
- $3\text{x}10^{12}$ / 300 x 10^6 = 10 kW per person in US

  (about 5-6 kW in Europe)
- That is the same as lifting 1 ton by 1 meter every second!!



Some thoughts on how much we (USA) consume:

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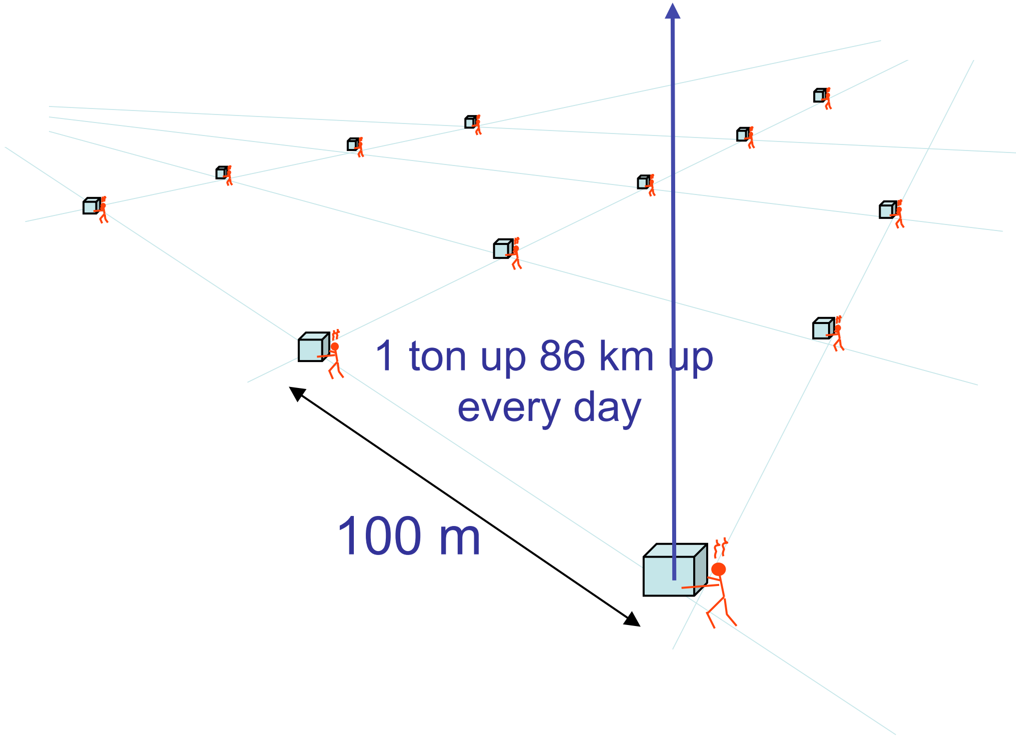
- Back to entire US (lower 48): 3.7 Million km²
- $$\sqrt{\frac{3.7\times10^{6}\times10^{6}}{300\times10^{6}}} \approx 100\text{ m}$$



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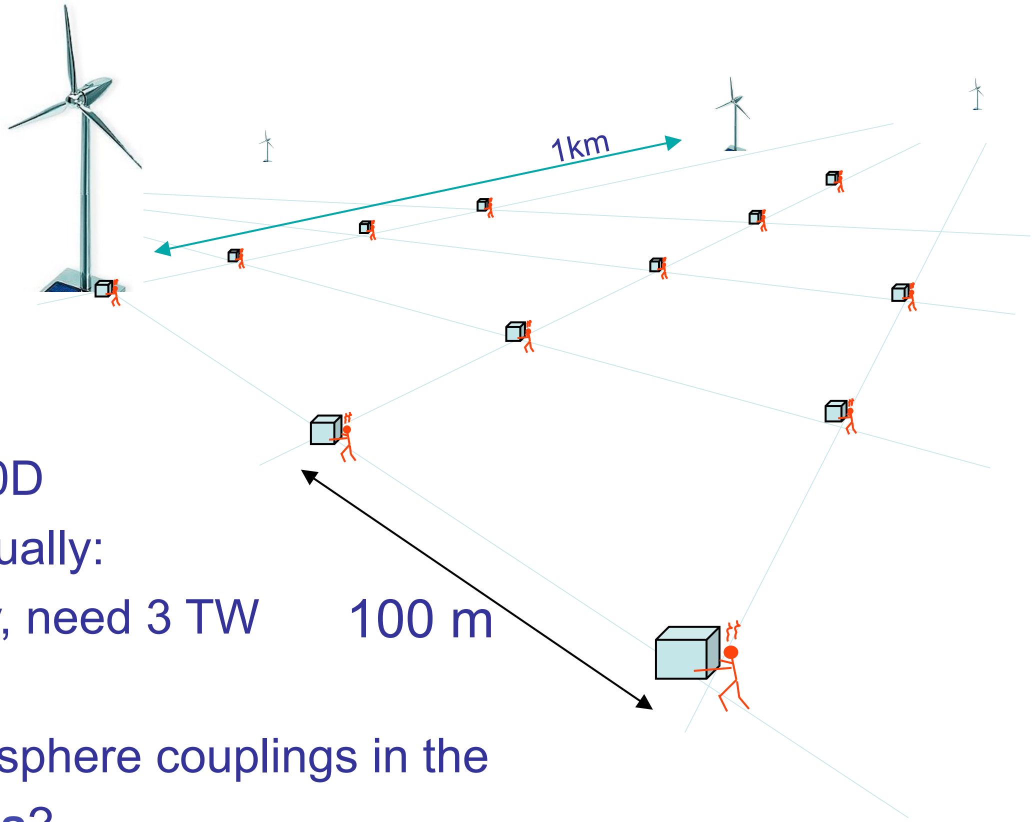


- Need one 1MW WindTurbine for 100 people (100 x 10 kW)
- 1 WindTurbine every 1km …. at 10D
- 3 Million wind turbines (doable actually: now US: 6 GW, av. power capacity, need 3 TW factor 500 = 2^9 - 9x3 = 27 years)
- What can we say about land-atmosphere couplings in the presence of **very large wind farms**?

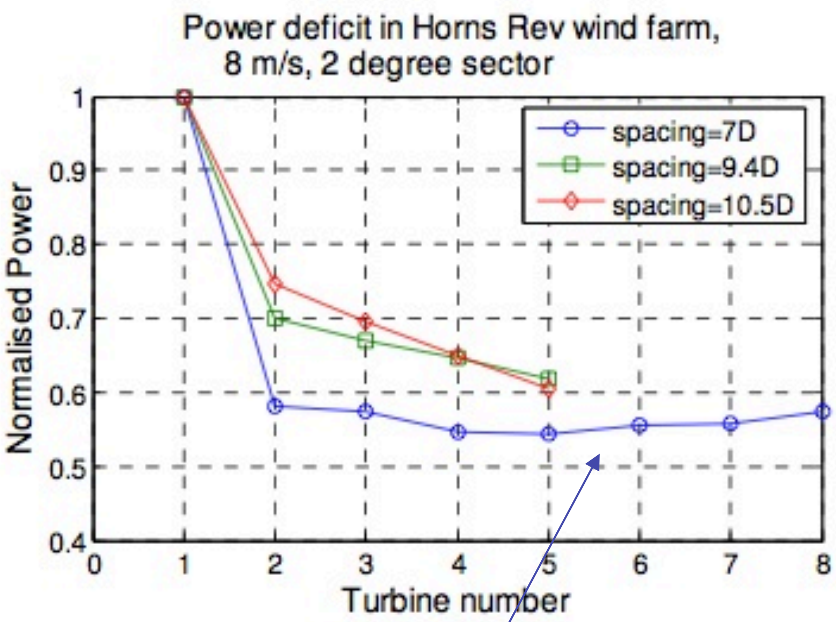
The windturbine-array boundary layer (WTABL)

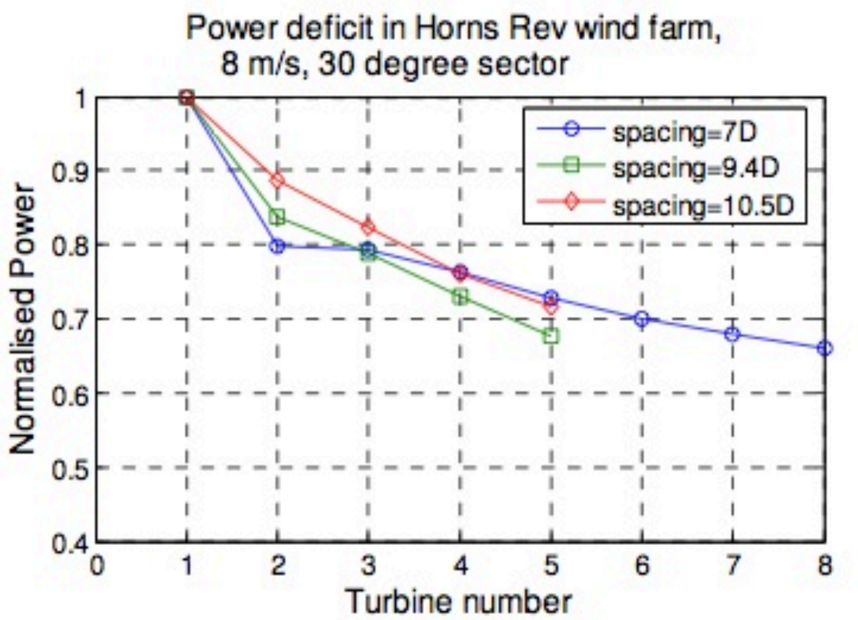
Horns Rev 1 owned by Vattenfall.
Photographer Christian Steiness

Arrays are getting bigger: when L > 10 H (H: height of ABL),
approach "fully developed" **FD-WTABL**

Related problem: Wind farm power degradation

(Wind farm operators: “10-15% wind farm underperformance problem”)



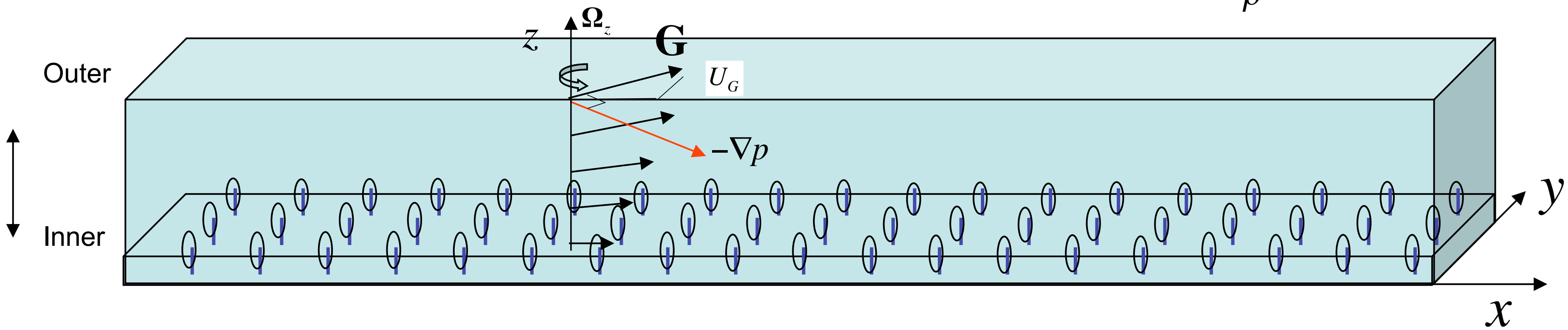


- asymptote ??
- how fast?
- is it really around 50%?
- mechanisms ?

“Modelling and measurements of wakes in large wind farms
Barthelemie, Rathmann, Frandsen, Hansen et al…
J. Physics Conf. Series 75 (2007), 012049

WTABL: Forcing by geostrophic wind

Above ABL (in mid-latitudes): geostrophic balance $2\mathbf{\Omega}\times\mathbf{G}-\frac{1}{\rho}\nabla P\approx 0$



Coupled through a stress $(u_*)^2$:

Outer length-scale: $H=\frac{u_*}{f}$ $\qquad f=2\Omega\sin\phi\approx 10^{-4}s^{-1}$ (mid-latitudes)

Inner length-scale: z_0

Inner-outer matching: $\frac{u(z)}{u_*}=\frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right)$ $\qquad \frac{G}{u_*}=\sqrt{A^2+\left[\frac{1}{\kappa}\ln\left(\frac{u_*}{fz_0}\right)-C\right]^2}$

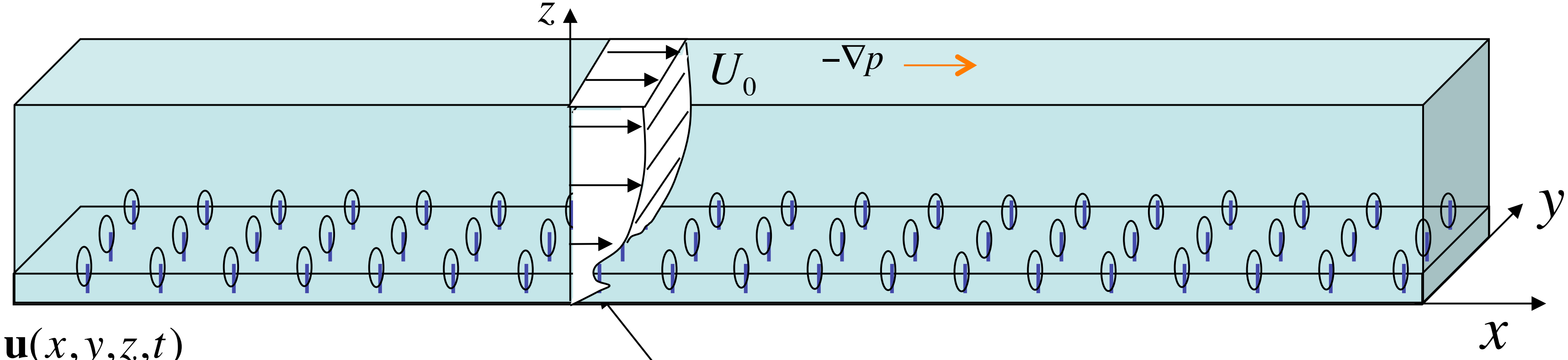
Given G and z_0 -> find u_* and H

+ effects of thermal stratification (will not be focused upon in this talk)

The "fully developed" pressure-grad-driven WTABL:

What is the generic structure of this specific type of boundary layer?

Pressure-gradient driven (half-channel flow)



$\mathbf{u}(x,y,z,t)$
DNS

$\tilde{\mathbf{u}}(x,y,z,t)$
LES

$\bar{\mathbf{u}}(x,y,z)$
RANS

$$U(z) = \langle \bar{u}(x,y,z) \rangle_{xy}$$

horizontal (canopy) average

What is the "averaged" velocity distribution? $U(z) = \langle \bar{u}(x,y,z) \rangle_{xy}$

Is there a "universal" WTABL profile?

What are profiles of shear stresses? $\tau_{xz}(z) = -\langle \overline{u'w'} \rangle_{xy}$

Near-surface turbulent boundary layer structure

classic turbulent boundary layer: momentum theory (Prandtl - von Karman)

$$0 = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \frac{d}{dz}\left(-\left\langle \overline{u'w'} \right\rangle_{xy} + viscous\ stress\right)$$

Integrate in z-direction and near surface: $\frac{dp_\infty}{dx} z \approx 0$

$$-\left\langle \overline{u'w'} \right\rangle_{xy}(z) \approx \text{constant} = viscous\ stress(z=0) \equiv u_*^2$$

Eddy-viscosity and mixing length model (Prandtl)

$$\nu_T \frac{d\langle u \rangle}{dz} = u_*^2 \;\rightarrow\; (\kappa u_* z)\frac{d\langle u \rangle}{dz} = u_*^2 \;\rightarrow\; \langle u \rangle = \frac{u_*}{\kappa}\ln(z) + C$$

Find C: For rough boundaries: u=0 at $z=z_0$ (effective roughness length):

$$\langle u \rangle = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_0}\right)$$

Example application of fully developed WTABL concepts and z_0: GCMs, mesoscale models, etc…

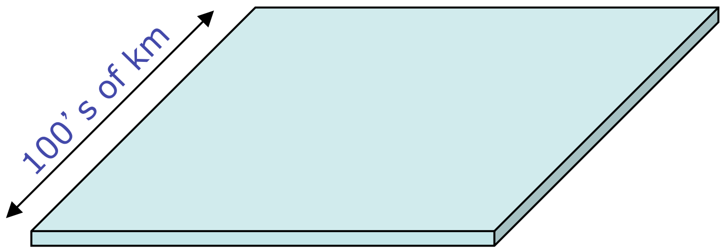
Keith et al. "The influence of large-scale wind power on climate" PNAS (2004)

Barrie & Kirk-Davidoff: "Weather response to management of large Wind turbine array", Atmos. Chem. Phys. **10**, 769-775, 2010

Use z_0 ~ 0.8 m - using "Lettau's formula" (ad-hoc geometric arguments…)

Grid-spacings 100's of km, first vertical point ~ 80m

"horizontally averaged structure"



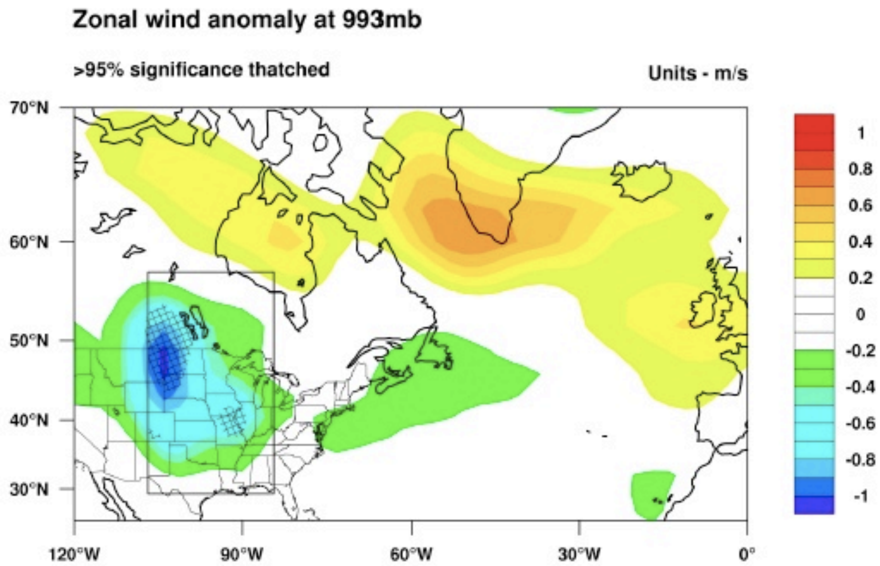


Fig. 1. 993 mbar zonal wind anomaly. The mean difference in the eastward wind in the lowest model level between the control and perturbed model runs highlights regions of atmospheric modification. Regions where significance exceeds 95%, as determined by a Student's t-test, are thatched. The wind farm is located within the rectangular box over the central United States and central Canada. Areas of the wind farm located over water are masked out during the model runs.

