Formation of columnar baroclinic vorticies in thermally stratified nonlinear spin-up*

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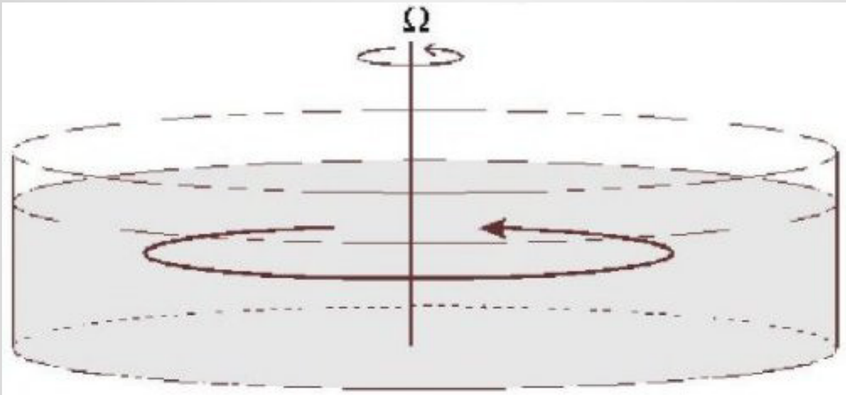
Outline

- Overview
  - What are spin-up and baroclic flows?
- Background
  - Laboratory experiments
  - Numerical simulations
- Results
- Conclusions
- Problems to consider in the future

Overview

Definition of spin-up/spin-down: an impulsive change of the rotation rate of a rigid container filled with a liquid



- fluid assumed to be in solid body rotation
- rotation rate of the container modified: increased (spin-up) or decreased (spin-down)
- adjustment of the primary circulation to the new rotation rate

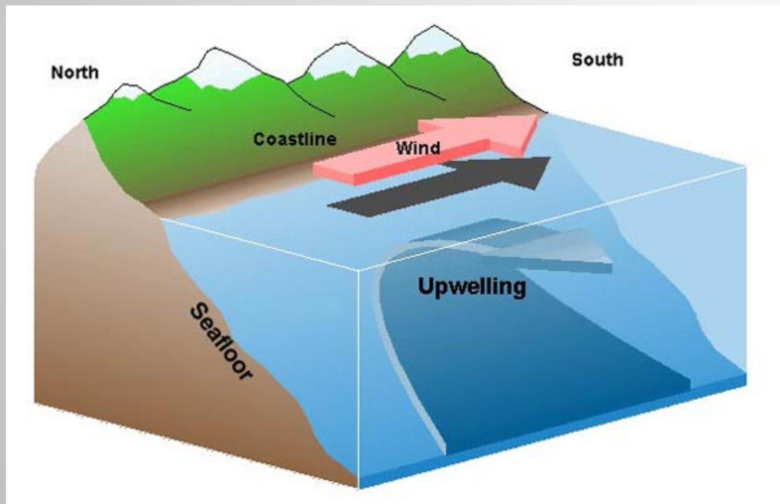
Background

- Existence of separated time scales: Ekman number $E = \nu / \Omega H^2$
  - Ekman boundary layer development: $\tau_e = \Omega^{-1}$
  - Spin-up due to secondary circulation: $\tau_s = \Omega^{-1} E^{-1/2}$
  - Viscous dissipation of residual motions: $\tau_v = \Omega^{-1} E^{-1}$

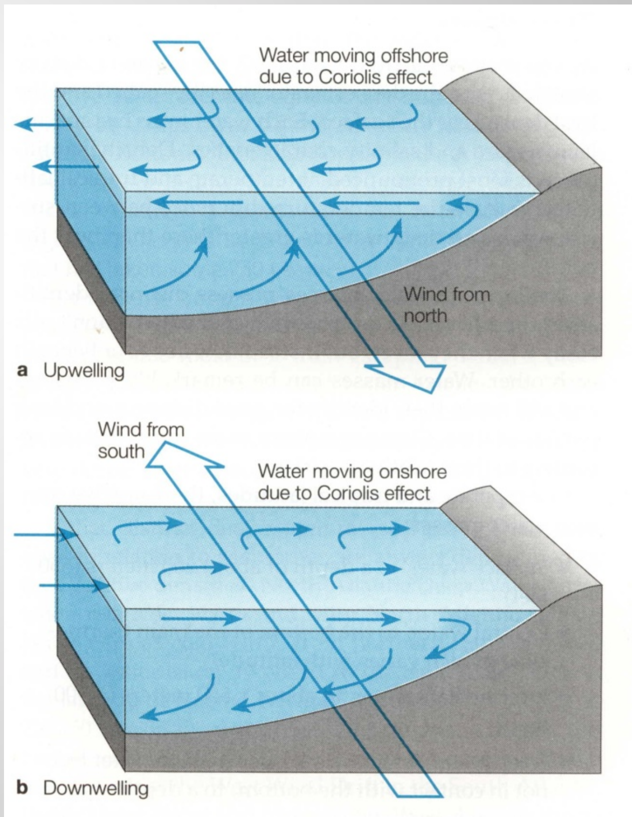
- Large body of work on the axisymmetric stratified spin-up, but not much on the non-axisymmetric counterpart
  - Numerical models: axisymmetric in nature
  - Experimental investigations: mostly in small facilities

Geophysical flows

Response mechanism to external forcing which arises in oceans

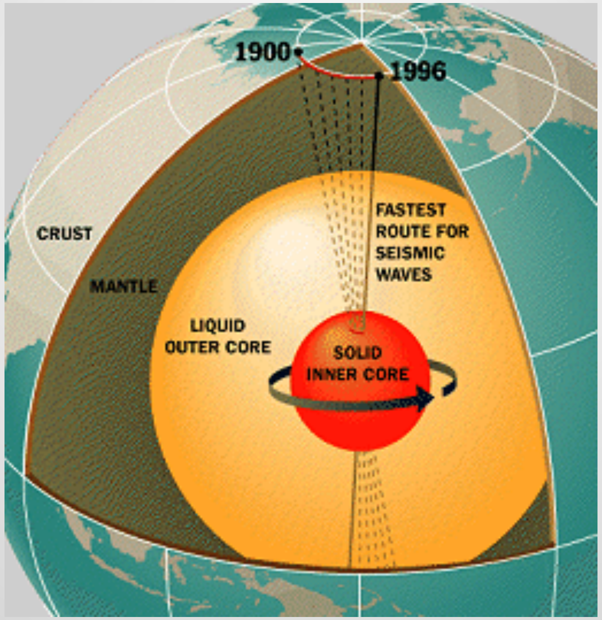


Ekman transport moves surface waters away from the coast; these are replaced by denser water that wells up from below



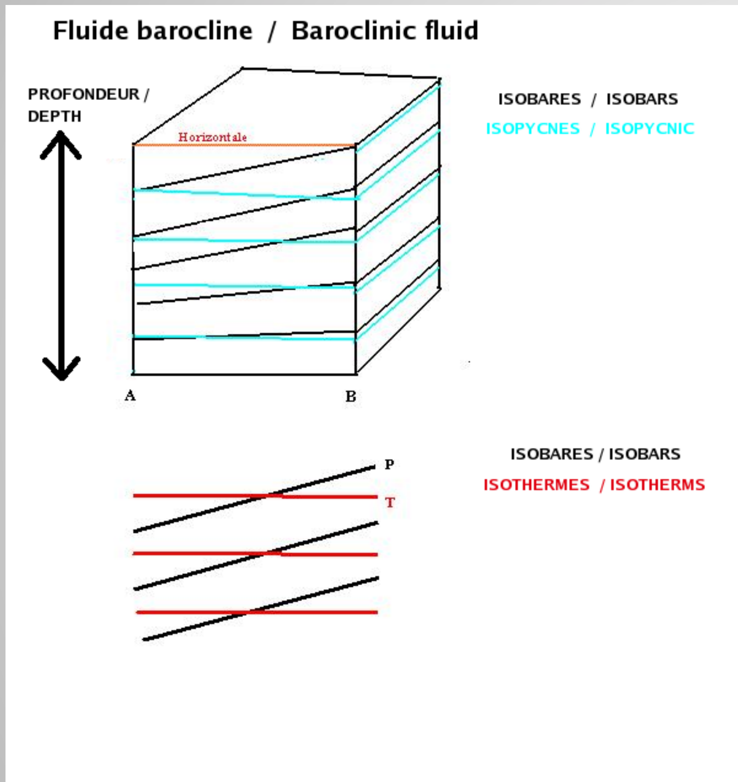
Magnetohydrodynamic flows

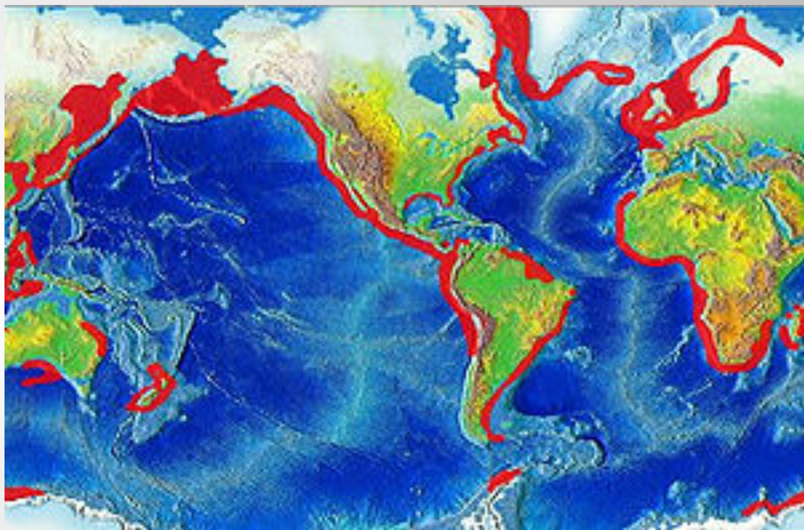
Response mechanism to magnetic forcing which arises in the inner core of the earth



Example: Mercury is known to have an at least partially fluid core, and during its orbit its rotation rate increases and decreases periodically.