Outline of the talk: 3 related topics

- **Unraveling the generic ABL structure in presence of large wind farms – the WTABL.**

- **Surface fluxes of scalars in presence of large wind farms (more drying? heating? ..)**

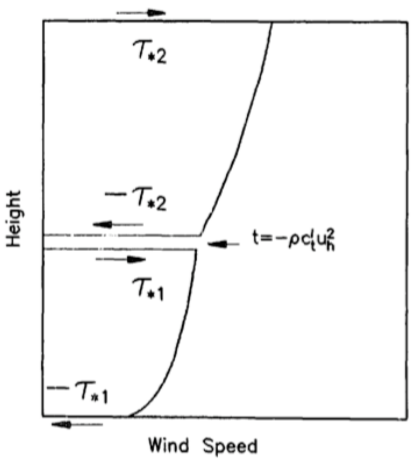
- **Fluxes of kinetic energy – how does kinetic energy get to the wind turbines? We propose a new flow-viz approach: energy transport tubes..**
  (presented also at EFMC-9 Rome, 2012)

Review: The fully developed WTABL & momentum theory

Horizontally averaged variables

Sten Frandsen, (J. Wind Eng & Ind Appl **39**, 1992):

$$0 = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \frac{d}{dz}\left(-\left\langle\overline{u'w'}\right\rangle_{xy} - \left\langle\bar{u}''\bar{w}''\right\rangle_{xy}\right) + \left\langle f_x\right\rangle_{xy}$$



$$s_x = \frac{L_x}{D}$$

$$s_y = \frac{L_y}{D}$$

Integrate in z-direction:

If top of WT canopy still

falls in the "surface layer", where $\frac{dp}{dx}z \approx 0$

and if wakes have "diffused" so that $\left\langle\bar{u}''\bar{w}''\right\rangle_{xy} \approx 0$

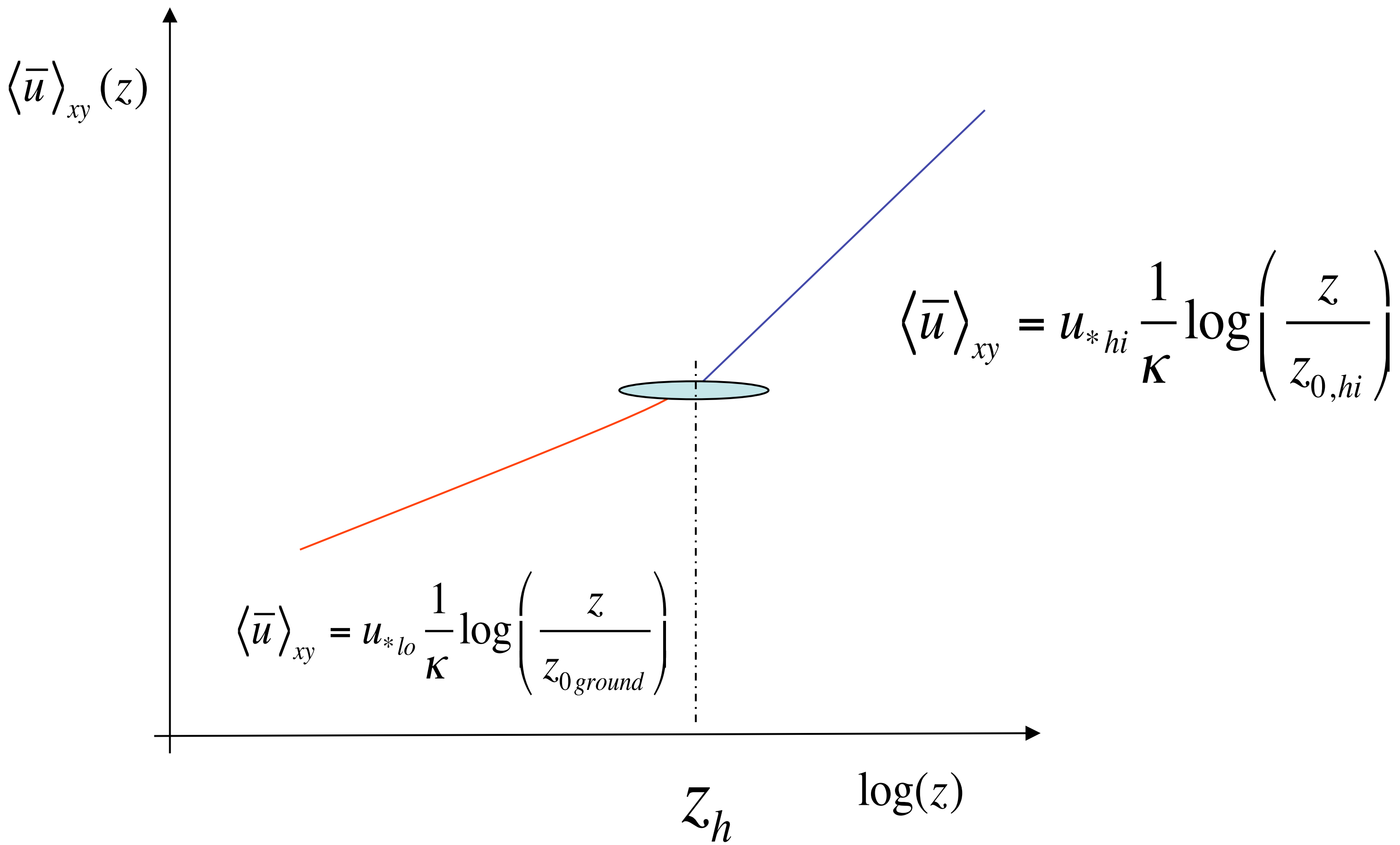
$$-\left\langle\overline{u'w'}\right\rangle_{xy}(z_{top}) \approx -\left\langle\overline{u'w'}\right\rangle_{xy}(z_{bottom}) + \frac{1}{2}C_T\frac{A_{disk}}{A_{xy}}U_h^2$$

$$u_{*\,hi}^2 \approx u_{*\,lo}^2 + \frac{1}{2}C_T\frac{\pi}{4s_xs_y}U_h^2$$

Review: The fully developed WTABL & momentum theory

$$u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2$$

Frandsen 1992 (also Newman 1977): postulated the existence of 2 log laws



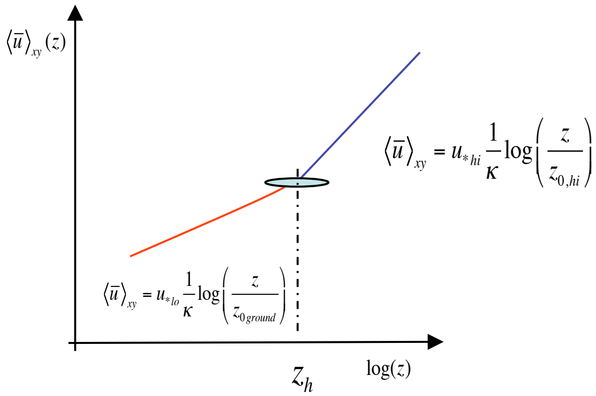
Review: The fully developed WTABL & momentum theory

S. Frandsen 1992, Frandsen et al. 2006:

Knowns: $u_{*hi},\ z_{0,ground},\ C_T,\ s_x,\ s_y$

3 unknowns: $z_{0,hi},\ U_h,\ u_{*lo}$

$$\begin{cases} u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2 \\ U_h = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,hi}}\right) \\ u_{*hi} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,hi}}\right) = u_{*lo} \frac{1}{\kappa} \log\left(\frac{z_h}{z_{0,ground}}\right) \end{cases}$$



Solve for effective roughness:

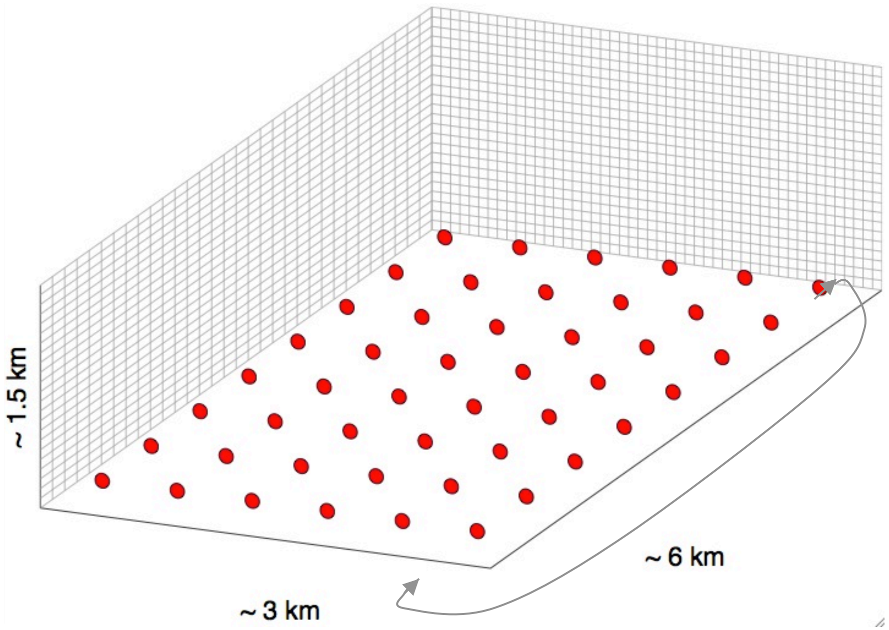
$$z_{0,hi} = z_h \exp\left(-\kappa \left[\frac{\pi C_T}{8 s_x s_y} + \left(\frac{\kappa}{\ln(z_h / z_{0,ground})} \right) \right]^{-1/2} \right)$$

Next: performe Large Eddy Simulations (LES) of WTABL
Simulations setup:

- LES code: horizontal pseudo-spectral (periodic B.C.), vertical: centered 2nd order FD (Moeng 1984, Albertson & Parlange 1999, Porté-Agel et al. 2000, Bou-Zeid et al. 2005)

$$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$$

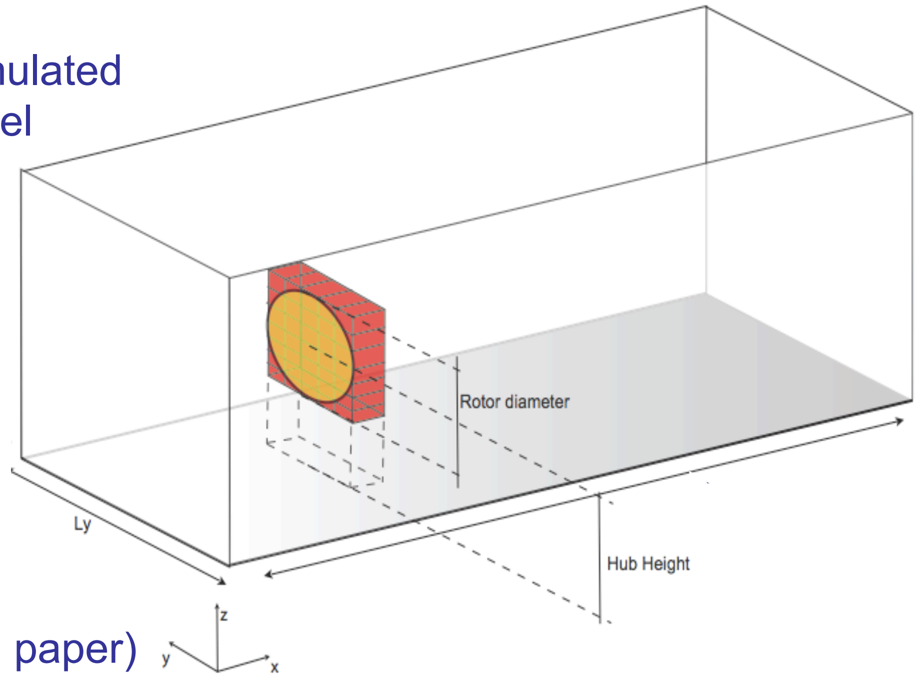
$$(N_x \times N_y \times N_z) = 128 \times 128 \times 128$$



- Horizontal periodic boundary conditions (only good for FD-WTABL)
- Top surface: zero stress, zero w
- Bottom surface B.C.: Zero w + Wall stress: Standard wall function relating wall stress to first grid-point velocity
- Scale-dependent dynamic Lagrangian model - eddy-viscosity closure but (*no* adjustable parameters)
- More details: Calaf, Meneveau & Meyers, "Large eddy simulation study of fully developed wind-turbine array boundary layers" Phys. Fluids. **22** (2010) 015110

Actuator disk modeling of turbines in LES

Jimenez et al., J. Phys. Conf. Ser. **75** (2007) simulated single turbine in LES using dynamic Smag. model



They used fixed reference (undisturbed) velocity:

$$f_{Tx} = -\frac{1}{2} C_T U_{ref}^2 \frac{\delta A_{yz}}{\delta V}, \quad C_T = 0.75$$

Here we use disk-averaged and time-averaged velocity, but local at the disk (see Meyers & Meneveau 2010, 48th AIAA conf., paper)

$$f_{Tx} = -\frac{1}{2} C_T \left(\frac{1}{1-a} \bar{U} \right)^2 \frac{\delta A_{yz}}{\delta V} = -\frac{1}{2} C_T' \bar{U}^2 \frac{\delta A_{yz}}{\delta V}$$

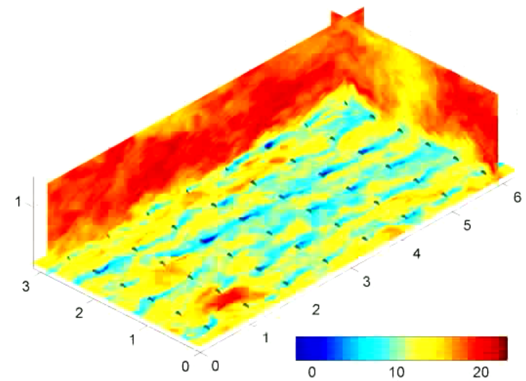
$$C_T = 0.75 \Rightarrow a \approx 0.25 \rightarrow C_T' = 1.33$$

Also, use first-order relax process to time-average:

$$\bar{U}(t) = (1-\varepsilon)\bar{U}(t-dt) + \varepsilon U_{disk}(t)$$

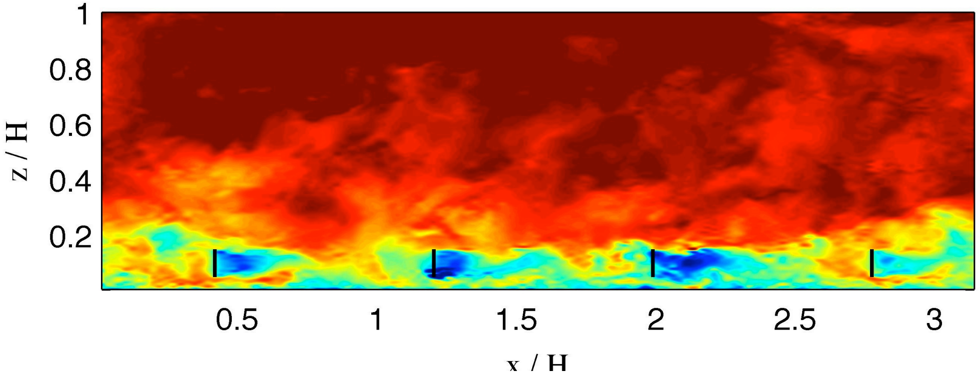
Suite of LES cases,

see Calaf et al. 2010, Phys. Fluids

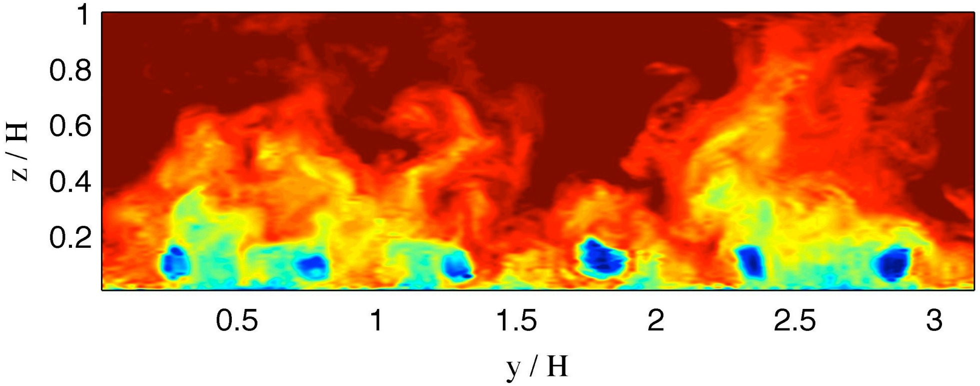


Instantaneous stream-wise velocity contours:

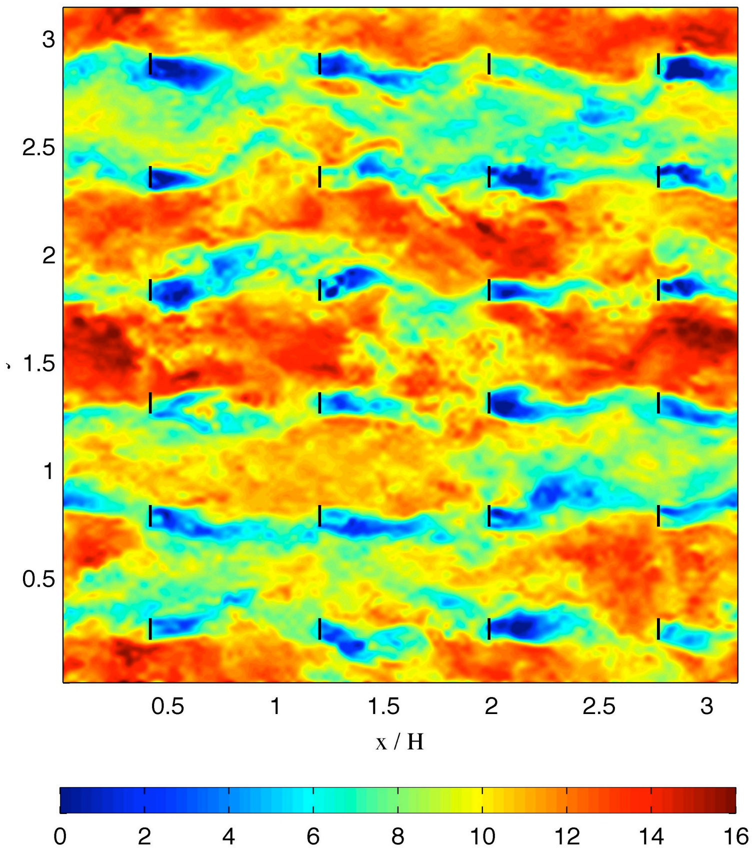
side-view



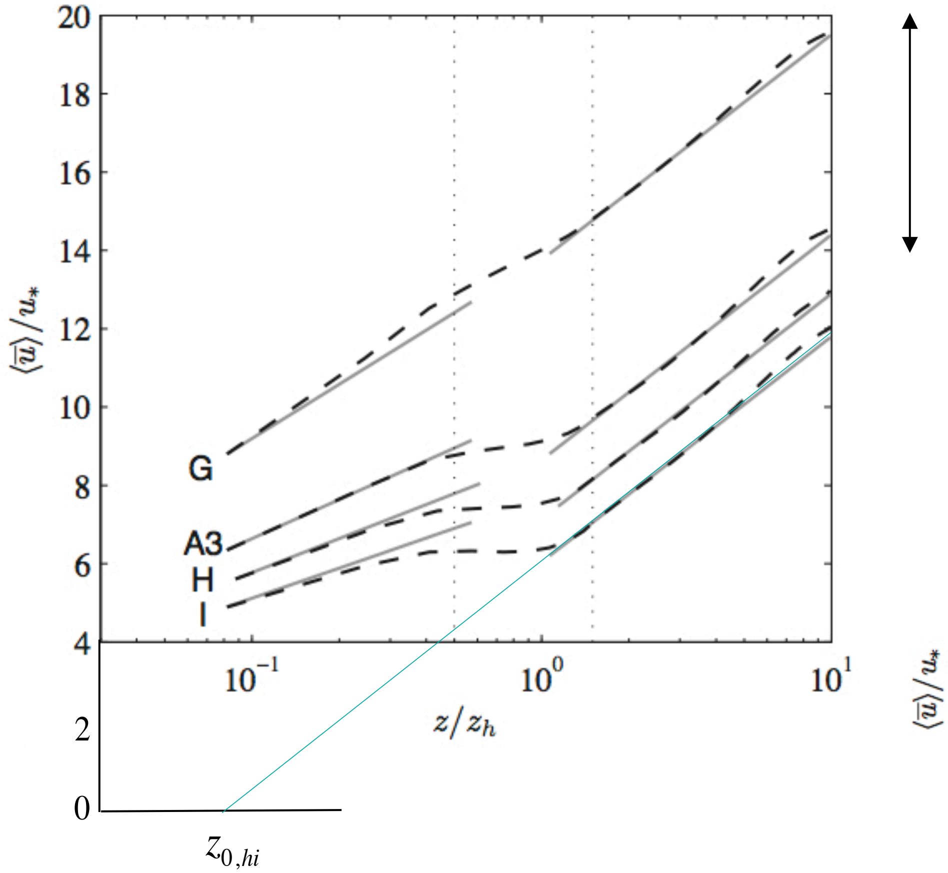
front-view



top-view



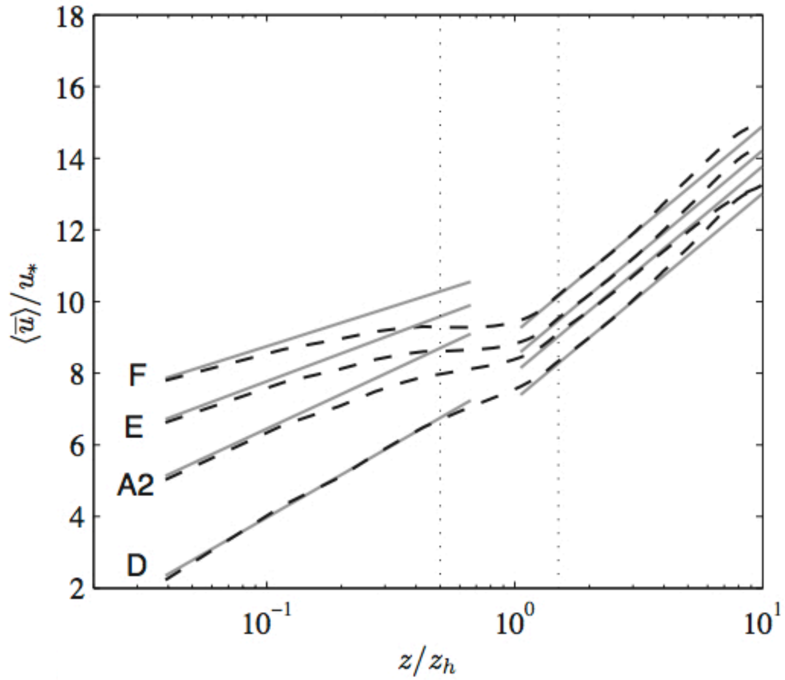
Measuring z_0 from LES (horizontally averaged)