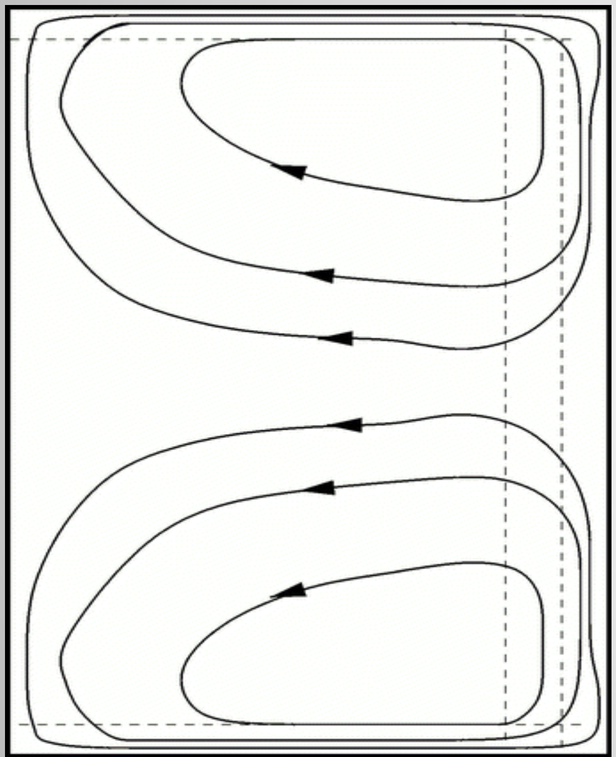
Baroclinic fluid





Spin-up homogeneous fluid

centerline *sidewall*



Streamlines for homogenous spin-up

Dashed lines – Ekman layers & sidewall Stewartson layers (all of which transport mass)

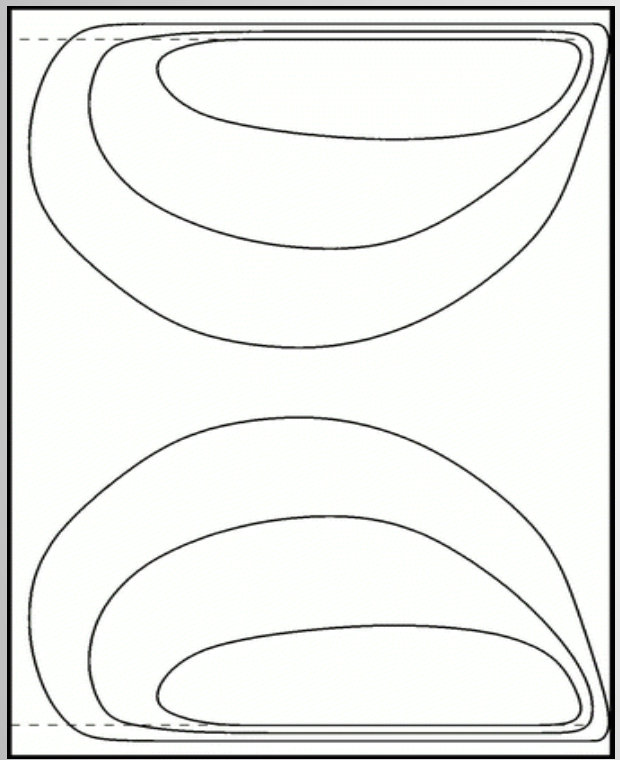
Stewartson (1957)
Greenspan & Howard (1963)

$$\varepsilon = \frac{\Delta\Omega}{\Omega} \qquad E = \frac{\nu}{\Omega H^2}$$

$$\tau_s = \Omega^{-1} E^{-1/2}$$

Spin-up stratified fluid

centerline *sidewall*



Streamlines for stratified spin-up

- Inhibition of vertical motions by buoyancy force
- Presence of horizontal density gradients
- Baroclinic instability (Bu<1)
  $Bu \equiv N^2H^2/\,\Omega^2R^2$
- Vertical shear of the horizontal velocity
- Ekman layers have the same structure as in homogeneous case

Stratified spin-up

Impulsive

Entire system

Local forcing

Incremental

Buzina & Veronis (1971) •
Saunders & Beardsley (1975) •
Lee (1975) •
Barcilon et al. (1975) +
Hyun et al (1982) +
Kanda (2004) •
Smirnov et al. (2005) •

From rest

Greenspan (1980) •
Hyun et al (1983) +
Flor et al. (2002) • +
Flor et al. (2004) •
Kanda (2004) •

Incremental

$R_l \ll R$

Moulin & Flor (2004) •

$R_l \sim R$

Pedlosky (1970) •
Linden (1977) •
Spence et al. (1992) •
Munro & Davies (2006) •

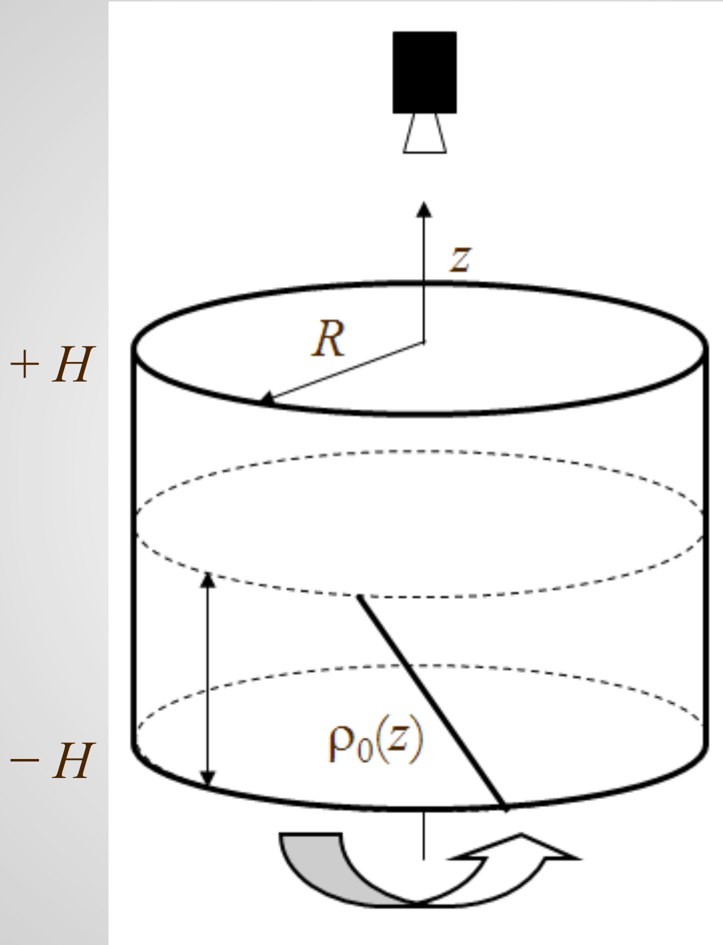
+ Smirnov, Pacheco and Verzicco *Physics of Fluids* (2010) +

axisymmetric flows
non-axisymmetric flows

• experimental study
+ numerical work

R_l = radius of lid
R = radius of horizontal boundaries

Experiments of Smirnov et al. (2005)
Salinity stratification



$\Omega(1 - \varepsilon) \rightarrow \Omega \qquad \varepsilon = \Delta\Omega/\Omega$

Stratified spin-up

Parameters:

$\varepsilon = 0.24 \quad B_u = 0.24$

$f = 0.5\ \mathrm{s}^{-1} \quad R_d = 22$ cm

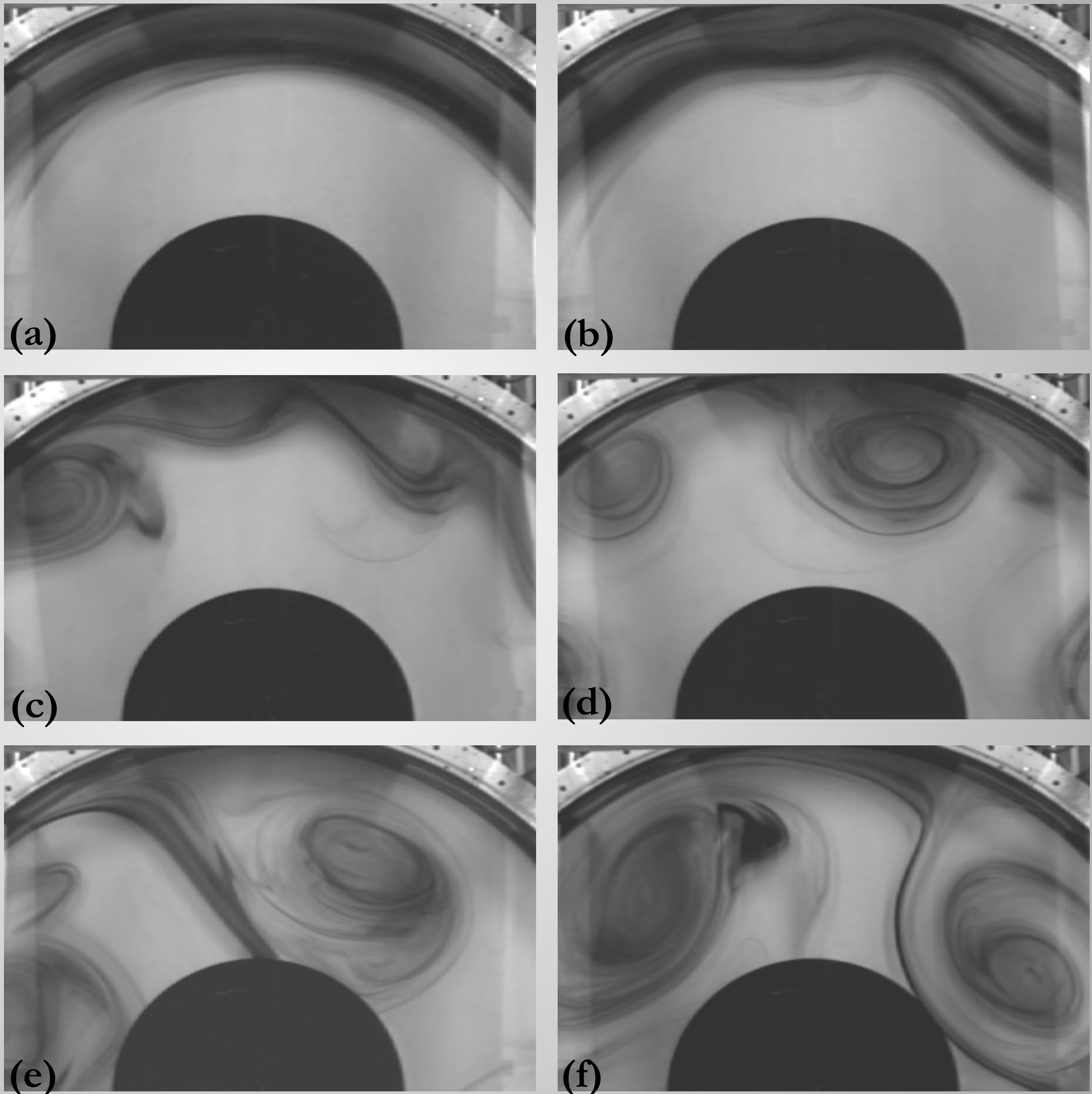
$\Omega^{-1}E^{-1} \sim 4.5$ hours

t/T

(a) 11.6 (b) 20.2

(c) 30 (d) 32.6

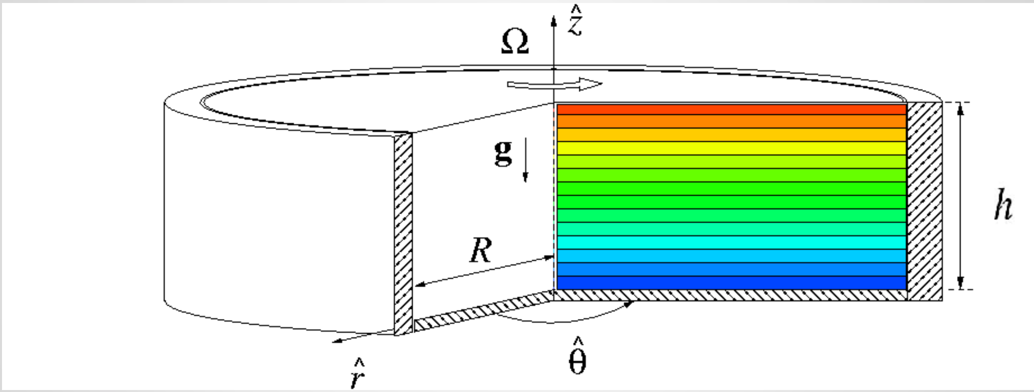
(e) 39.2 (f) 43.2



Description of the problem

Consider the flow in a cylindrical region with radius R and height h.

At $t = 0$ the system is accelerated from its initial state of solid body rotation with angular velocity Ω_i to a new rotation rate $\Omega = \Omega_i + \Delta\Omega$.



Governing equations

$$\frac{D\vec{u}}{Dt} + 2\vec{\Omega}\times\vec{u} = -\frac{1}{\rho_o}\nabla p - \frac{\rho}{\rho_o} g\vec{e}_z + \frac{\rho}{\rho_o}\vec{\Omega}^2 r\vec{e}_r + \nu\nabla^2\vec{u},$$

$$\frac{DT}{Dt} = \kappa\nabla^2 T, \qquad \nabla\bullet\vec{u} = 0, \qquad \rho = \rho_o(1-\alpha T).$$

Initial conditions

$$u = w = 0, v = -\Delta\Omega r$$

$$\frac{\partial T}{\partial z} = \frac{\Delta T}{h}$$

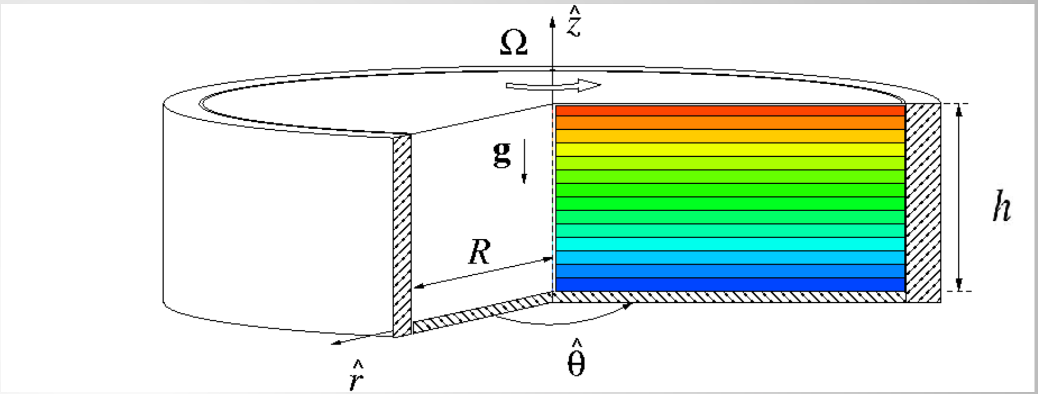
First 3D numerical simulations of spin-up with thermal stratification and adiabatic/isothermal endwalls*

Boundary conditions (BC):

$$u = v = w = \frac{\partial T}{\partial r} = 0, \quad \text{at} \quad r = R$$

$$u = v = w = 0, \quad \text{at} \quad z = 0$$

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = w = 0, \quad \text{at} \quad z = h$$



Adiabatic (Neumann) BC

$$\frac{\partial T}{\partial z} = 0, \quad \text{at} \quad z = 0, h$$

Isothermal (Dirichlet) BC

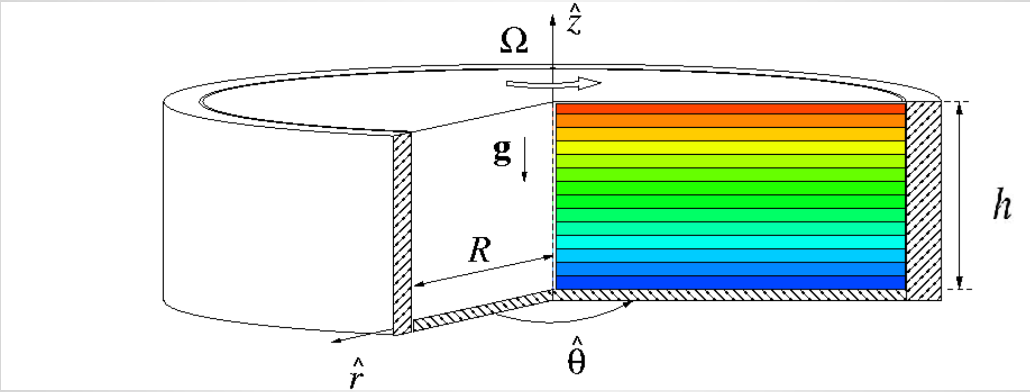
$$T = T_b, \quad \text{at} \quad z = 0,$$

$$T = T_t, \quad \text{at} \quad z = h$$

*Smirnov, Pacheco and Verzicco *Physics of Fluids* 22 (11): (2010)