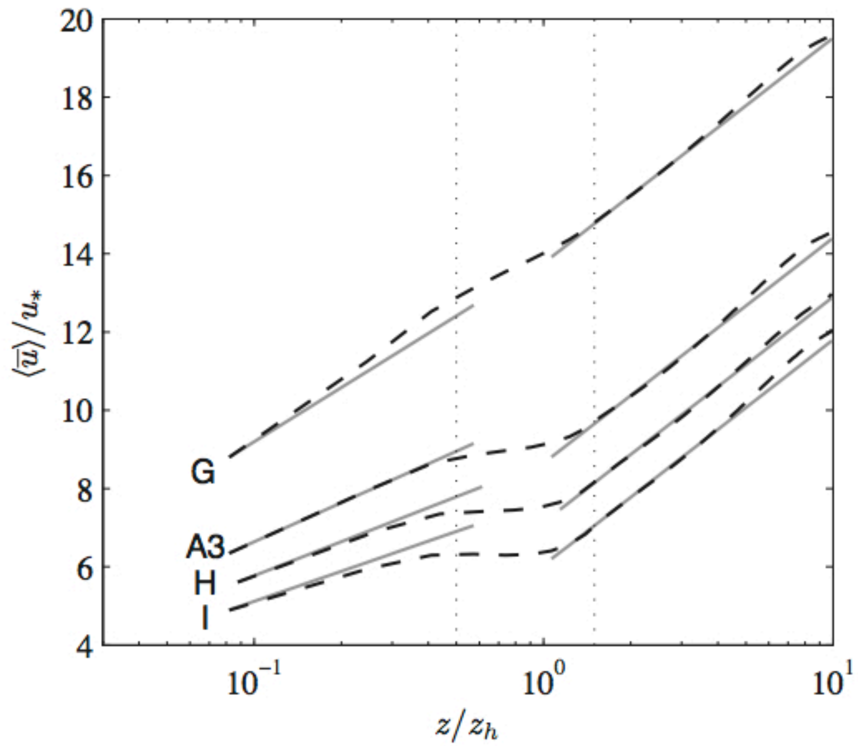
measure $z_{0,hi}$ from intercept

$$\langle \overline{u} \rangle_{xy} = u_{*hi} \frac{1}{\kappa} \log\left(\frac{z}{z_{0,hi}} \right)$$

(essentially the "Clauser plot" method)



"Wake upgrade" to Frandsen's top-down model



$$\left(\kappa u_* z_h + \nu_w\right)\frac{\partial\langle\overline{u}\rangle}{\partial z} = u_*^2$$

In wake, reduced slope:

$$\frac{\partial\langle\overline{u}\rangle}{\partial z} = \frac{1}{\kappa u_* z_h + \nu_w} u_*^2$$

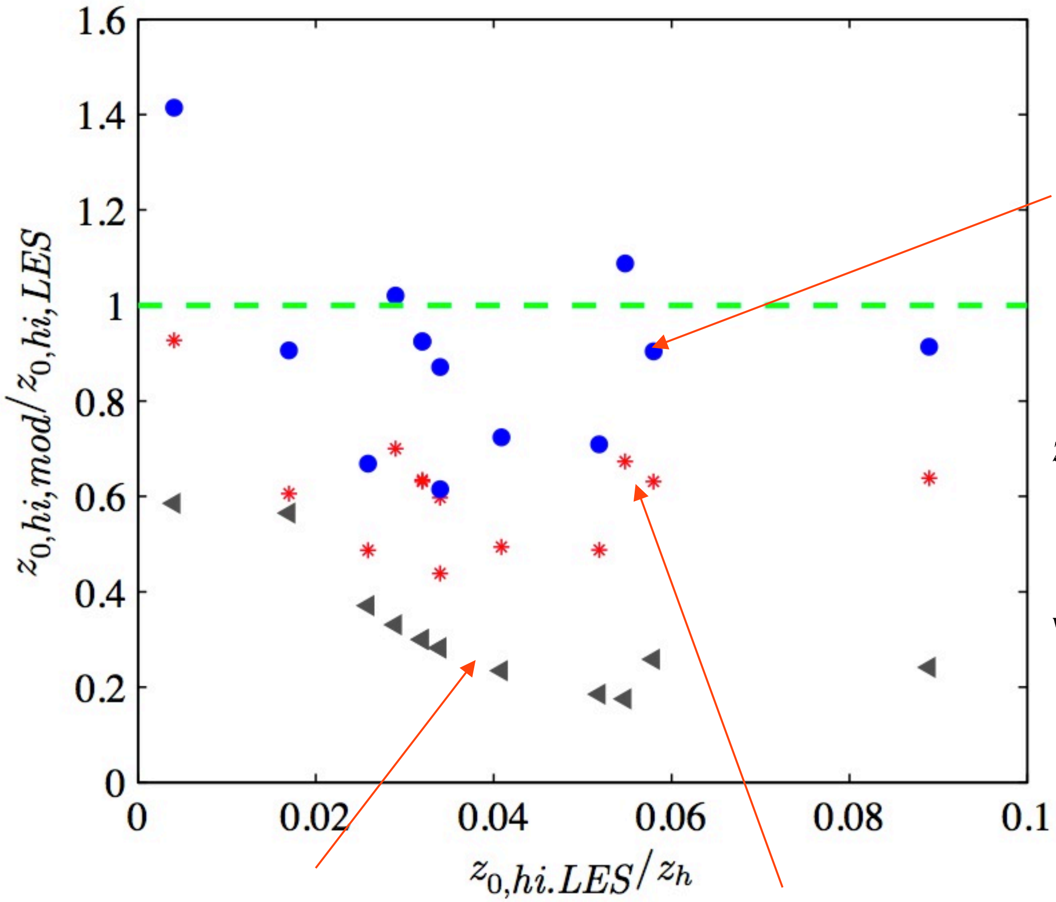
$$\nu_w = \sqrt{\tfrac{1}{2} c_{ft}} \langle\overline{u}\rangle D$$

$$\nu_w^* = \frac{\sqrt{\tfrac{1}{2} c_{ft}} \langle\overline{u}(z_h)\rangle D}{\kappa u_* z_h} \approx 28\sqrt{\tfrac{1}{2} c_{ft}}$$

$$z_{0,hi} = z_h\left(1+\frac{D}{2z_h}\right)^{\beta}\exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1-\frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

where $\beta = \dfrac{28\sqrt{\tfrac{1}{2} c_{ft}}}{1+28\sqrt{\tfrac{1}{2} c_{ft}}}$,

Comparison of LES results with models



Circles: improved Frandsen model
Calaf, Meneveau & Meyers,
(Phys. Fluids 2010, **22**)

$$z_{0,hi} = z_h\left(1+\frac{D}{2z_h}\right)^{\beta}\exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y}+\left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1-\frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

where $\beta = \frac{\nu_w^*}{1+\nu_w^*}$, and $\nu_w^* = \frac{\nu_T}{\kappa u_* z_h}$, eddy viscosity due to wake

Triangles: Lettau formula

Asterisks: Frandsen et al. (2006) formula

$$z_{0,hi} = z_h \exp\left(-\kappa\left[\frac{\pi C_T}{8 s_x s_y}+\left(\frac{\kappa}{\ln(z_h/z_{0,ground})}\right)^2\right]^{-1/2}\right)$$

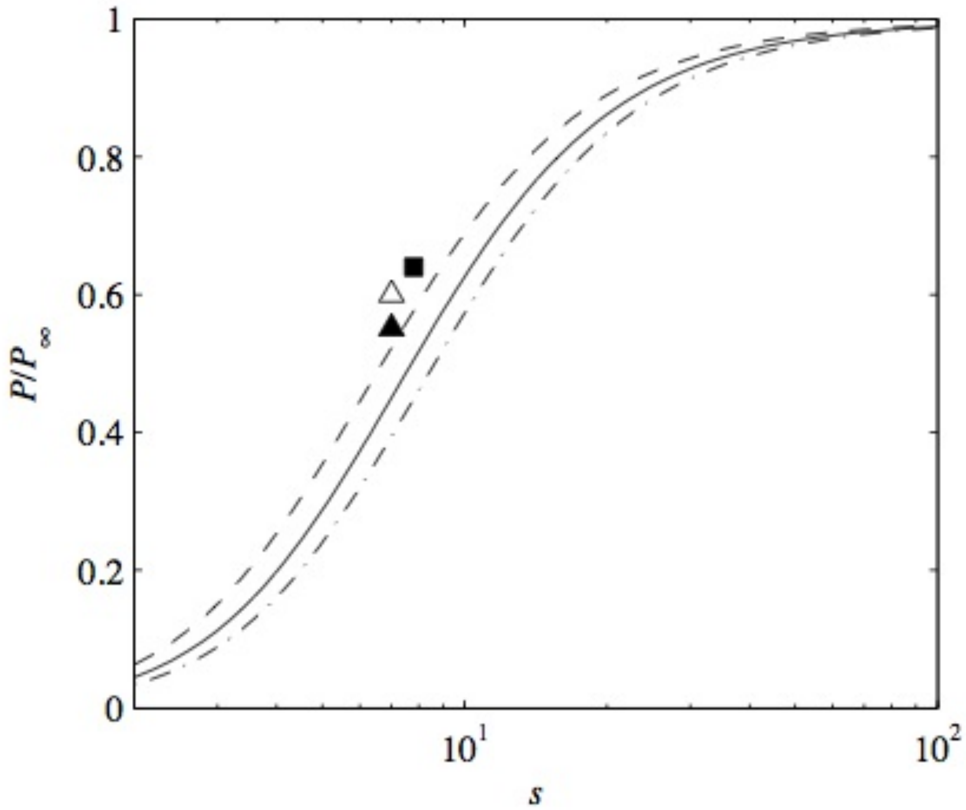
Sample application: What is the most optimal spacing s_{opt} between wind turbines in the fully developed WTABL?

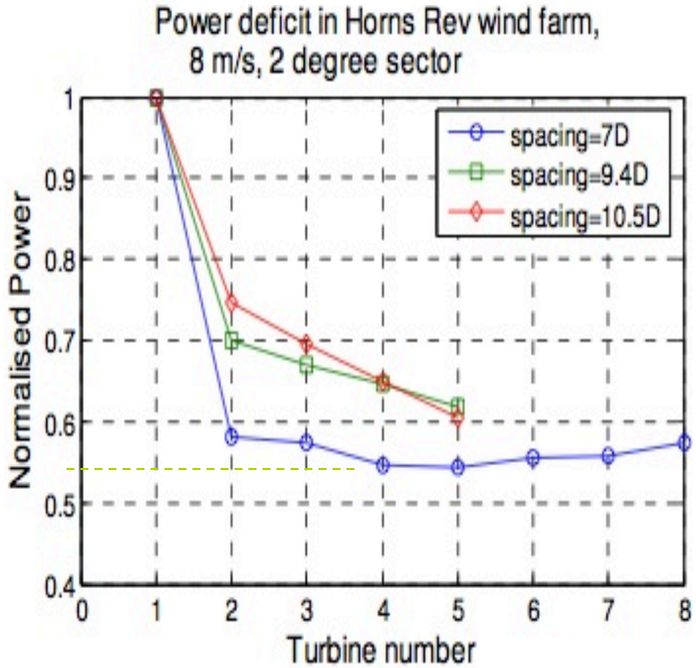
Meyers & Meneveau (2012), Wind Energy **15**, 305-317

Horns Rev 1 owned by Vattenfall.
Photographer Christian Steiness

Use of "top-down" model ($z_{0,hi}$) to find s_{opt}

For given s, $z_{0,lo}$, D, z_h, C_T evaluate P
divide by P_∞ of single WT ($z_{0,hi} = z_{0,lo}$ case)





From: Barthelmie et al.
J. of Phys. Conf. (2007)

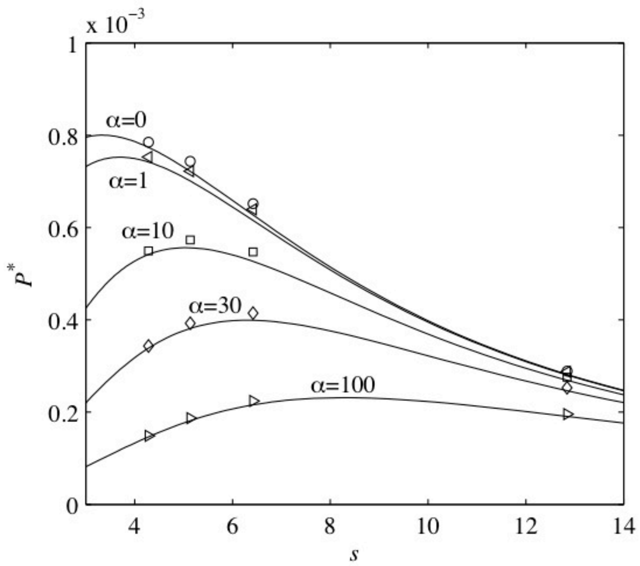
Use of "top-down" model ($z_{0,hi}$) to find s_{opt}

Power per unit cost:

$$P^* = \frac{Power-per-turbine}{Cost-per-turbine} = \frac{C_P \frac{\rho}{2} U_h^3 \frac{\pi}{4} D^2}{Cost_{land}(\$/m^2) \times s_x s_y D^2 + Cost_{turb}(\$)}$$

Define dimensionless ratio: $\alpha = \dfrac{Cost_{turb} / \left(\frac{\pi}{4} D^2\right)}{Cost_{land}}$

$$P^* \propto \frac{C_P}{4 s_x s_y / \pi + \alpha} \left(\frac{u_{*,hi}}{G}\right)^3 \left(\frac{U_h}{u_{*,hi}}\right)^3$$

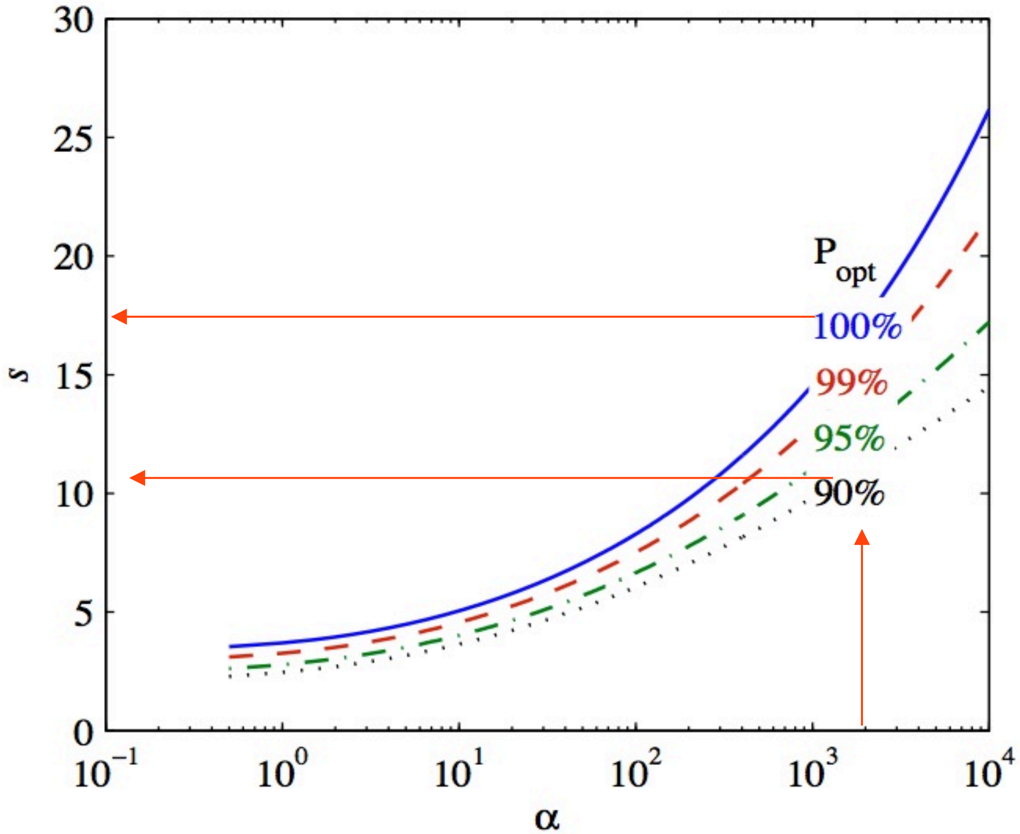


Meyers & Meneveau (2012), Wind Energy **15**, 305-317

$$z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1 - \frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$$

where $\beta = \dfrac{28\sqrt{\frac{1}{2}c_{ft}}}{1 + 28\sqrt{\frac{1}{2}c_{ft}}}$,

Use of "top-down" model ($z_{0,hi}$) to find s_{opt}



Typical **alpha**~ 2,000 (Texas land costs …)

Valid for:
Neutral stability and
"region II" operation: C_T and C_P independent of U_h

At common s ~ 7D, 10-20% suboptimal - "use ~15 D instead"

Meyers & Meneveau (2012), Wind Energy **15**, 305-317

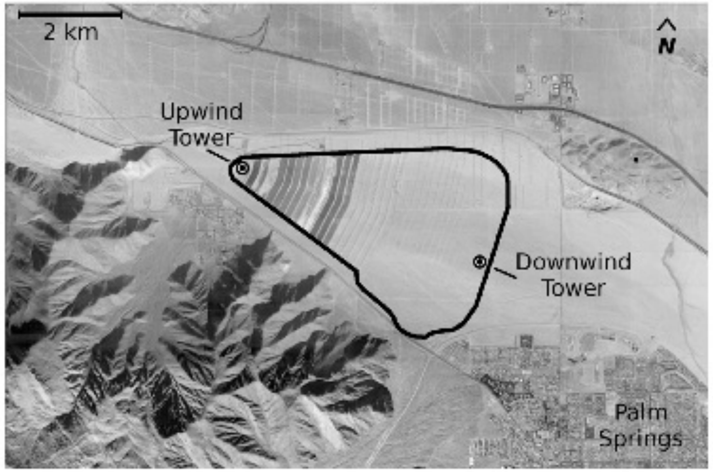
Effects of large wind farms on scalar fluxes:
Heat and moisture

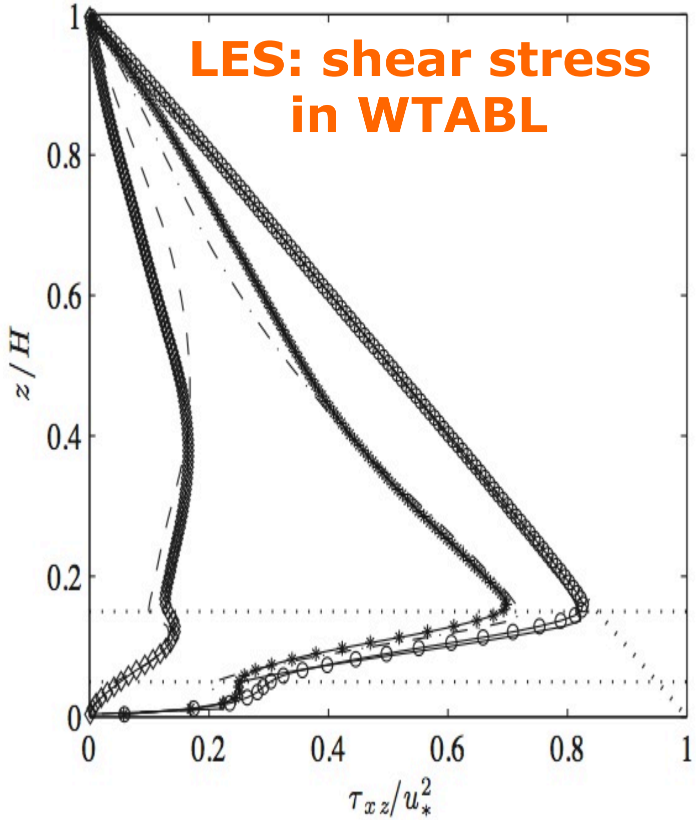
Observations: increased fluxes

(evaporation, drying, ??)

Baidya-Roy & Traiteur PNAS 2010

in San Gorgonio wind farm (CA)





But: Farm increases turbulence in wakes and $u_{*,hi}$ is increased, but $u_{*,lo}$ is DECREASED. Net effect?

First step: Passive scalar LES

(no Boussinesq term in momentum equations)

(M. Calaf, Parlange & M, Physics of Fluids December 2011)

Velocity (hub-height)

Temperature

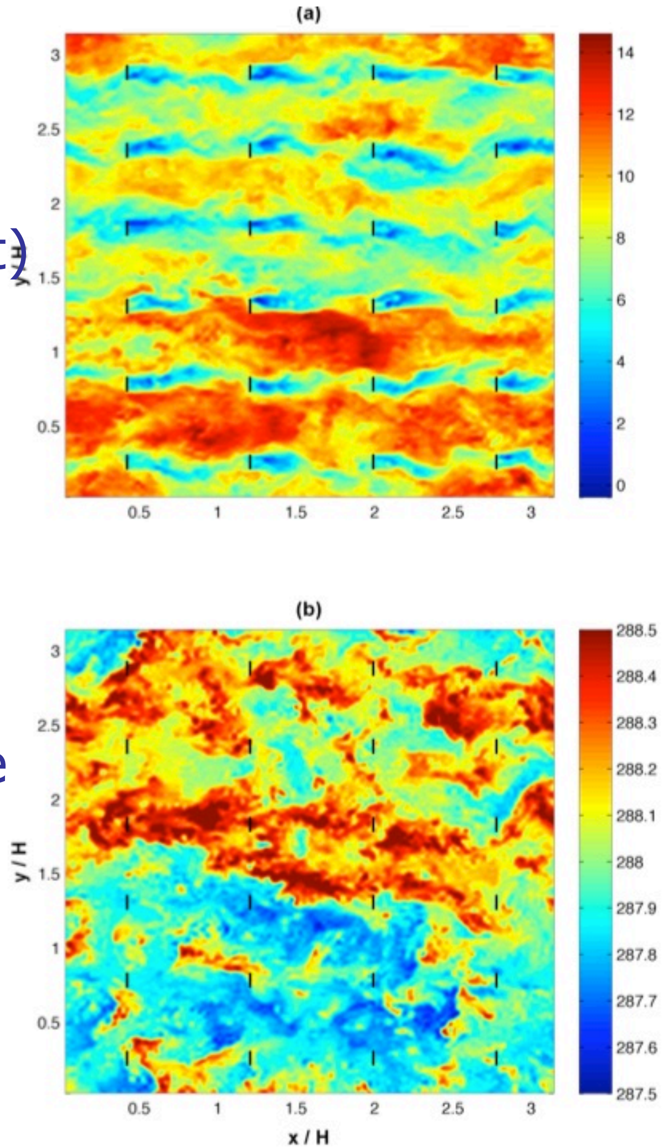
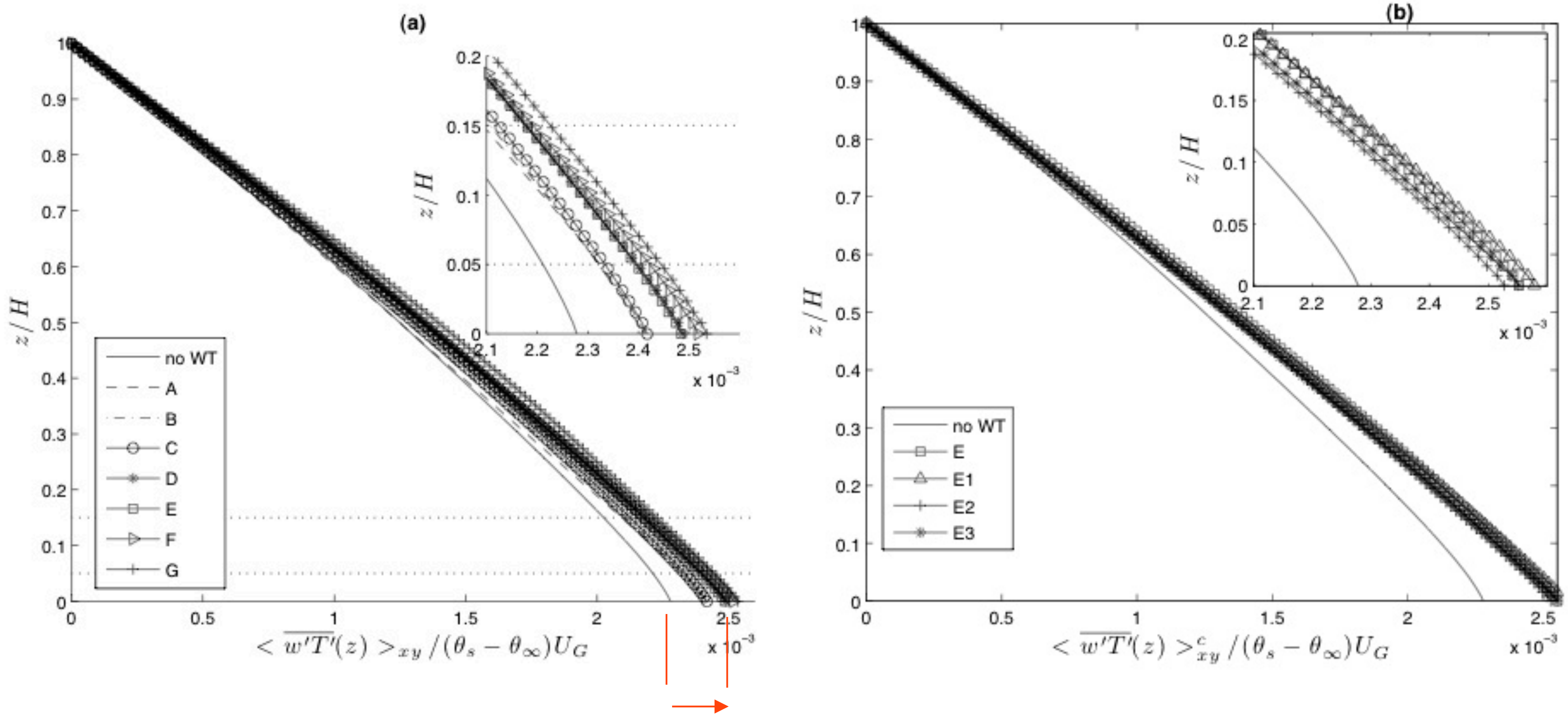


TABLE I: Table summarizing parameters of the various LES cases.

	s_x	s_y	$4s_xs_y/\pi$	N_t	C_T	C'_T	c_{ft}	c'_{ft}
A	7.85	$s_x/1.5$	52.3	4×6	0.45	0.6	0.009	0.011
B	7.85	$s_x/1.5$	52.3	4×6	0.52	0.7	0.01	0.013
C	7.85	$s_x/1.5$	52.3	4×6	0.6	0.88	0.012	0.017
D	7.85	$s_x/1.5$	52.3	4×6	0.68	1.13	0.013	0.022
E	7.85	$s_x/1.5$	52.3	4×6	0.75	1.33	0.014	0.025
F	7.85	$s_x/1.5$	52.3	4×6	0.82	1.63	0.016	0.031
G	7.85	$s_x/1.5$	52.3	4×6	0.88	2	0.017	0.038
$E1$	7.85/2	7.85/1.5	26.15	8×6	0.75	1.33	0.029	0.051
$E2$	7.85	$s_x/3$	26.15	4×12	0.75	1.33	0.029	0.051
$E3$	7.85/2	$s_x/1.5$	13.1	8×12	0.75	1.33	0.057	0.1

Horizontally averaged scalar flux from LES



10-15% increase, not strongly dependent on loading

Horizontally averaged scalar balance: constant flux

$$q_H^{WT} = \begin{cases} \frac{u_{*,lo}\kappa z}{Pr_T^{WT}} \frac{d[\theta_s - \tilde{\theta(z)}]}{dz} & (z_{0,s} < \mathrm{z} < z_h\text{-D}/2) \\ \frac{(u_{*,lo}\kappa z + \sqrt{c_{ft}/2}\langle\tilde{u}(z_h)\rangle D)}{Pr_T^{WT}} \frac{d[\theta_s - \tilde{\theta(z)}]}{dz} & (z_h - D/2 < \mathrm{z} < z_h) \\ \frac{(u_{*,hi}\kappa z + \sqrt{c_{ft}/2}\langle\tilde{u}(z_h)\rangle D)}{Pr_T^{WT}} \frac{d[\theta_s - \tilde{\theta(z)}]}{dz} & (z_h < \mathrm{z} < z_h + D/2) \\ \frac{u_{*,hi}\kappa z}{Pr_T^{WT}} \frac{d[\theta_s - \tilde{\theta(z)}]}{dz} & (z_h + D/2 < \mathrm{z} < H) \end{cases}$$

Horizontally averaged scalar balance: constant flux

For imposed geostrophic wind,
ratio of scalar flux with and without wind farm

$$\frac{q_H^{WT}}{q_H^0} = \frac{u_{*,hi}}{u_*}\frac{Pr_T^0}{Pr_T^{WT}}\left\{\frac{ln\left(\frac{u_{*,hi}}{fz_{0,s}}\right) - kC + \frac{u_{*,hi}}{u_{*,lo}} ln\left[\frac{z_h}{z_{0,s}}\left(1-\frac{D}{2z_h}\right)^{\beta}\right] - ln\left[\frac{z_h}{z_{0,s}}\left(1+\frac{D}{2z_h}\right)^{\beta}\right]}{ln\left(\frac{u_*}{fz_{0,s}}\right) - kC}\right\}$$

Term 1: factor due to
increased turbulence in wake

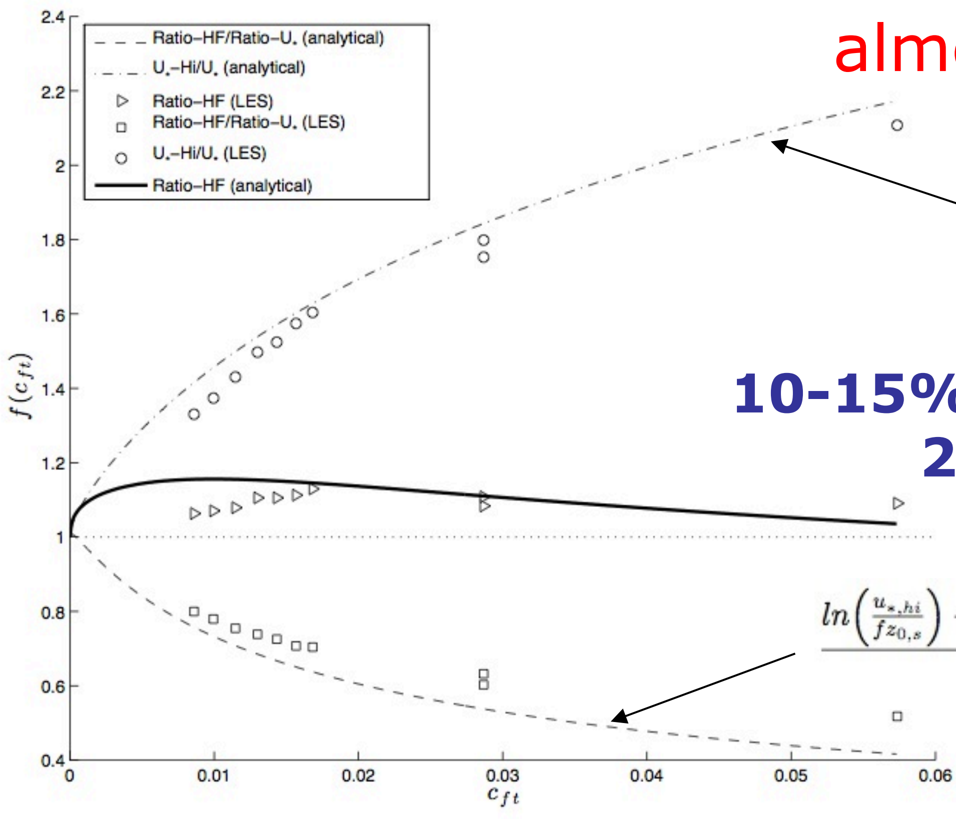
Term 2: factor due to
“dead water region”, screened
region below WT

$$\frac{u_{*,hi}}{u_*} = \left[1 - \frac{\ln\left(\frac{z_{0,hi}}{z_{0,lo}}\right)}{\ln\left(\frac{U_G}{fz_{0,lo}}\right) - C_*}\right]^{-1}$$

LES measured and model terms as function of loading

(neutral stratification)

For imposed geostrophic wind,
ratio of scalar flux with and without wind farm (symbols=LES)



almost double of u* above

$$\frac{u_{*,hi}}{u_*} = \left[1 - \frac{\ln\left(\frac{z_{0,hi}}{z_{0,lo}}\right)}{\ln\left(\frac{U_G}{f z_{0,lo}}\right) - C_*} \right]^{-1}$$

10-15% increase arises due to 2 opposing trends

$$\frac{ln\left(\frac{u_{*,hi}}{f z_{0,s}}\right) - kC + \frac{u_{*,hi}}{u_{*,lo}} ln\left[\frac{z_h}{z_{0,s}}\left(1 - \frac{D}{2z_h}\right)^{\beta}\right] - ln\left[\frac{z_h}{z_{0,s}}\left(1 + \frac{D}{2z_h}\right)^{\beta}\right]}{ln\left(\frac{u_*}{f z_{0,s}}\right) - kC}$$

almost half of u* below

Where does the kinetic energy at wind turbines come from?

Examine fluxes of kinetic energy

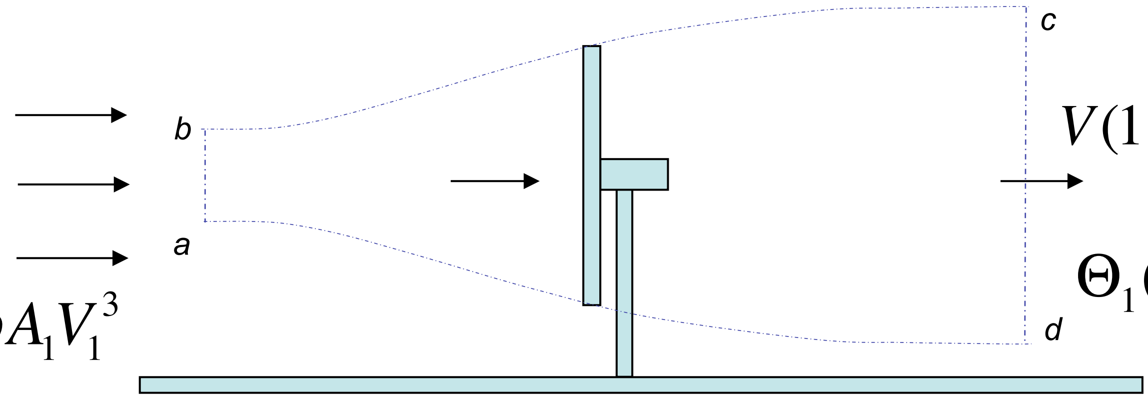
Classic Betz analysis and limit:
Horizontal fluxes of kinetic energy

ideal potential flow

$V(1-a)$

ideal potential flow

V



$V(1-2a)$

$$\Theta_1(x_1) = \frac{1}{2}\rho A_1 V_1^3$$

$$\Theta_1(x_2) = \frac{1}{2}\rho A_2 V_2^3$$

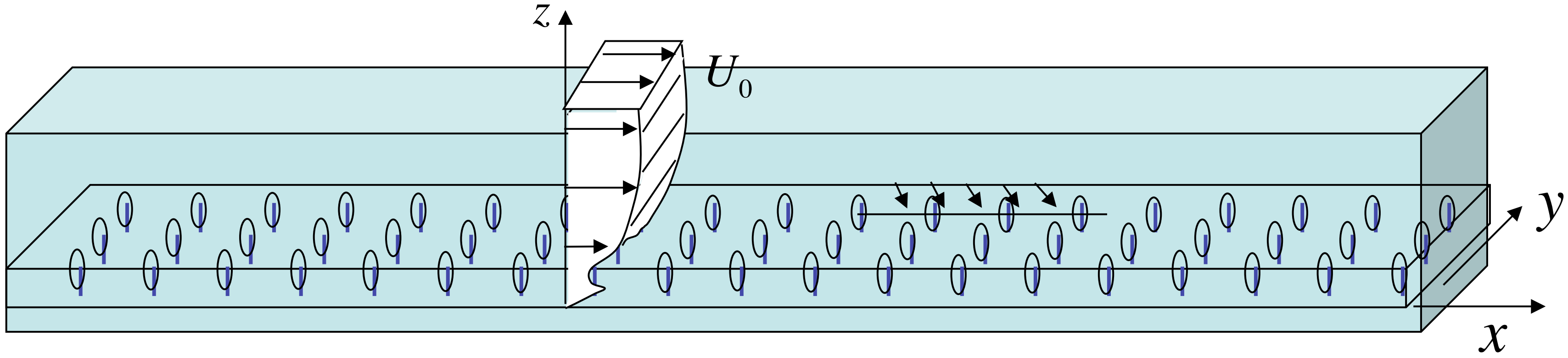
For **single** wind turbine, extracted power =
difference in front and back fluxes of kinetic energy

$$P = \frac{1}{2}\rho\left(A_1V_1^3 - A_2V_2^3\right) = \frac{1}{2}C_P\rho A_d V_1^3$$

$$a_{opt} = \frac{1}{3}, \qquad C_{P-\max} = \frac{16}{27}$$

+ wake models, etc..