Navier Stokes equations

$$(\partial_t + \boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = \boldsymbol{\nabla}p + B^2\Theta\boldsymbol{e}_z + 2\boldsymbol{u}\times\boldsymbol{e}_z - F\,B^2\Theta r\boldsymbol{e}_r + E\nabla^2\boldsymbol{u},$$
$$(\partial_t + \boldsymbol{u}\cdot\boldsymbol{\nabla})\Theta = \sigma^{-1}E\nabla^2\Theta, \quad \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0,$$



Ekman number: $E = \nu/\Omega h^2$,

Froude number: $F = \Omega^2 h/g$,

Burger number: $B = N/\Omega$,

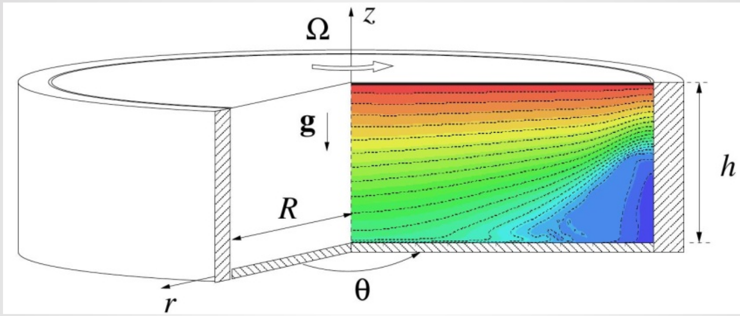
Prandtl number: $\sigma = \nu/\kappa$,

aspect ratio: $\Gamma = R/h$,

Rossby number: $\epsilon = \Delta\Omega/\Omega$,

Results of numerical simulations

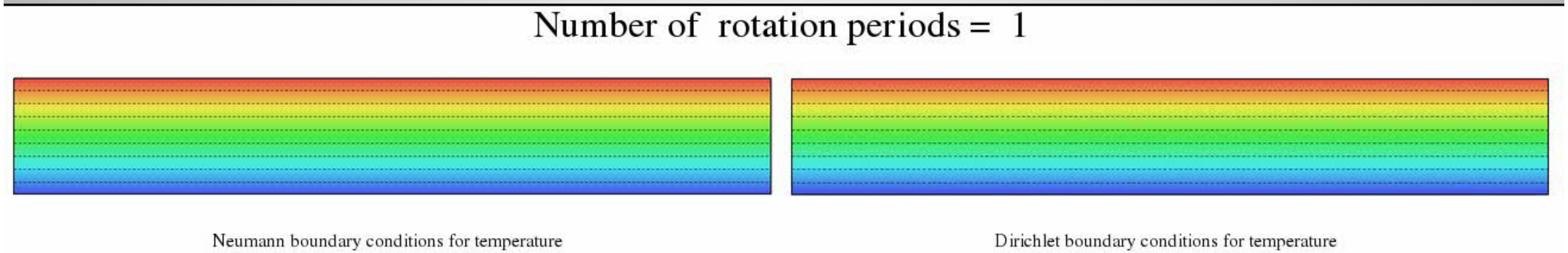
Look at the isotherms on the planes $\theta = 0 - \pi$