Mechanical Energy in horizontally averaged horizontal flow and fluxes in FD-WTABL



For **multiple** (∞) wind turbines in fully developed WTABL,
extracted power = must be brought to wind turbine
by vertical fluxes of kinetic energy:

$$\frac{d\frac{1}{2}\langle u\rangle_{xy}^{2}}{dt} = -\varepsilon_{turb} - \varepsilon_{disp} - \frac{d}{dz}\left(\langle\overline{u'w'}\rangle_{xy}\langle u\rangle_{xy} + \langle\overline{u}''\overline{w}''\rangle_{xy}\langle u\rangle_{xy}\right) - \langle u\rangle_{xy}\frac{1}{\rho}\frac{dp_\infty}{dx} - P_T(z)$$

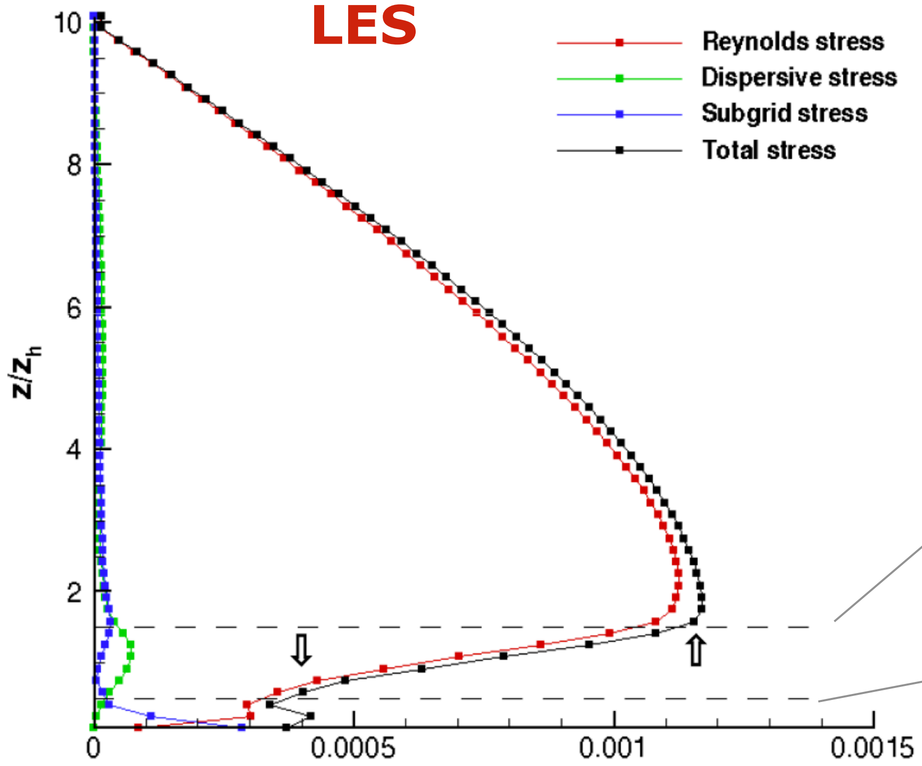
Turbulence-mediated flux of
Kinetic energy in horizontally
averaged mean flow

$$\Theta(z) = -\langle\overline{u'w'}\rangle_{xy}\langle\overline{u}\rangle_{xy}$$

Measure fluxes of KE from LES (+ experiments):

$$\Theta(z) = -\langle u'w' \rangle_{xy} \langle \overline{u} \rangle_{xy}$$

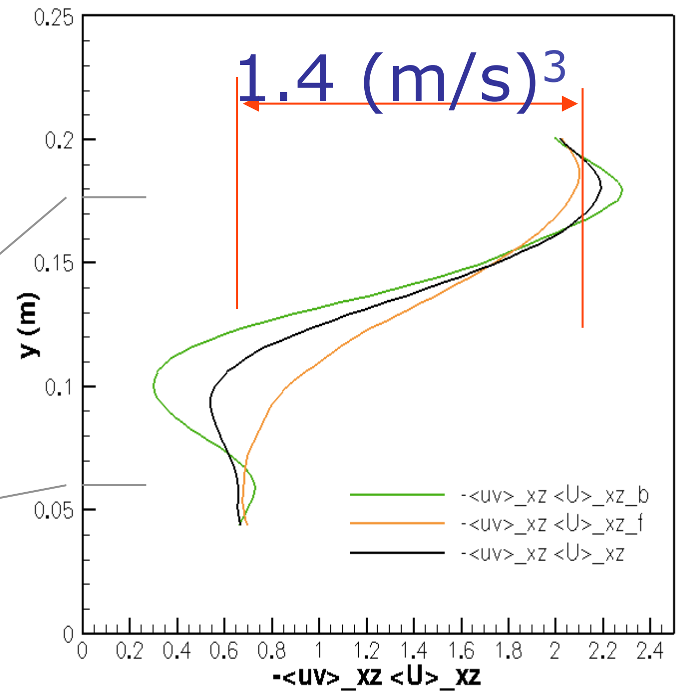
LES



$$\Theta(z) = -\langle u'w' \rangle_{xy} \langle \overline{u} \rangle_{xy}$$

Wind tunnel measurements

Cal et al.: J. Renewable and Sustainable Energy **2**, 2010



"Flow visualization using momentum and energy transport tubes and applications to turbulent flow in wind farms"
J. Meyers & C.M. 2013 (JFM)

Consider linear momentum transport of mean flow, in statistically steady turbulent flow

$$F_{m,j} = \overline{u}_1 \; \overline{u}_j + \overline{u_1' u_j'} \; - \; 2\nu \overline{S}_{1j}$$

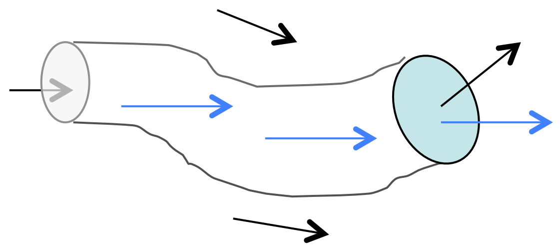
Mean-flow x_1-momentum transport **vector field**

↓

Tangent lines – bundles – tubes

$$\frac{\partial}{\partial x_j}(\rho F_{m,j}) = -\frac{\partial \overline{p}}{\partial x_1} + \rho \overline{f}_1$$

$$\iint_{A_2} \rho F_{m,j} n_j \, \mathrm{d}\mathbf{x} + \iint_{A_1} \rho F_{m,j} n_j \, \mathrm{d}\mathbf{x} = -\iiint_{\Omega} \left(\frac{\partial \overline{p}}{\partial x_1} \right) \mathrm{d}\mathbf{x} + \iiint_{\Omega} \rho \overline{f}_1 \mathrm{d}\mathbf{x}$$



Consider total mechanical energy transport of mean flow, in statistically steady turbulent flow

$$E = \frac{1}{2}\overline{u}_i\overline{u}_i + \frac{1}{\rho}\hat{p}$$

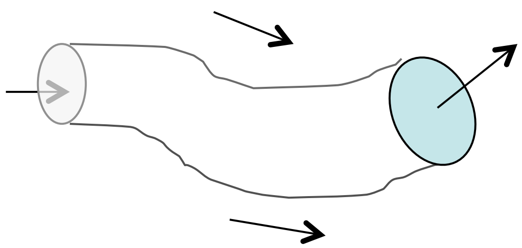
$$\overline{F}_{E,j} = E\ \overline{u}_j + \overline{u_i' u_j'}\ \overline{u}_i - 2\nu\overline{S}_{ij}\ \overline{u}_i$$

Mean-flow total energy transport vector field

Tangent lines – bundles – tubes

$$\frac{\partial}{\partial x_j}(\rho\overline{F}_{E,j}) = -\rho\overline{u_i' u_j'}\frac{\partial\overline{u}_i}{\partial x_j} - 2\mu\overline{S}_{ij}\overline{S}_{ij} + \rho\overline{u}_i\overline{f}_i$$

$$\iint_{A_2}\rho\overline{F}_{E,j}n_j\,\mathrm{d}\mathbf{x} + \iint_{A_1}\rho\overline{F}_{E,j}n_j\,\mathrm{d}\mathbf{x} = -\iiint_{\Omega}\left(2\mu\overline{S}_{ij}\overline{S}_{ij} - \rho\overline{u_i' u_j'}\frac{\partial u_i}{\partial x_j}\right)\mathrm{d}\mathbf{x} + \iiint_{\Omega}\rho\overline{u}_i(\overline{f}_i + f_{i,\infty})\mathrm{d}\mathbf{x}$$



Tutorial examples for laminar flows: momentum transport lines and tubes in **laminar Couette** flow:

$$F_{m,1} = (yU/h)^2$$

$$F_{m,2} = -\nu(\partial u/\partial y) = -\nu(U/h)$$

$$\frac{dy_m}{dx} = \frac{F_{m,2}}{F_{m,1}} = -(\nu h/U)y_m^{-2} = -Re^{-1}(y_m/h)^{-2} \qquad \mathrm{Re} = \frac{Uh}{\nu}$$

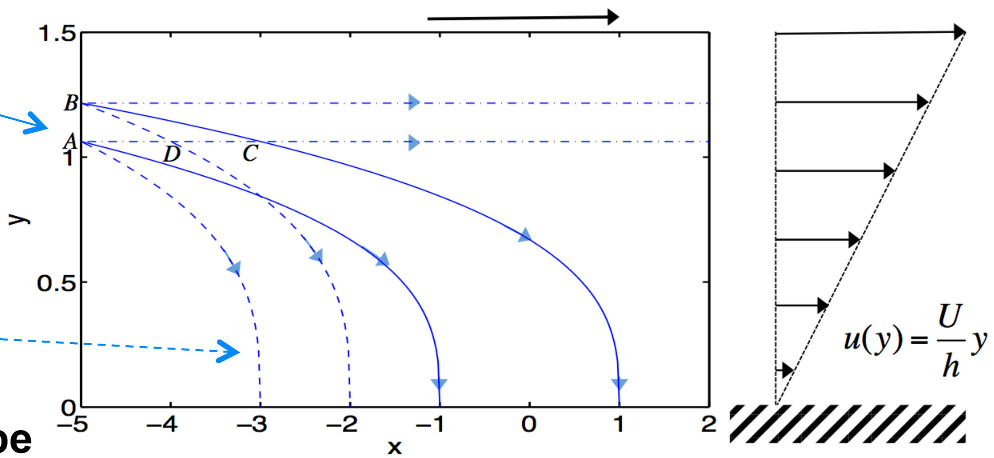
$$F_{m,j} = u_1 u_j - 2\nu S_{1j}$$

$$\frac{y_m(x)}{h} = \left[\frac{3}{Re}\frac{x_0 - x}{h}\right]^{1/3}$$

2D momentum transport tube

$$\frac{y_E(x)}{h} = \left[\frac{6}{Re}\frac{x_0 - x}{h}\right]^{1/3}$$

2D mechanical energy transport tube



Tutorial examples for laminar flows: momentum transport lines and tubes in **laminar Couette flow**:

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2D momentum transport tube

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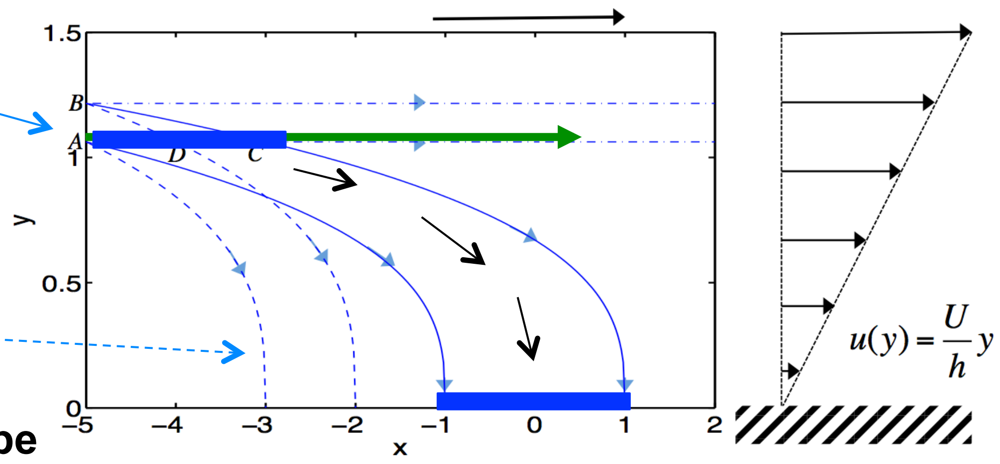
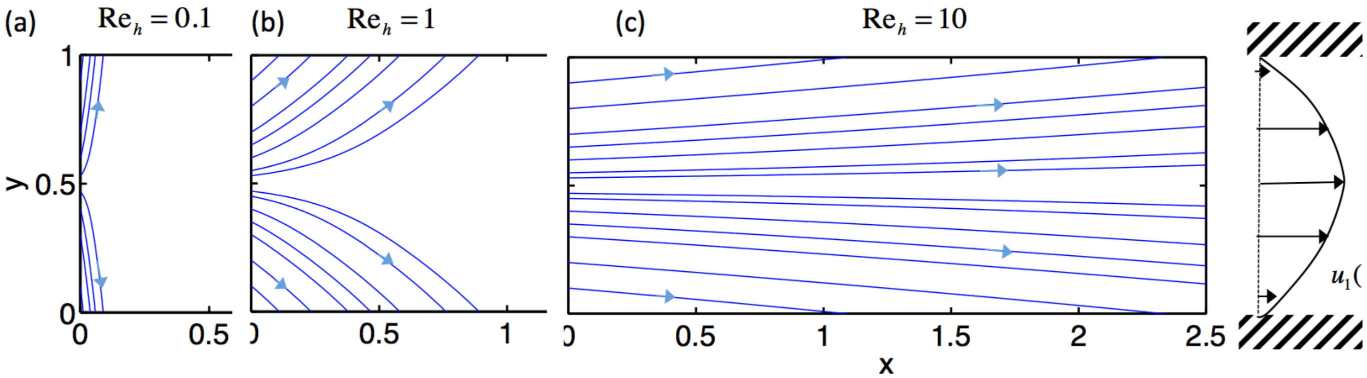
2D mechanical energy transport tube



laminar Poiseulle flow: $F_{m,j} = u_1\, u_j - 2\nu S_{1j}$



$$u(y) = \frac{Gh^2}{2\nu} y(1-y)$$

$$F_{m,1} = u(y)^2$$

$$F_{m,2} = -\frac{Gh^2}{2\nu}(1-2y)$$

$$\frac{dy_m}{dx} = -(8Re_h)^{-1}(1-2y)[y^2(1-y)^2]^{-1}$$

$$4\ln\left(\frac{y-\frac{1}{2}}{y_0-\frac{1}{2}}\right) + (2y-3)(2y+1)(1-2y)^2 - (2y_0-3)(2y_0+1)(1-2y_0)^2 = \frac{16}{Re_h}(x-x_0)$$

laminar round jet:

$$\psi = \nu x f(\eta), \quad \eta = r/x$$

$$f(\eta) = (c\eta)^2 \left[1 + (c\eta/2)^2\right]^{-1}$$

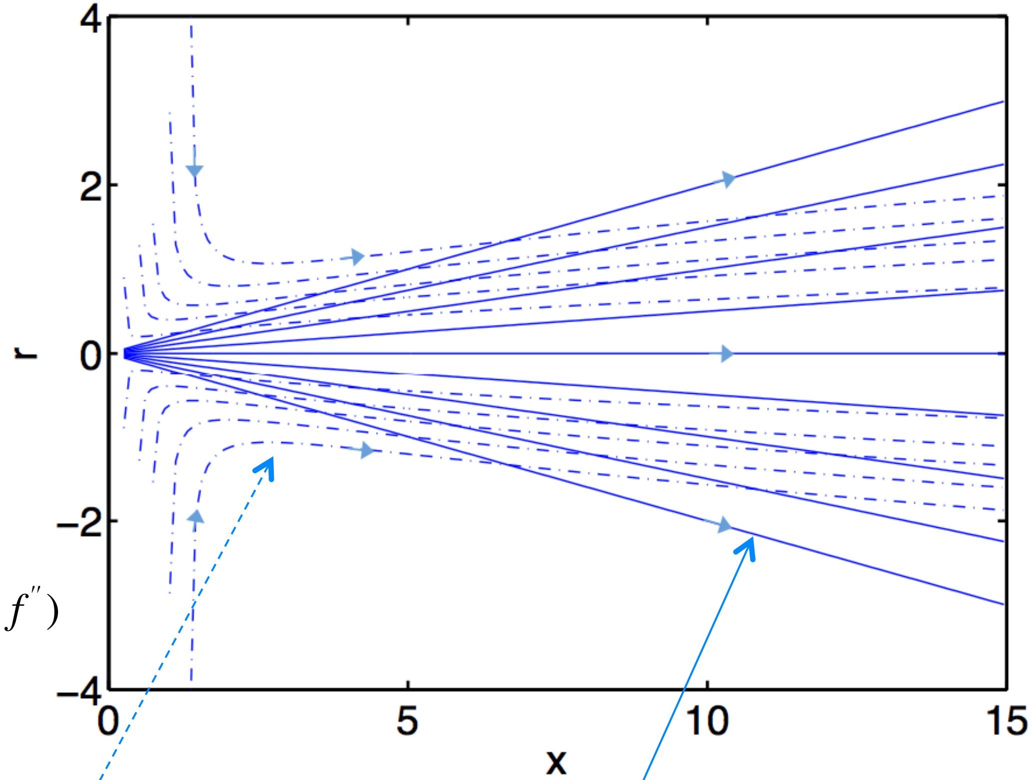
$$u = (\nu/r) f'$$

$$v = (\nu/r)(\eta f' - f)$$

$$F_{m,x} = u^2$$

$$F_{m,r} = uv - \nu \frac{\partial u}{\partial r} = (\nu/r)^2 f'(\eta f' - f + 1 - \eta f'/f'')$$

$$\frac{dr_m}{dx} = \frac{F_{m,r}}{F_{m,x}} = \eta + \frac{f' - ff' - \eta f''}{f^2} = \eta$$



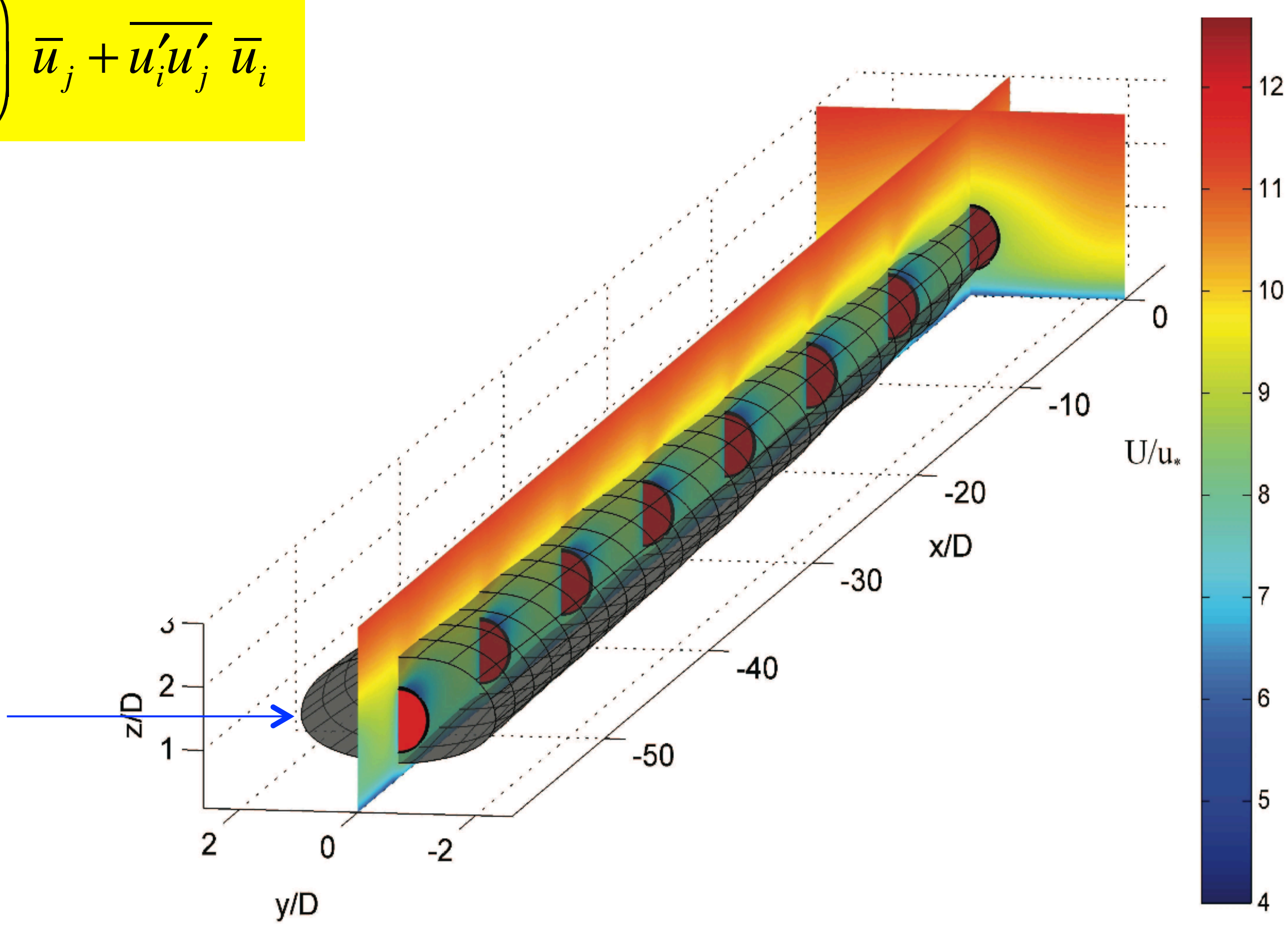
Viscous entrainment of fluid: Streamlines

No sources: Momentum transport lines along constant similarity variable

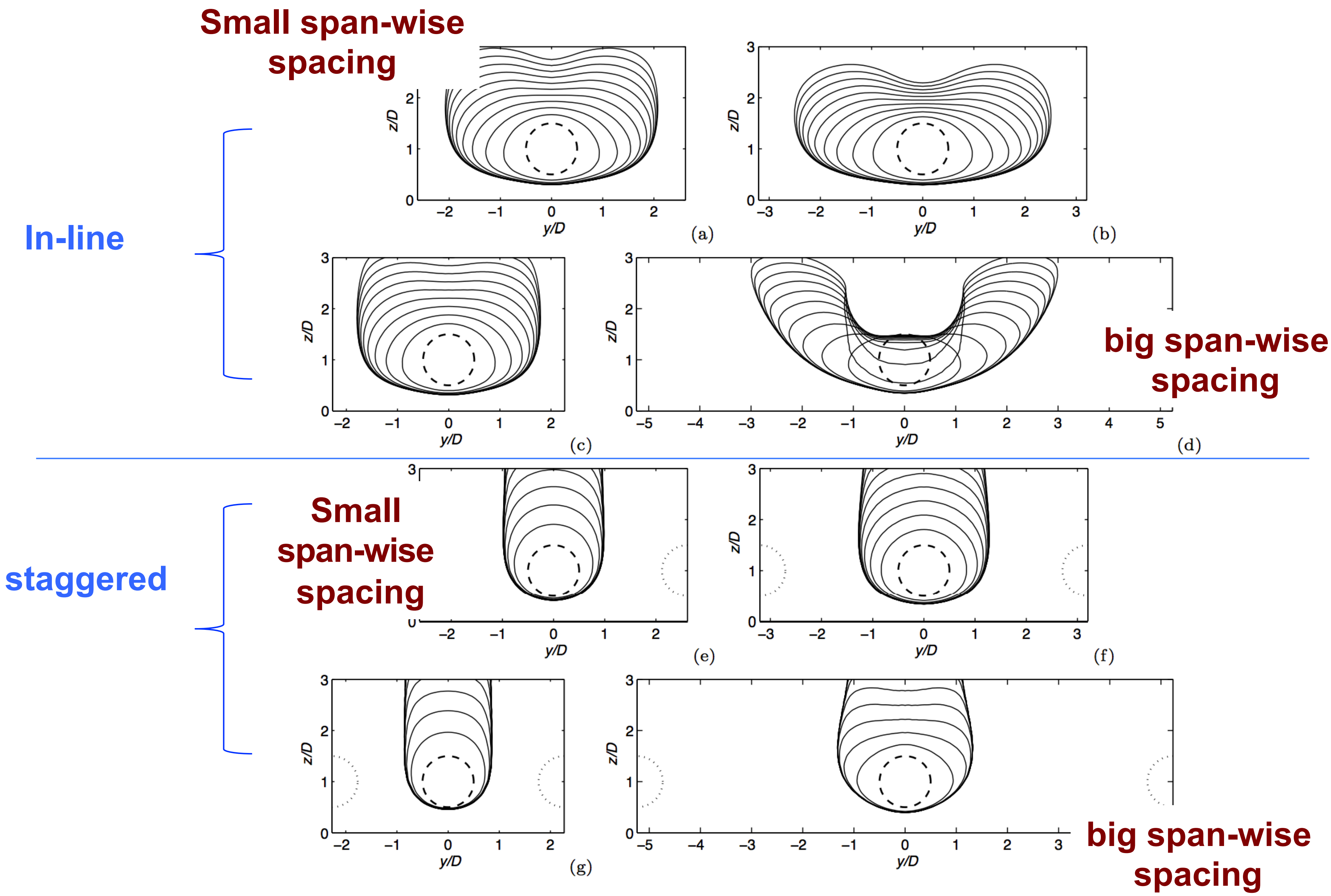
Energy transport tubes in wind farms:

$$\overline{F}_{E,j} = \left(\frac{1}{2}\overline{u}_i\overline{u}_i + \frac{1}{\rho}\hat{p} \right) \overline{u}_j + \overline{u_i' u_j'}\ \overline{u}_i$$

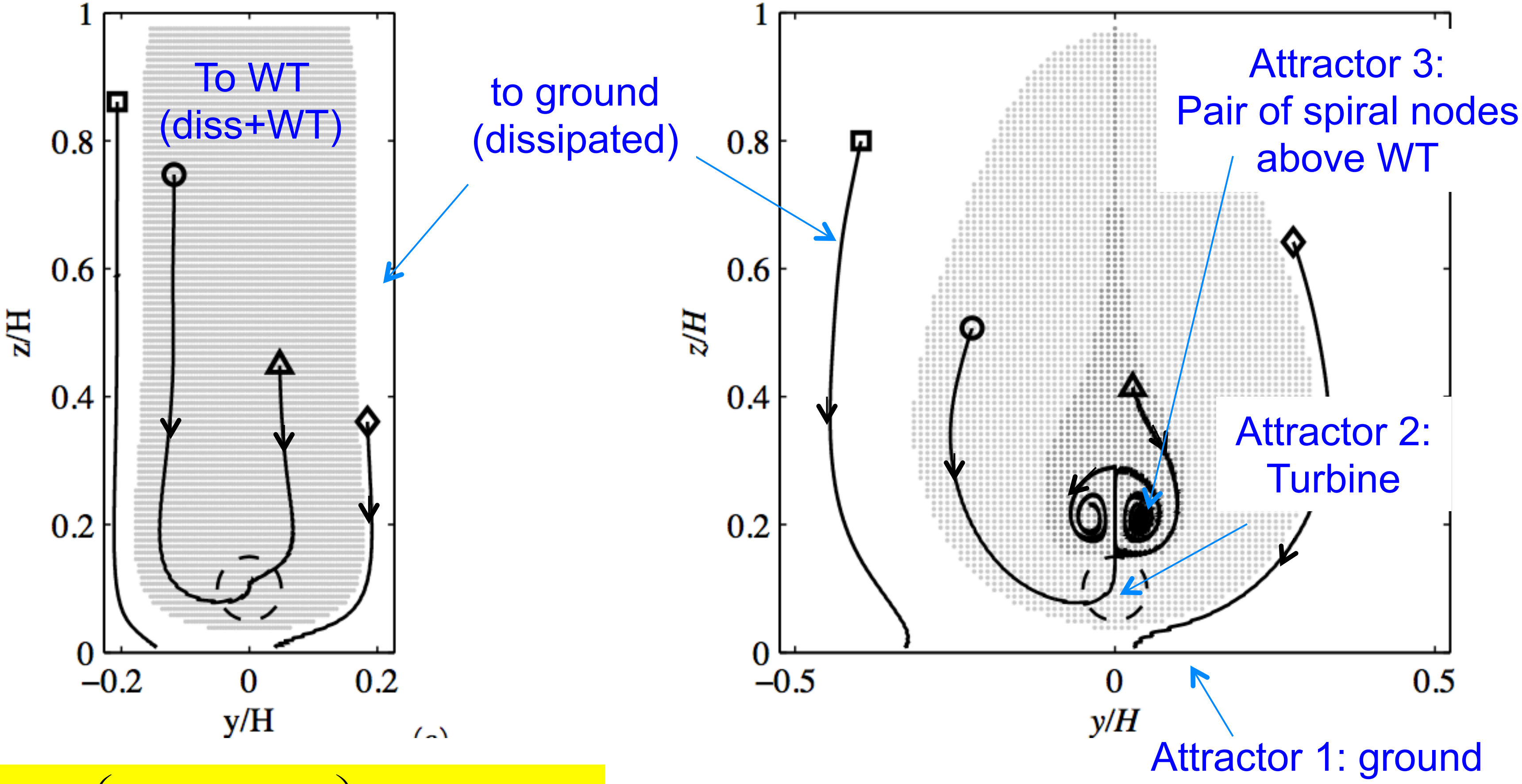
Total energy transport tube passing through last wind turbine



Effects of various wind farm layouts on transport tube geometry



Energy transport lines in "Poincaré sections" & attractors & basins of attraction



$$\overline{F}_{E,j} = \left(\frac{1}{2} \overline{u}_i \overline{u}_i + \frac{1}{\rho} \hat{p} \right) \overline{u}_j + \overline{u_i' u_j'} \; \overline{u}_i$$

"Flow visualization using momentum and energy transport tubes and applications to turbulent flow in wind farms"
J. Meyers & C.M. 2012 (JFM, in press)

Concluding remarks

- **LES of large wind farms (deep arrays) needed to better understand details of coupling of large man-made systems and the Atmospheric Boundary Layer.**

- **New, more realistic, effective roughness scale proposed.**

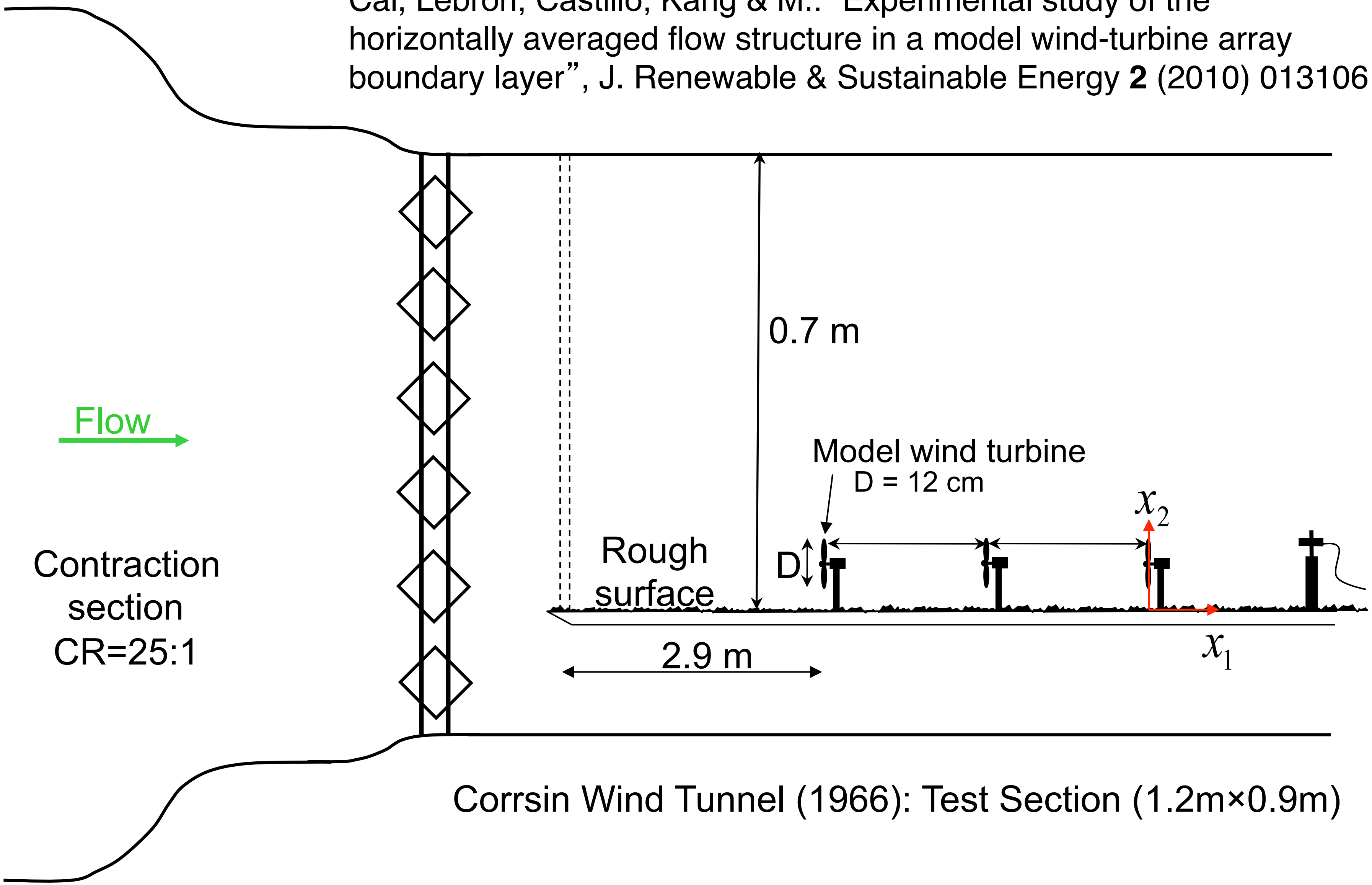
- **Surface fluxes of scalars: competing mechanisms increased turbulence above + screening below = 10-15% increases in fluxes**

- **New flow-viz approach: energy transport tubes..**

Q?

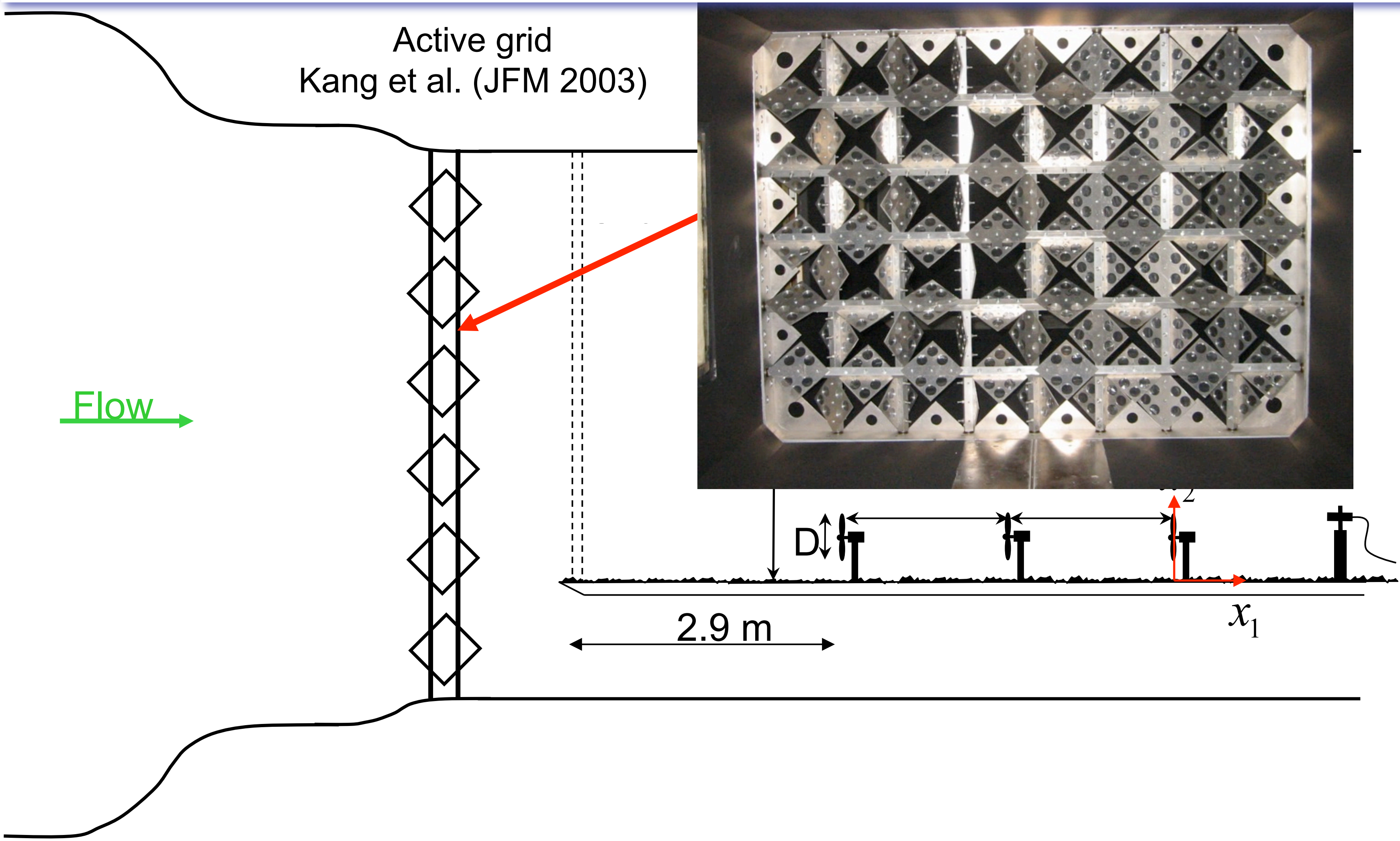
Wind-tunnel measurements: mechanics of vertical KE entrainment??

Cal, Lebrón, Castillo, Kang & M.: "Experimental study of the horizontally averaged flow structure in a model wind-turbine array boundary layer", J. Renewable & Sustainable Energy **2** (2010) 013106

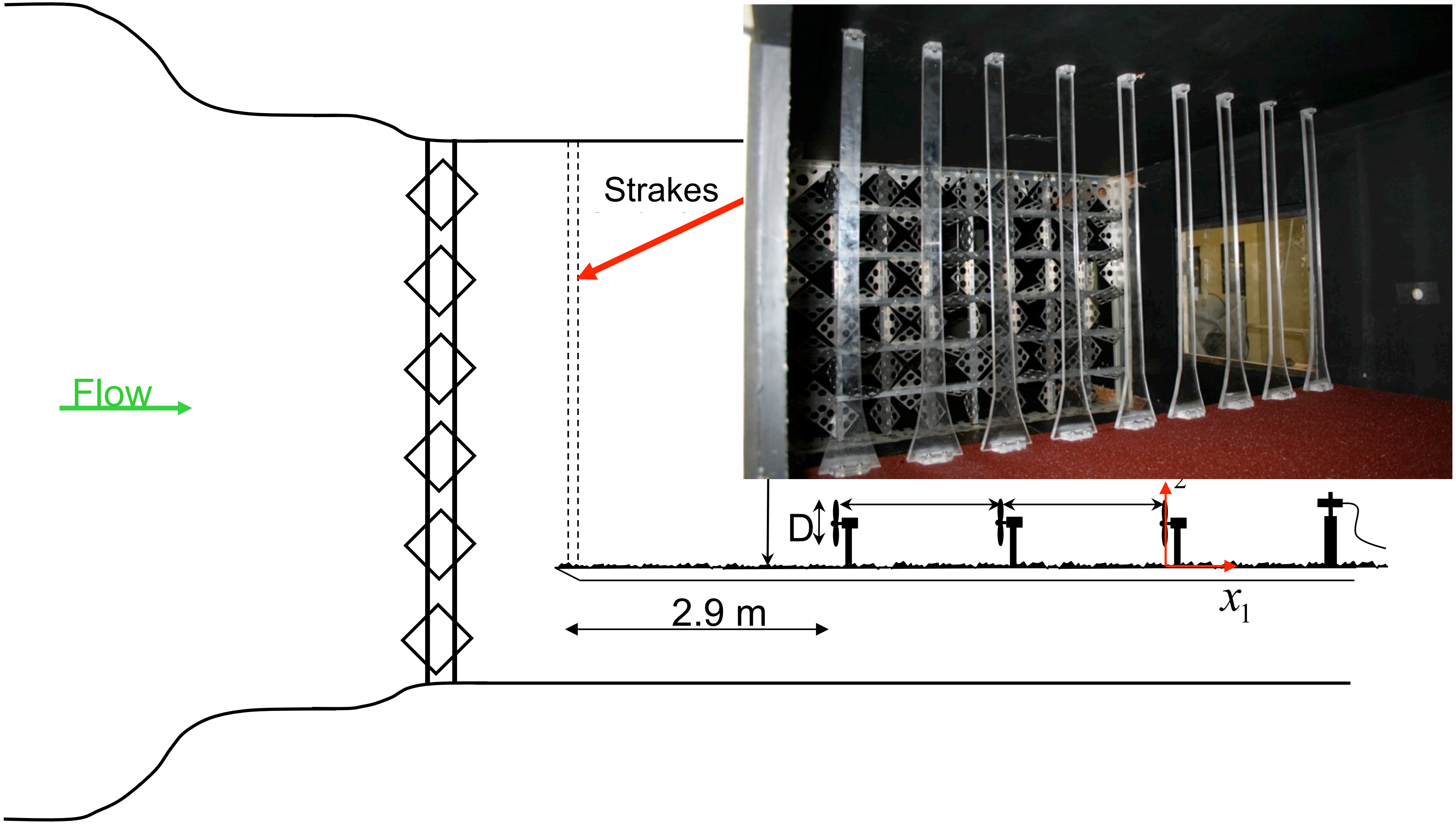


Corrsin Wind Tunnel (1966): Test Section (1.2m×0.9m)

Wind-tunnel measurements: mechanics of vertical KE entrainment??