R/h=3.3, ε=0.73

Adiabatic BC

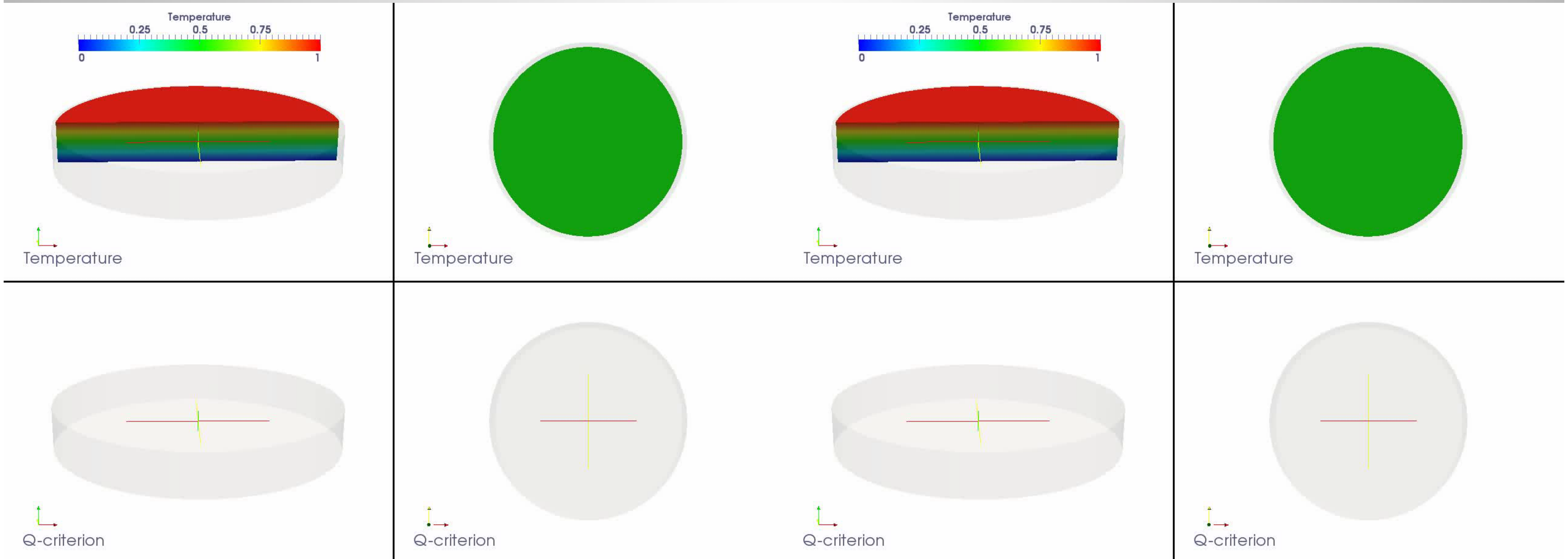
Isothermal BC



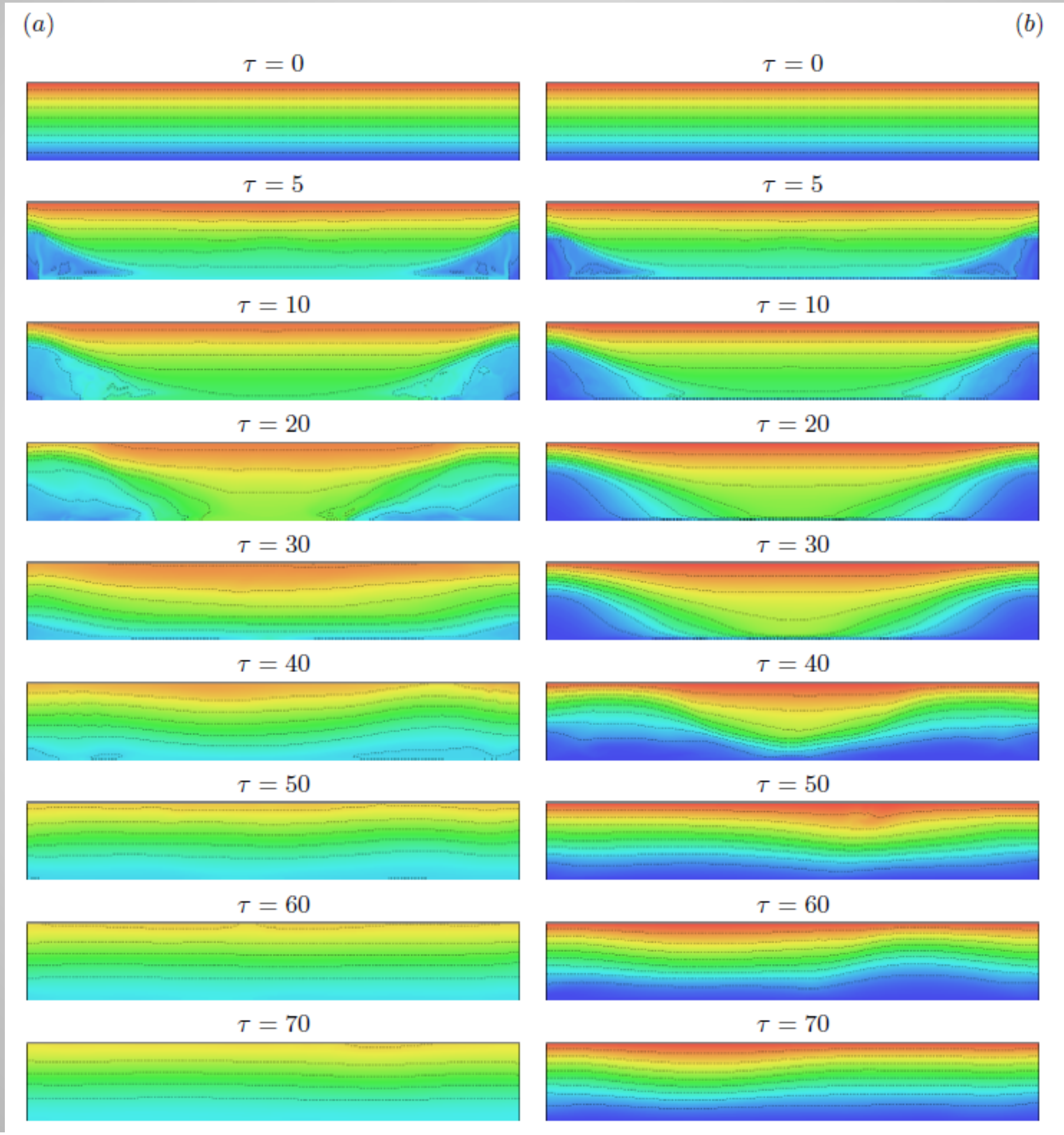
Vortex structure

Vortex structure identified by the iso-surfaces of $Q = 0$ (Hunt et al. 1988) colored by temperature

Adiabatic BC | Isothermal BC



Adiabatic BC R/h=3.3, ε=1   Isothermal BC



Observations:

- Columnar eddies seen in experiments are reproduced in a numerical setting
- The emergence of these columnar eddies are influenced by the type of boundary condition (adiabatic/isothermal)
- Forced stratification suppresses the instability, i.e. isothermal BC are more stable than adiabatic BC
- Columnar vortices with isothermal boundary conditions appear at a later time compared to those with adiabatic boundary conditions

Why?

Hypothesis: production of baroclinic vorticity is responsible for the formation of vertical structures

Vorticity: $(\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{\omega} = (\boldsymbol{\omega} + 2\boldsymbol{e}_z) \cdot \boldsymbol{\nabla}\boldsymbol{u} - F\,B^2\boldsymbol{\nabla}(r\Theta) \times \boldsymbol{e}_r + B^2\boldsymbol{\nabla}\Theta \times \boldsymbol{e}_z + E\,\boldsymbol{\nabla}^2\boldsymbol{\omega}$

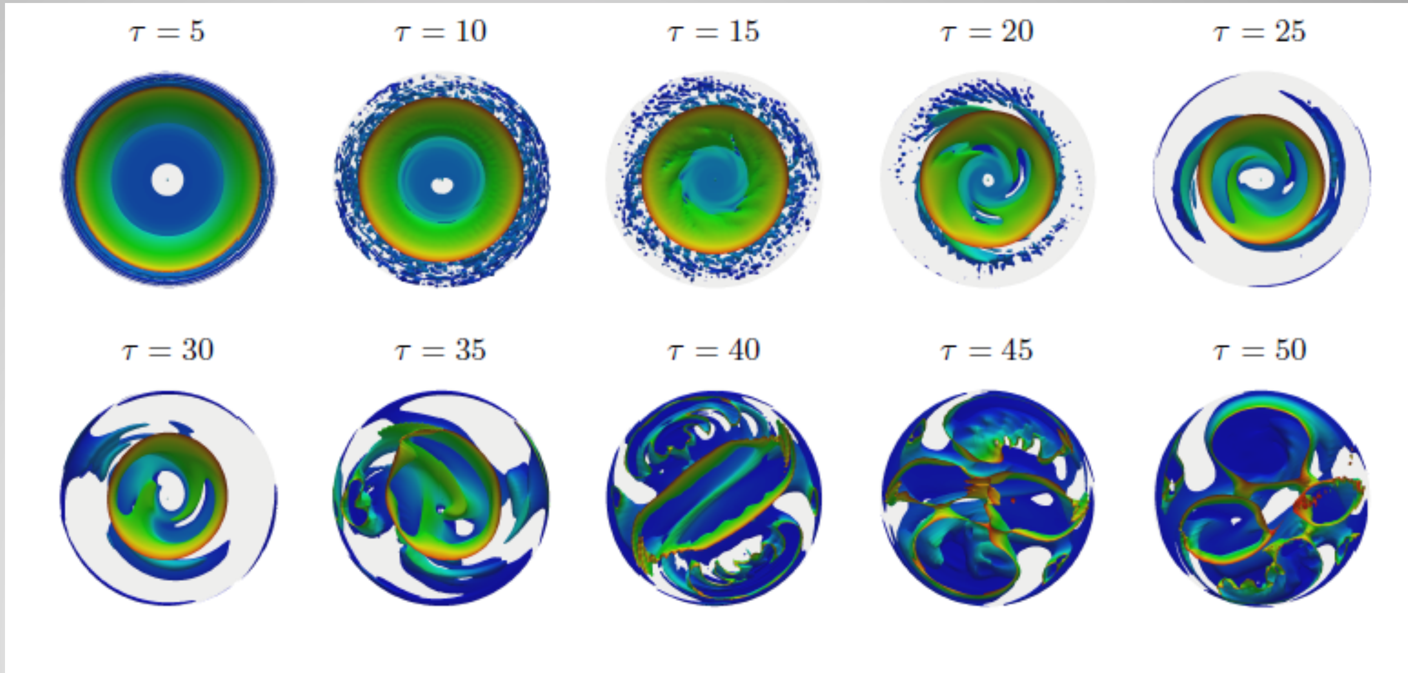
Baroclinic vorticity: $\boldsymbol{\omega}_b = \boldsymbol{\nabla}\Theta \times \boldsymbol{e}_z = \left(\frac{1}{r}\frac{\partial}{\partial\theta}\boldsymbol{e}_r - \frac{\partial}{\partial r}\boldsymbol{e}_\theta\right)\Theta.$

Radial variation cannot produce vertical structures!

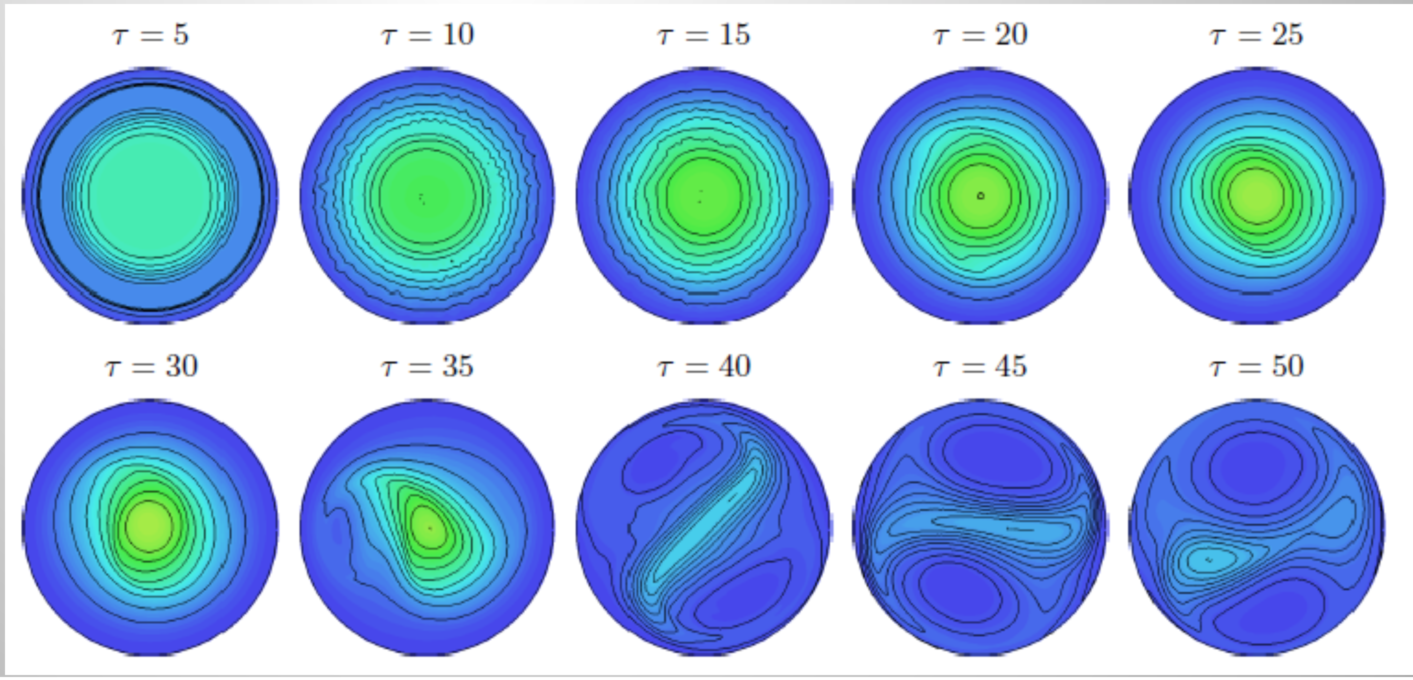
Azimuthal variation cannot produce vertical structures!