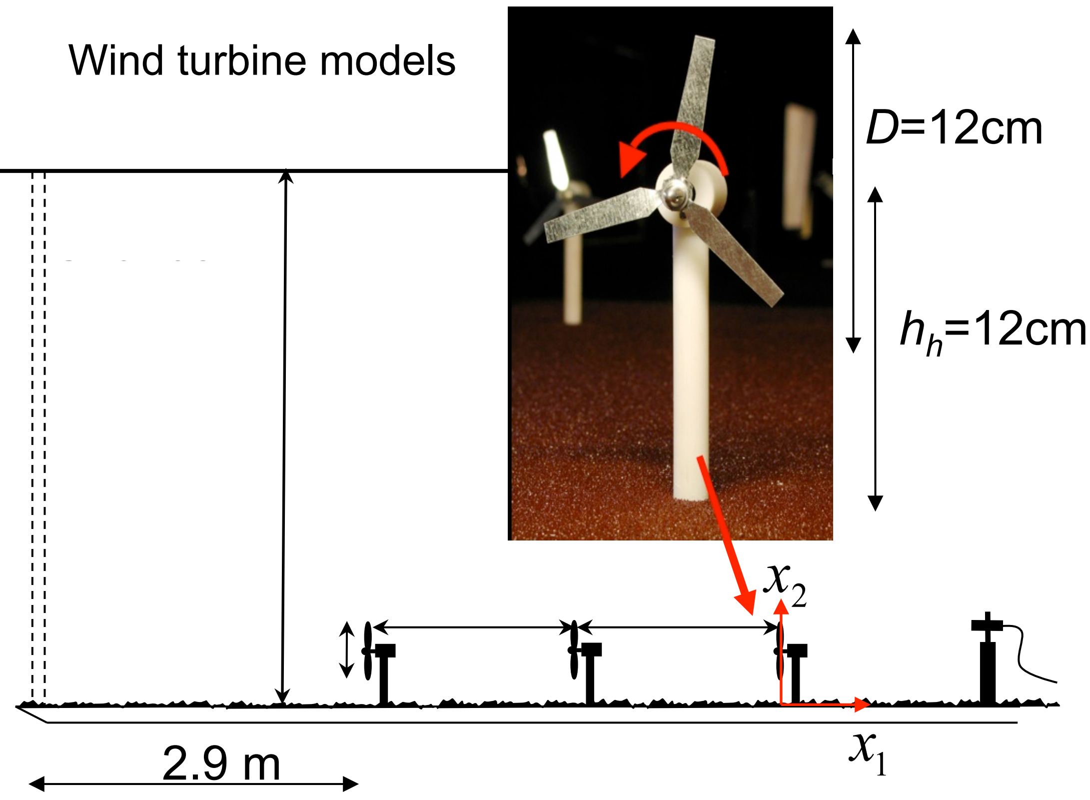


Wind-tunnel measurements

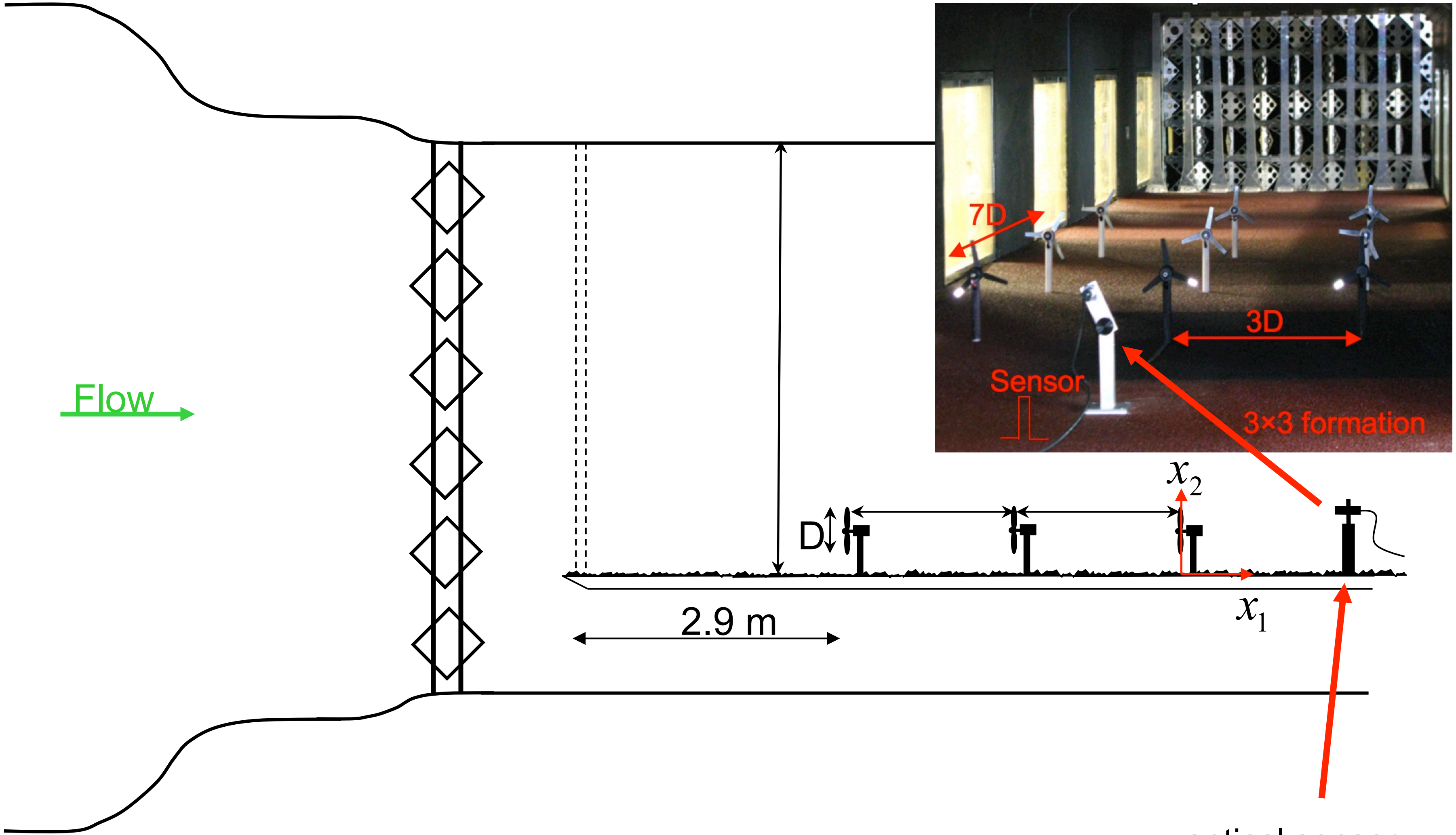


Wind-tunnel measurements

- **Turbine Models**
  - –Scaled down 850 times from typical real life length scales (real diameters of 100 m scaled down to 12 cm).
- **Rotors**
  - –Made from G28 galvanized sheet metal
  - –Twisted 1.1 degrees per cm, from 15º at the root to 10º at the tip
  - –Tip speed ratio, $\lambda = V_{tip}/U_{hub}$ is 5
  - –Rotate at 4800 RPM
- **Tower**
  - – Height of 12 cm
  - – Constructed using rapid prototyping



Wind-tunnel measurements

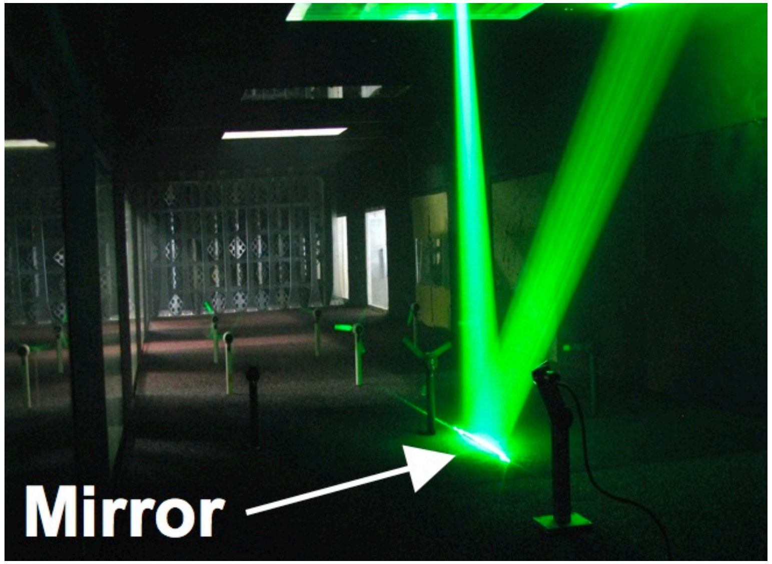


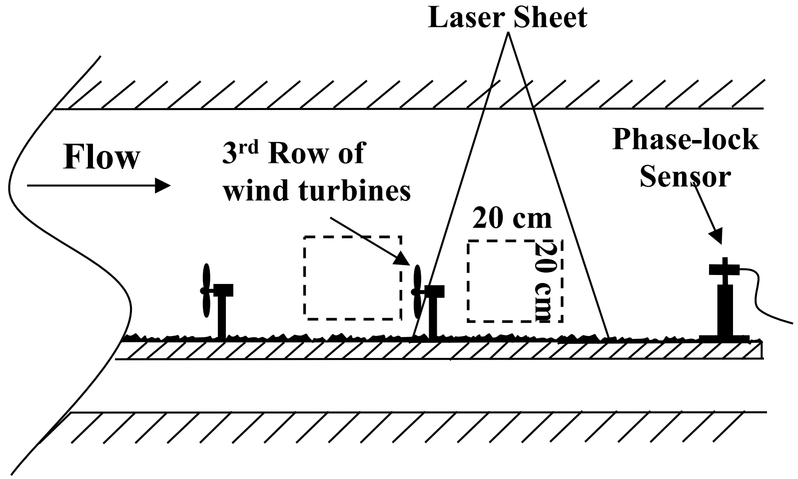
optical sensor
for phase-lock and
rpm measurements

Stereo-PIV system

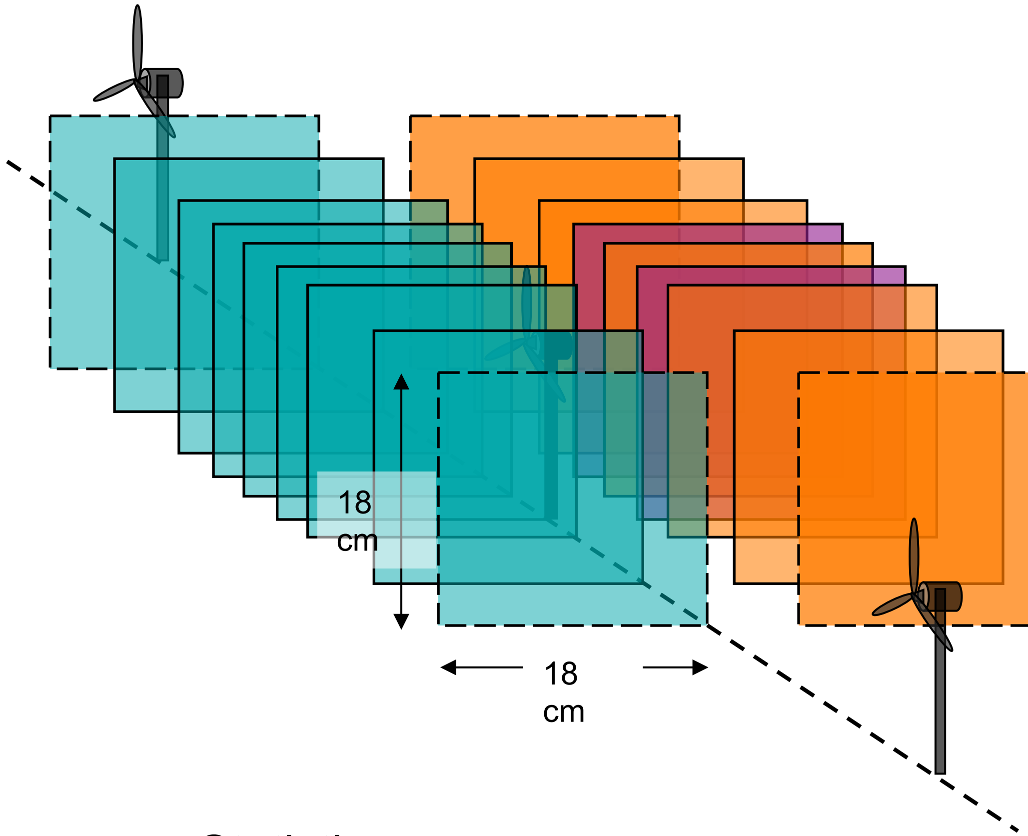
TSI System with:

- Double pulse Nd:YAG laser(120 mJ/ pulse)
  - Laser sheet thickness of 1.2 mm
  - Time between pulses of 50 ms
  - Optical sensor external trigger for phase lock measurements
- Two high resolution cross/auto correlation digital CCD cameras with
  - a frame rate of 16 frames/sec.
  - Interrogation area of 20 cm by 20 cm

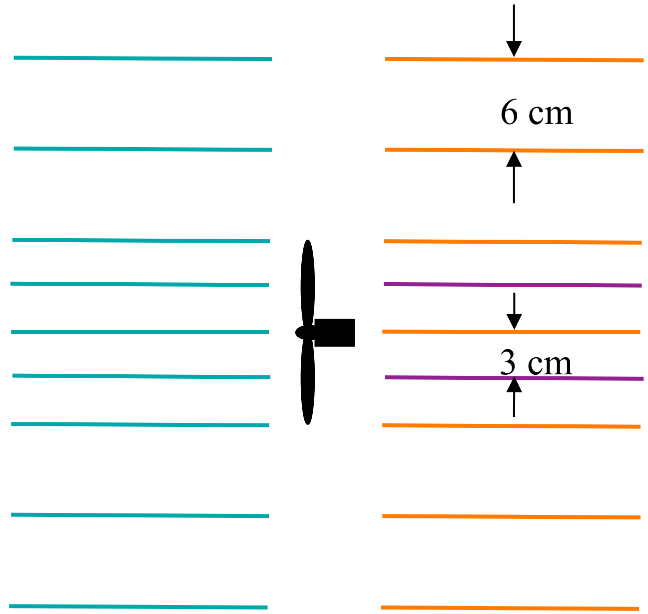




PIV data planes:



Top view:

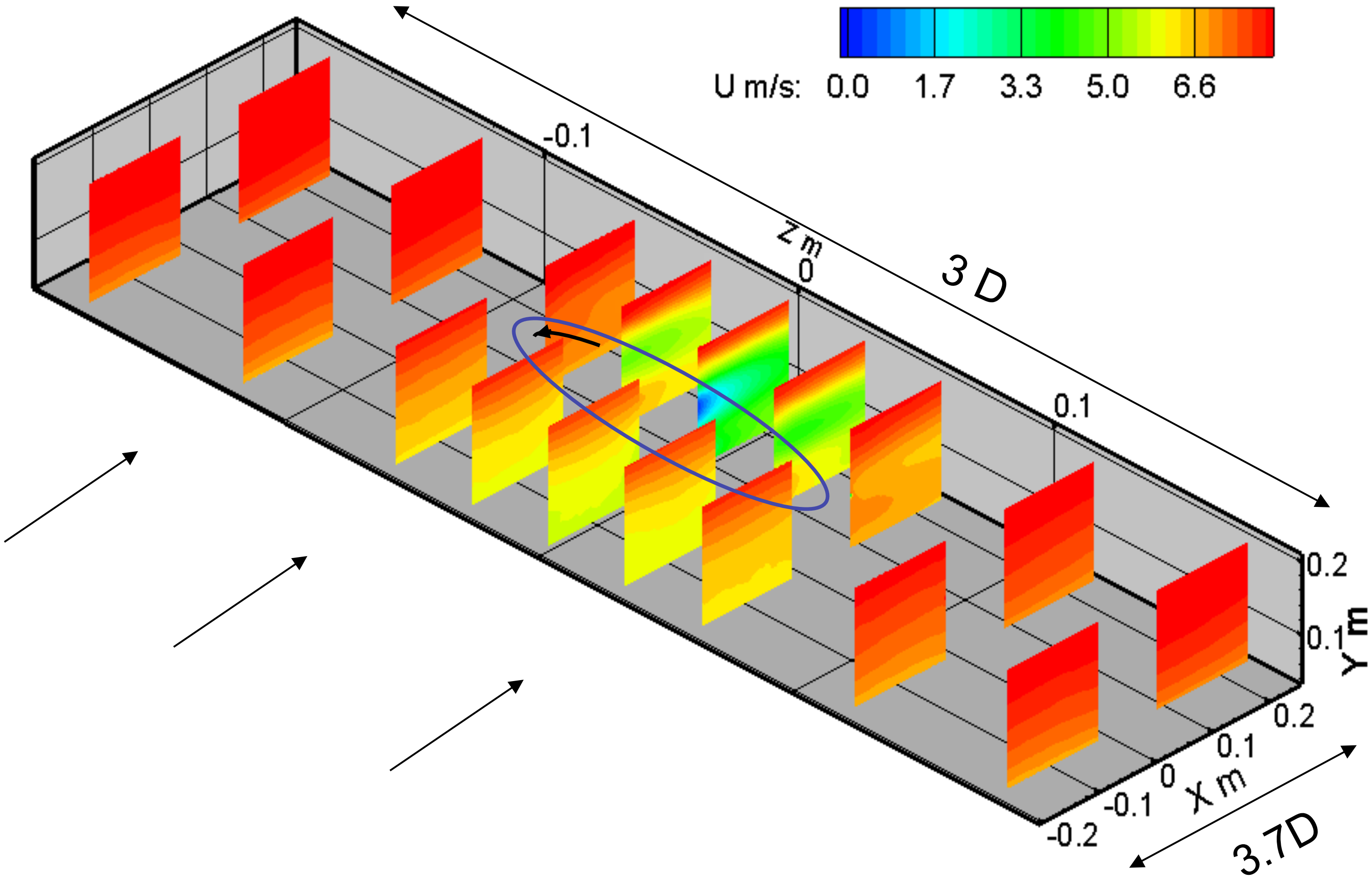


Statistics:

2000 vector maps for each front plane
12000 samples each back plane (6 phase-locked cases)