Feedback mechanism

Vortex structure:



Contours of temperature (z=0.15):



Kinetic energy growth rate of the azimuthal disturbance:

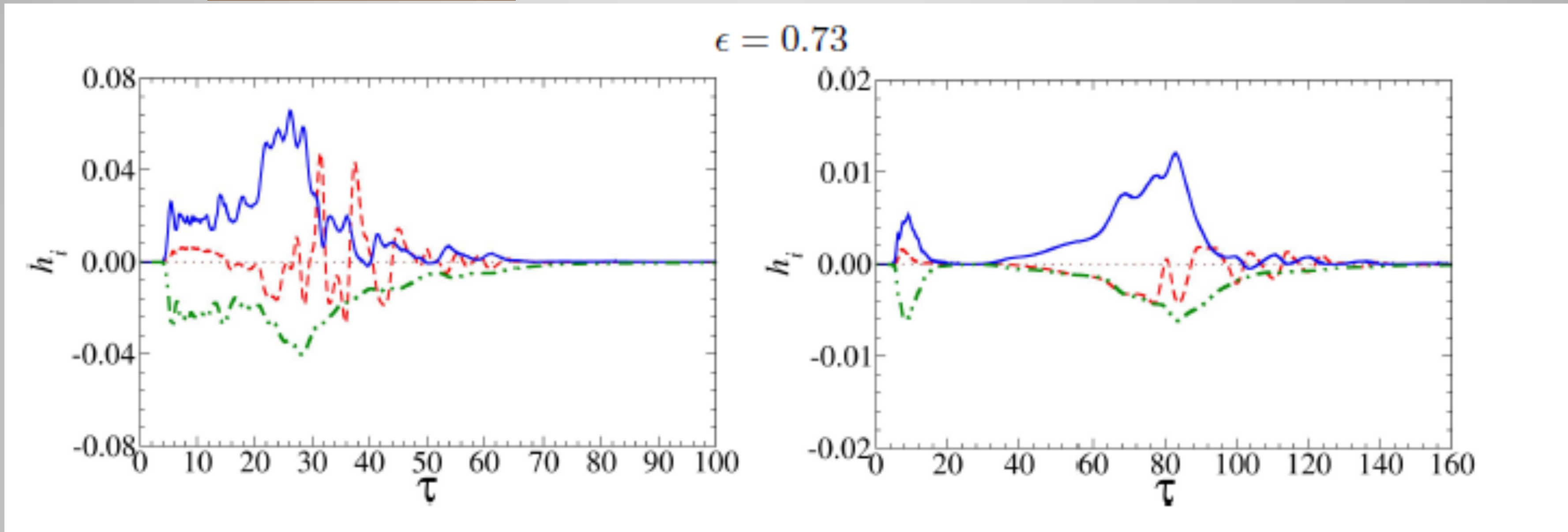
$$\frac{\mathbf{d}e}{\mathbf{d}t} = \frac{\mathbf{d}}{\mathbf{d}t}\int_V \frac{1}{2}|\vec{u}'|^2\mathbf{d}V = \underbrace{-\int_V \vec{u}'\bullet(\vec{u}'\bullet\nabla'\overline{\vec{u}})\mathbf{d}V}_{h_1} + \underbrace{B^2\int_V \Theta u_z'\mathbf{d}V}_{h_2} - \underbrace{FB^2\int_V \Theta r u_r'\mathbf{d}V}_{h_3} - \underbrace{E\int_V |\nabla'\vec{u}|^2\,\mathbf{d}V}_{h_4}$$

- h_1: shear of the mean axisymmetric flow (barotropic production)
- h_2: conversion of gravitational potential energy (baroclinic production)
- h_3: conversion of centrifugal potential energy
- h_4: viscous dissipation

Kinetic energy growth rate of the azimuthal disturbance

Adiabatic BC

Isothermal BC



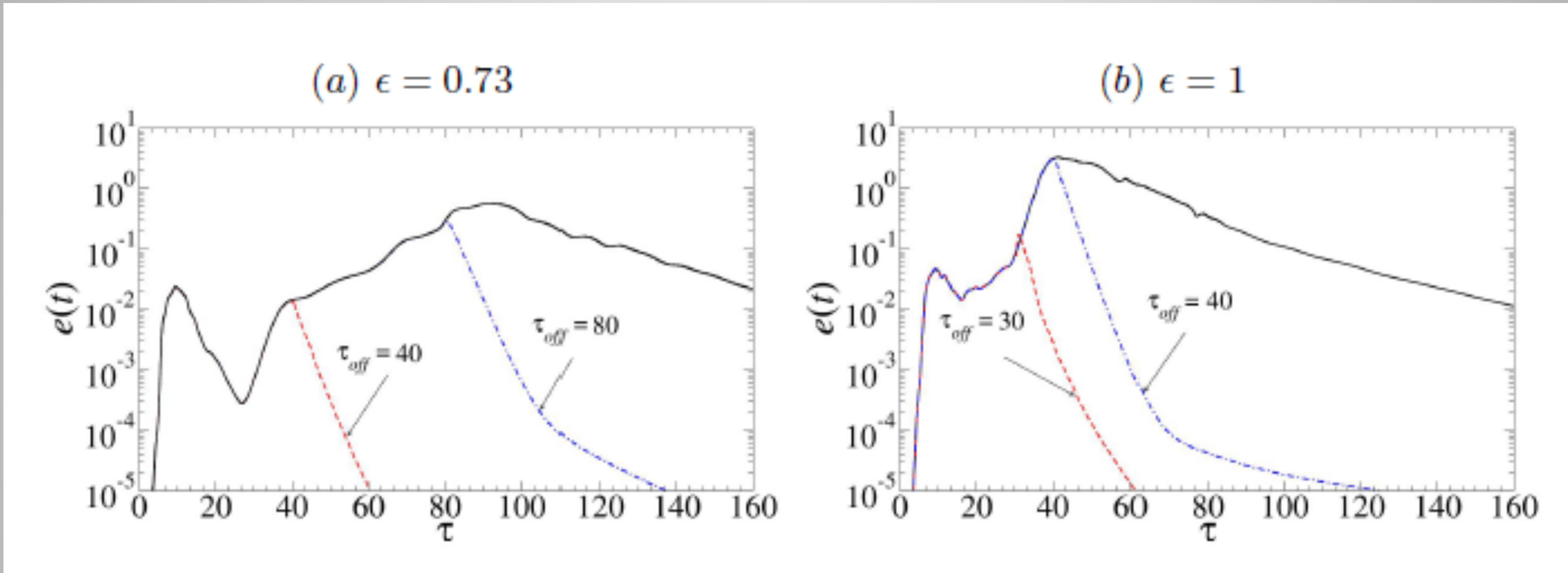
•h_1: shear of the mean axisymmetric flow (barotropic production)

•h_2: conversion of gravitational potential energy (baroclinic production)

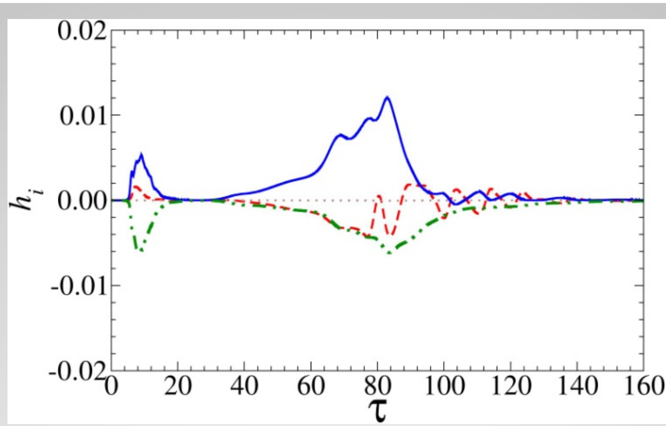
•h_3: conversion of centrifugal potential energy

•h_4: viscous dissipation

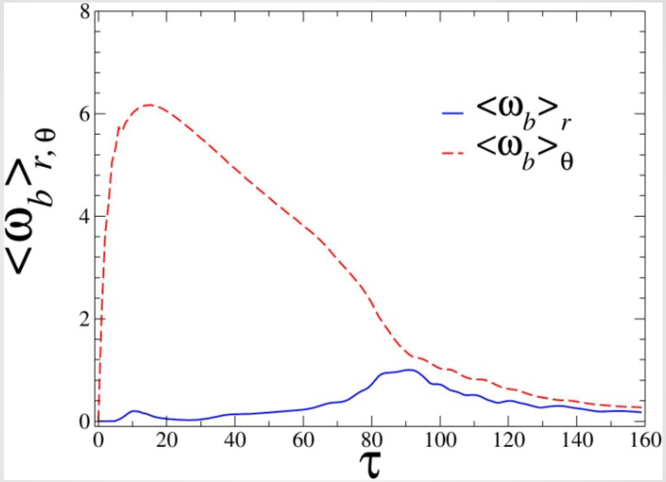
Turn off the baroclinic term



- h_i terms of the energy equation of azimuthal disturbances

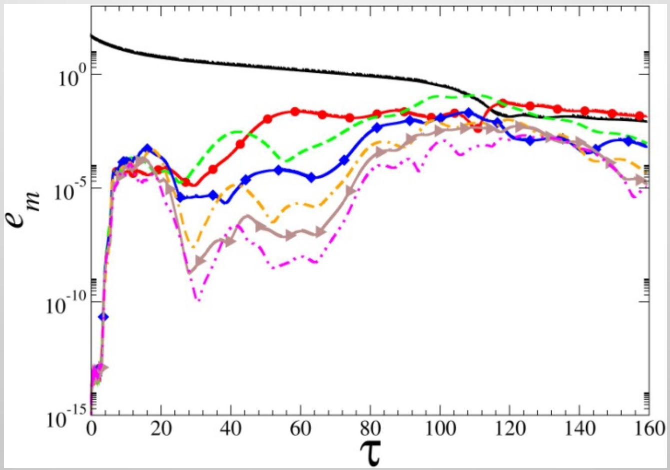


- Total baroclinic vorticity components



$$\omega_b = \nabla\Theta \times e_z = \left(\frac{1}{r}\frac{\partial}{\partial\theta}e_r - \frac{\partial}{\partial r}e_\theta\right)\Theta.$$

- Modal energy e_m



Conclusions*

- Radial variations of baroclinic vorticity produce an unstable system
- The azimuthal variations of baroclinic vorticity responsible for the formation of columnar vortices at late stages of flow development via feedback mechanism (enhances axial vorticity and increases vertical shear)
- Shear-free lateral boundary is more unstable
- Elliptical instability not important in this geometry

*Pacheco, J. R. and Verzicco, R. `Formation of columnar baroclinic vortices in thermally stratified non-linear spin-up,' *J. of Fluid Mech.* 2012

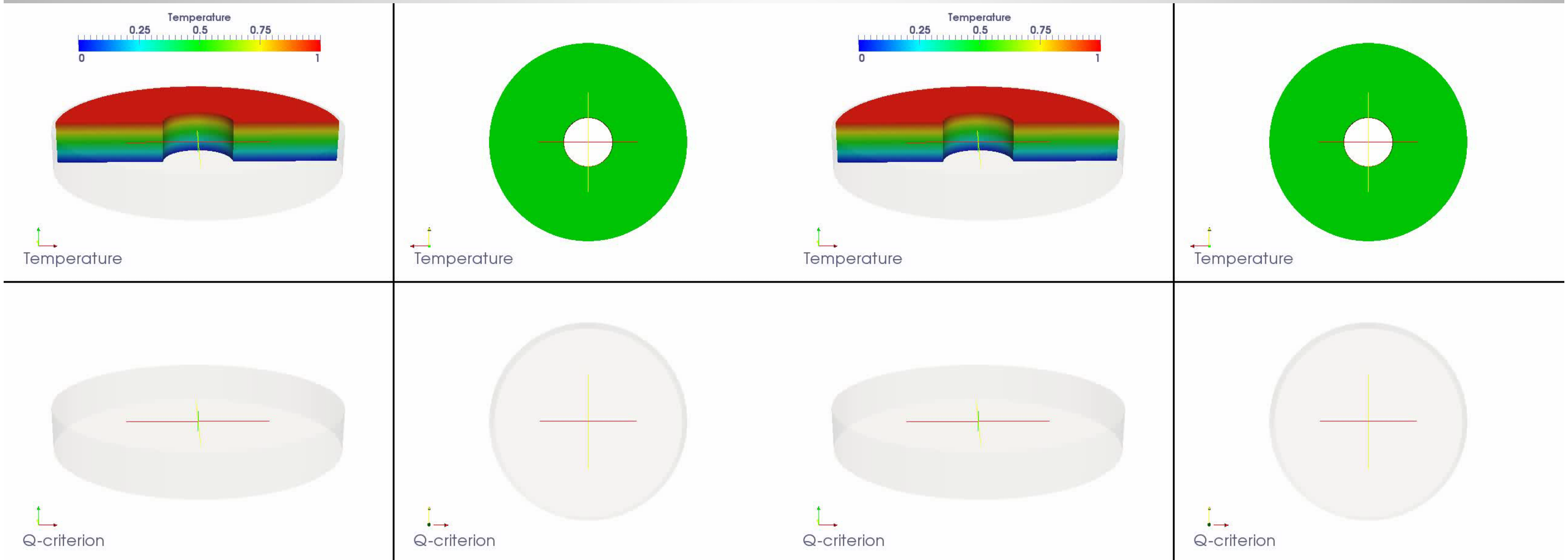
What other aspects of spin-up/down can be studied?

- Study in parameter space (Γ, B, ε, σ)
- Spin-up with sloping bottom (Ekman arrest)
- Uneven bottom boundary (wavy bottom)
- Heating/cooling on horizontal walls
- Non-Newtonian flows (lava flows)
- Different geometries: annulus, cone, half cone, half cylinder, etc.
- Spin-down
- Libration
- Turbulence!!!