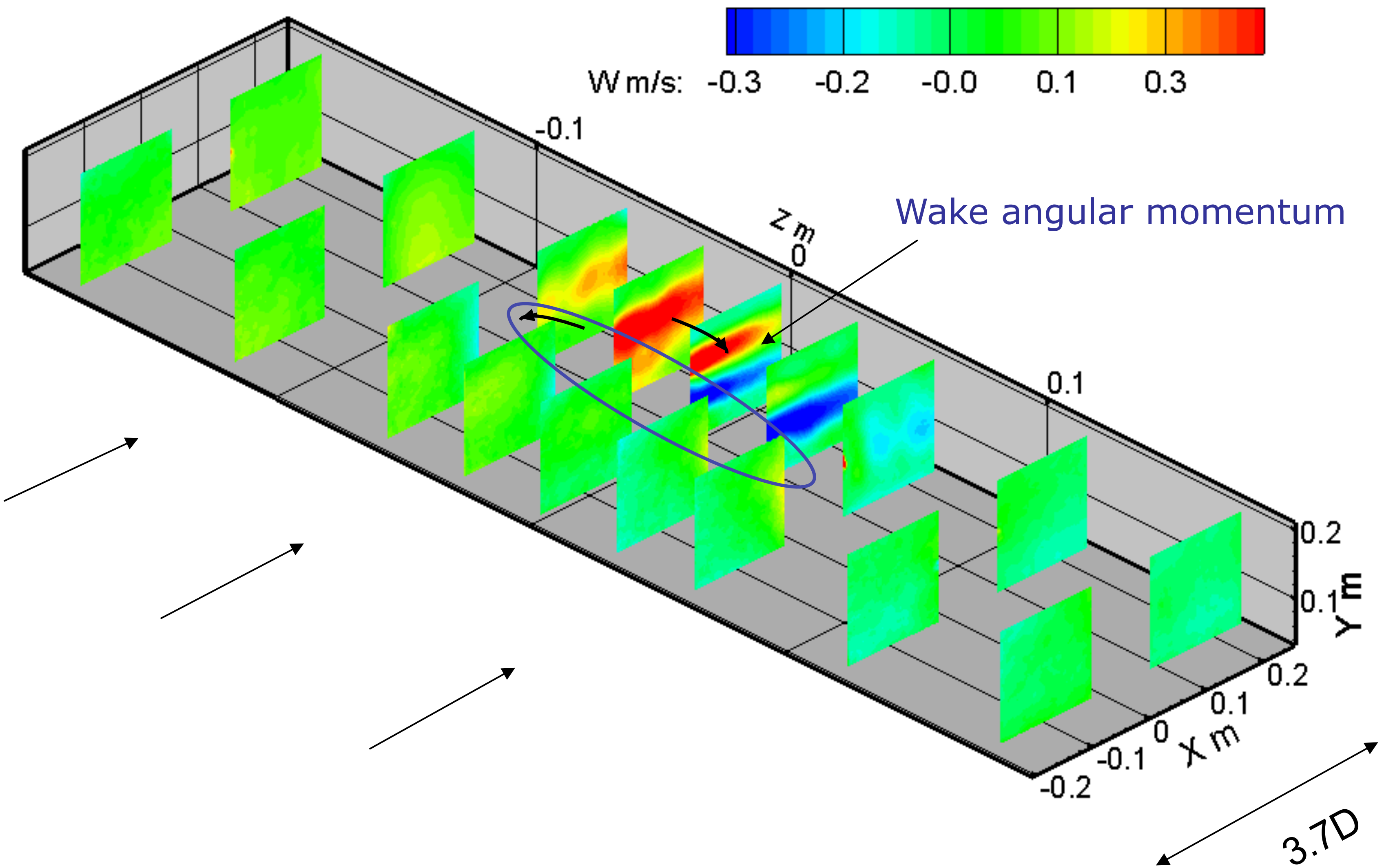
Velocity maps:

Mean streamwise velocity



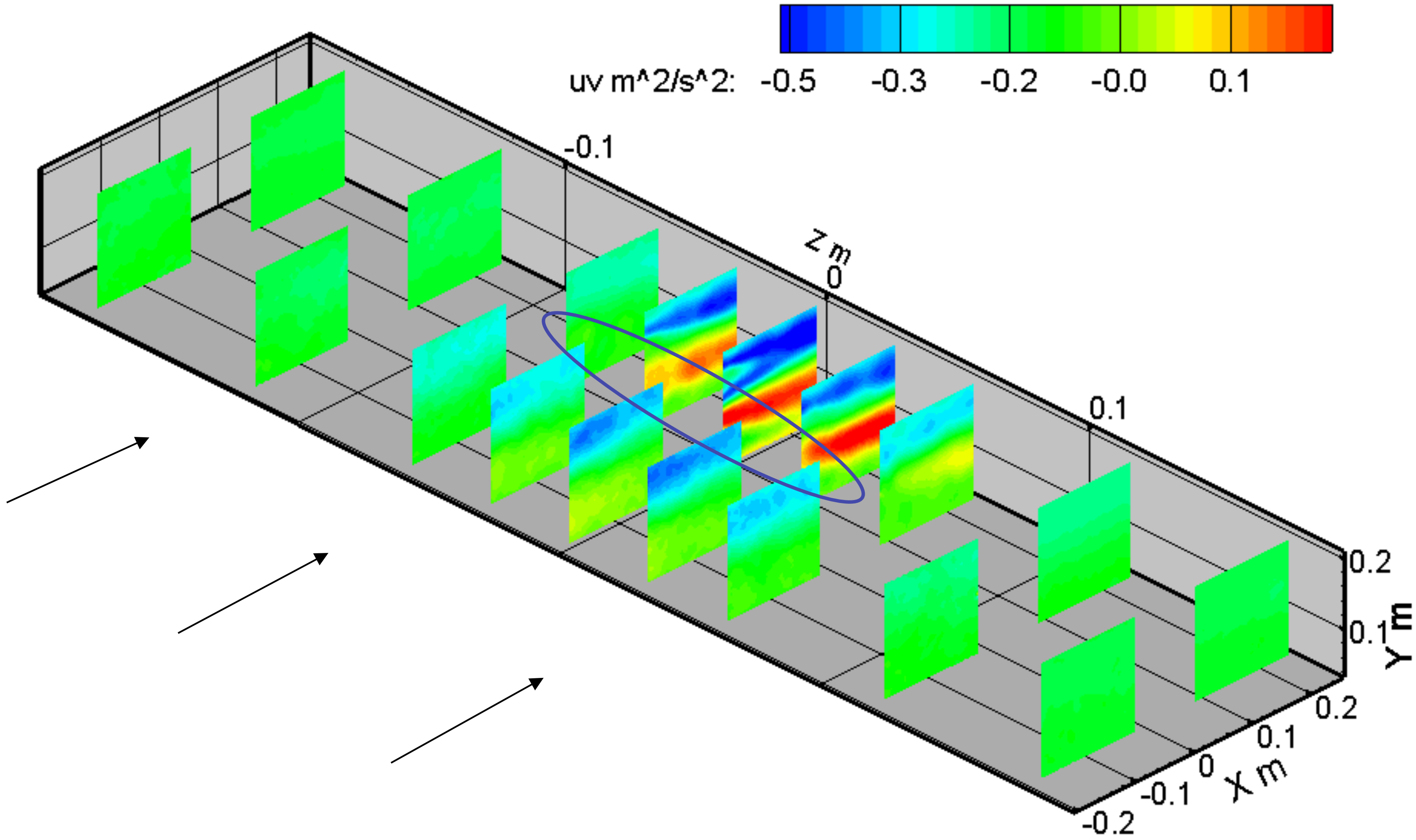
Velocity maps:

Mean transverse velocity



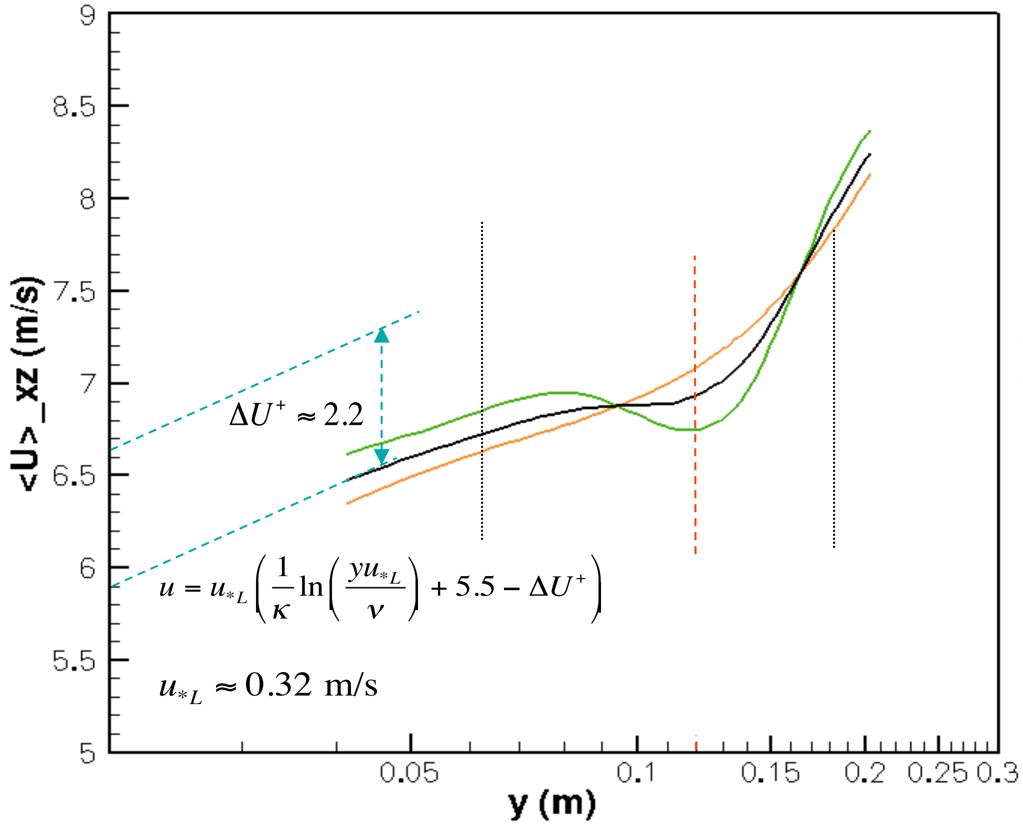
Velocity maps:

(negative) Reynolds shear stress

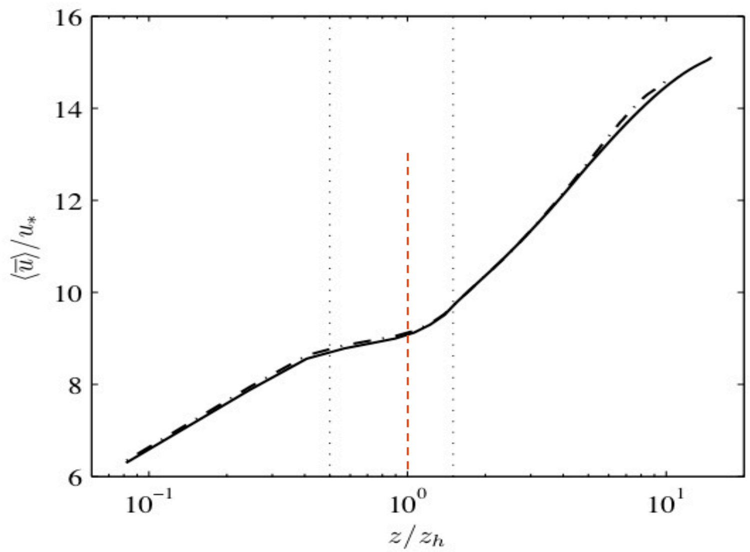


Horizontally averaged profiles:

experiment



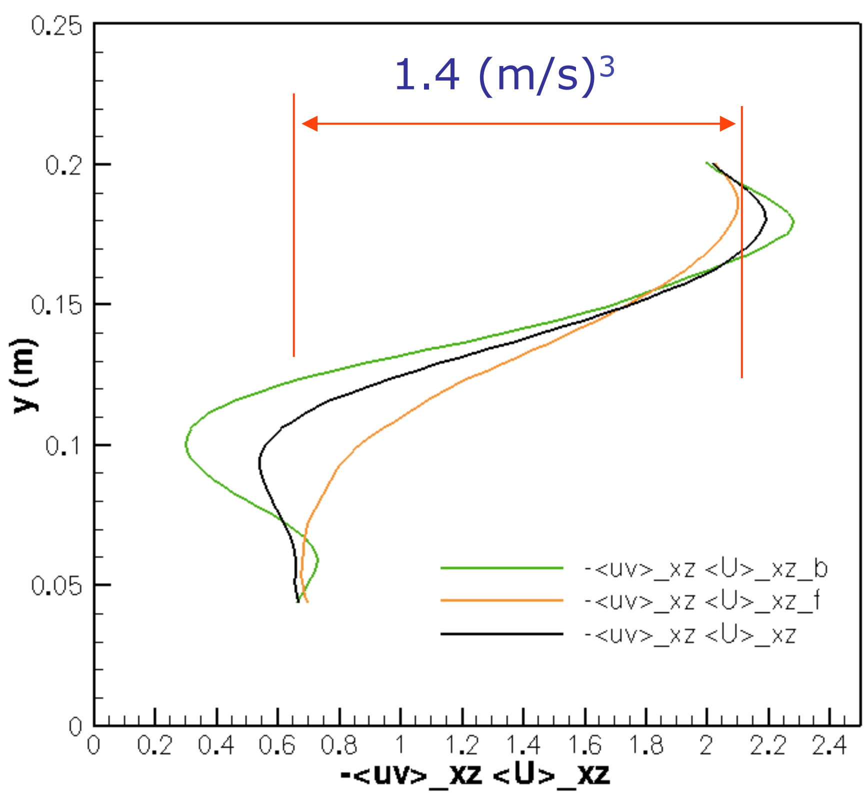
LES



Horizontally averaged profiles - kinetic energy terms:

$$\frac{d\frac{1}{2}\langle u\rangle_{xz}^2}{dt} = -\varepsilon_{turb} - \varepsilon_{canop} - \frac{d}{dy}\left(\langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz} + \langle\overline{u}''\overline{v}''\rangle_{xz}\langle u\rangle_{xz}\right) - \langle u\rangle_{xz}\frac{1}{\rho}\frac{dp_\infty}{dx} - P_T(y)$$

found to be negligible here



$$\langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz}(y_{top}) - \langle\overline{u'v'}\rangle_{xz}\langle u\rangle_{xz}(y_{bottom}) \approx 1.4\ \frac{W/m^2}{(kg/m^3)}$$

$$\Downarrow$$

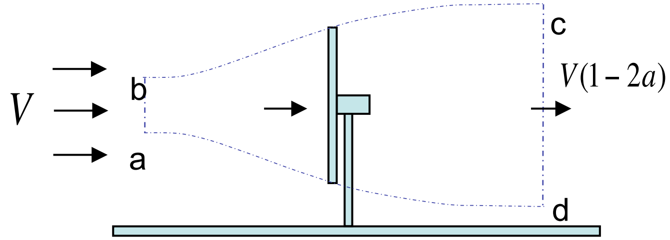
$$P^{turb-flux} = 1.4\rho A$$

$$P^{turb-flux} = 1.4\times 1.2\times(3\times 0.12)(7\times 0.12)$$

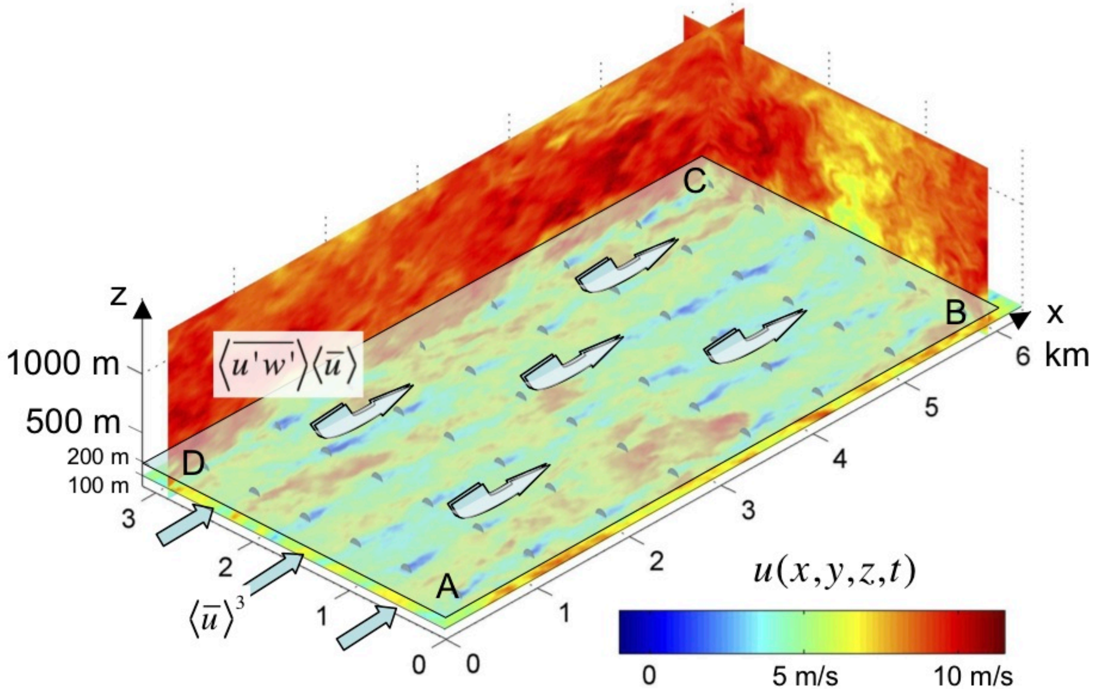
$$P^{turb-flux} = 0.51\ W$$

Analysis consistent with view that kinetic energy extracted by turbine (0.34W) is delivered vertically by turbulence fluxes (0.51W)
(rest goes into dissipation, etc…)

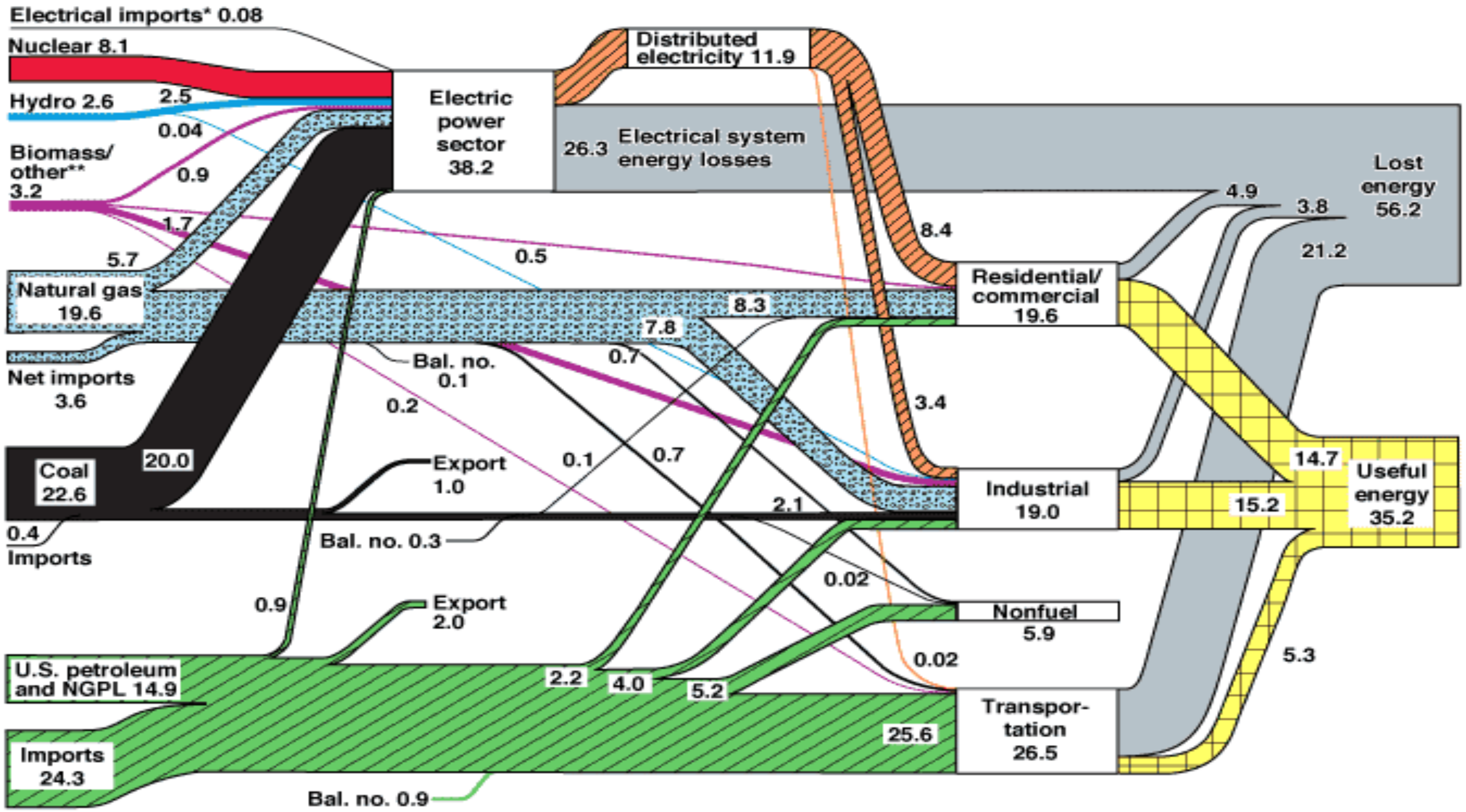
to scale: vertical entrainment (turbulence) dominates



Classical Betz analysis:
focused on "horizontal" fluxes



U.S. Energy Flow Trends
Net Primary Resource Consumption ~97 Quads



Source: Production and end-use data from Energy Information Administration, *Annual Energy Review 2002*.
*Net fossil-fuel electrical imports.
**Biomass/other includes wood, waste, alcohol, geothermal, solar, and wind.

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