Short annulus

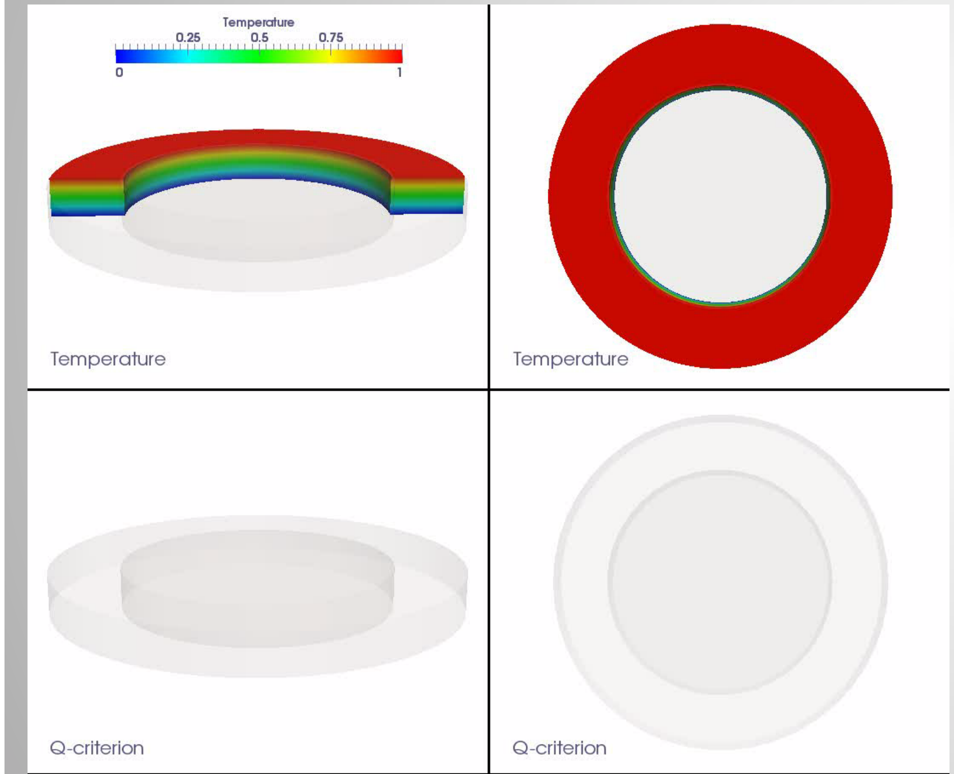
Neumann BC

Dirichlet BC

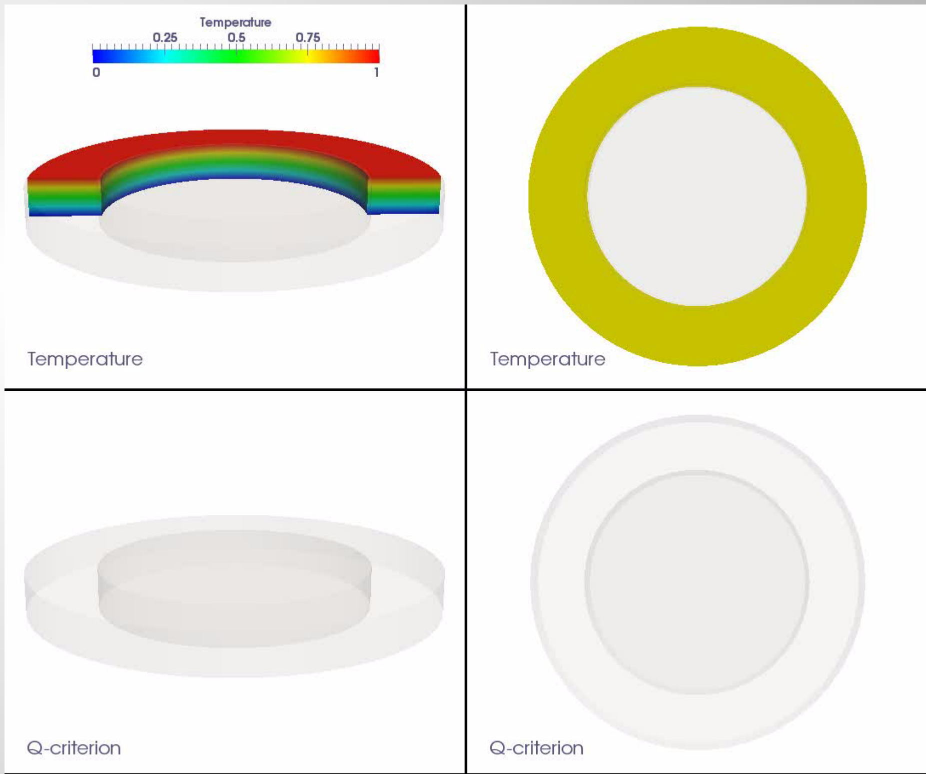


Thank you!

Neumann BC



Dirichlet BC



Ongoing

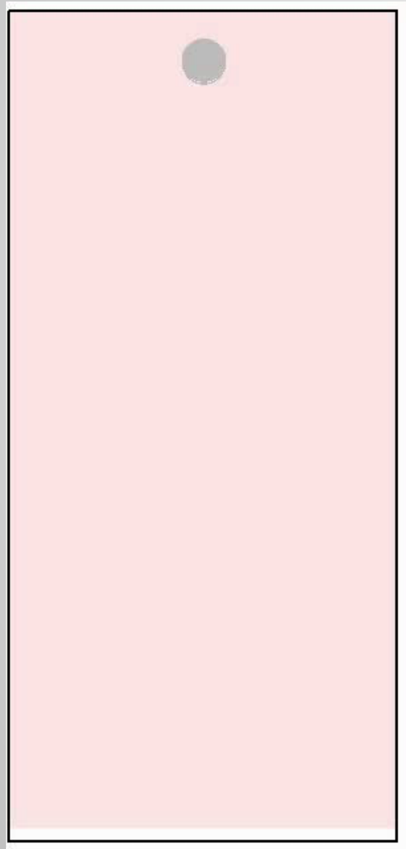
- Inertial waves
- Non-Boussinesq equations
- Non-Newtonian flows
- Particulate flows
- Gravity currents

Examples

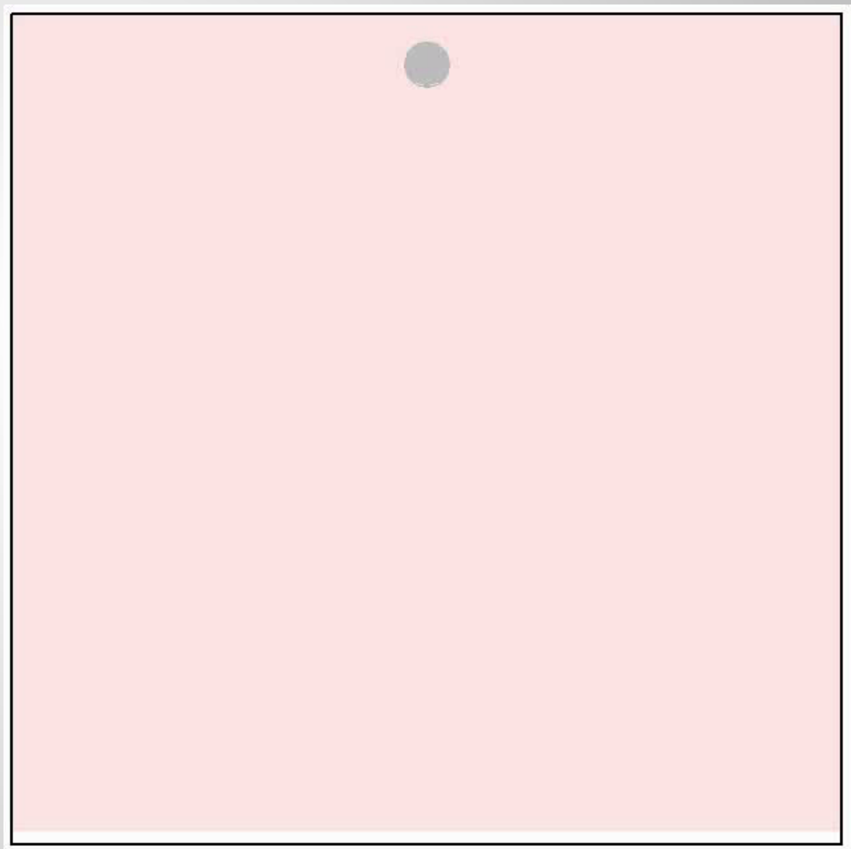
- Spin-up annulus
- Particulate flows
- Inertial waves

Particulate flows

No rotation*

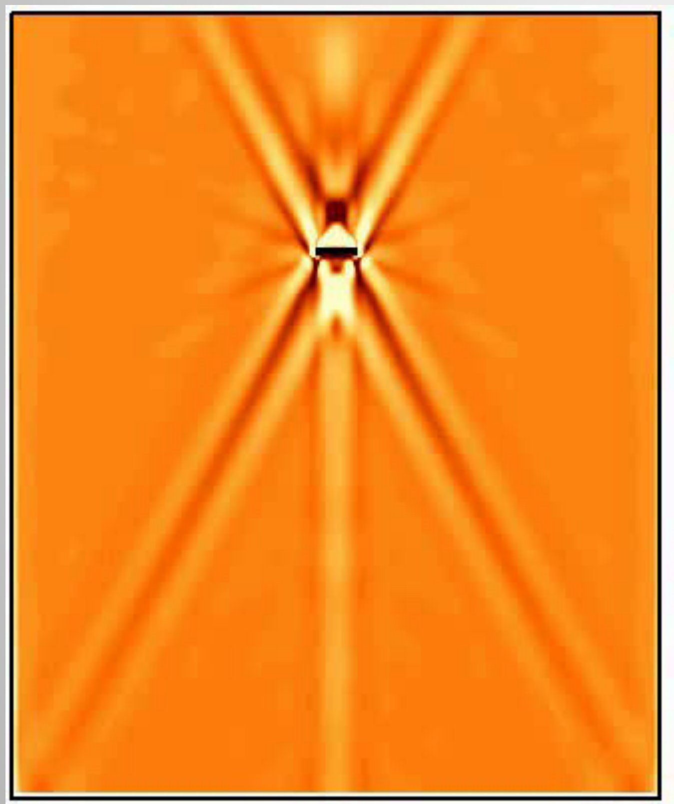


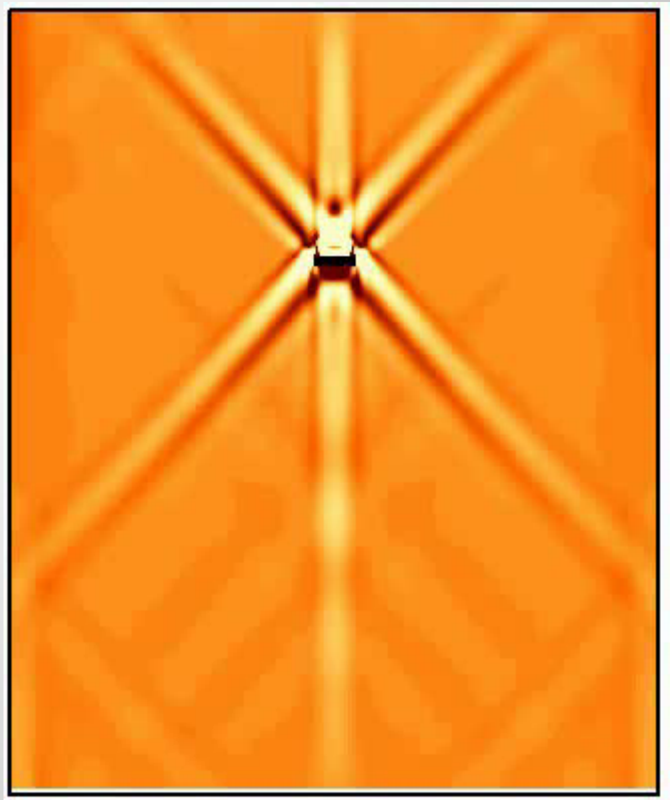
Rotation^



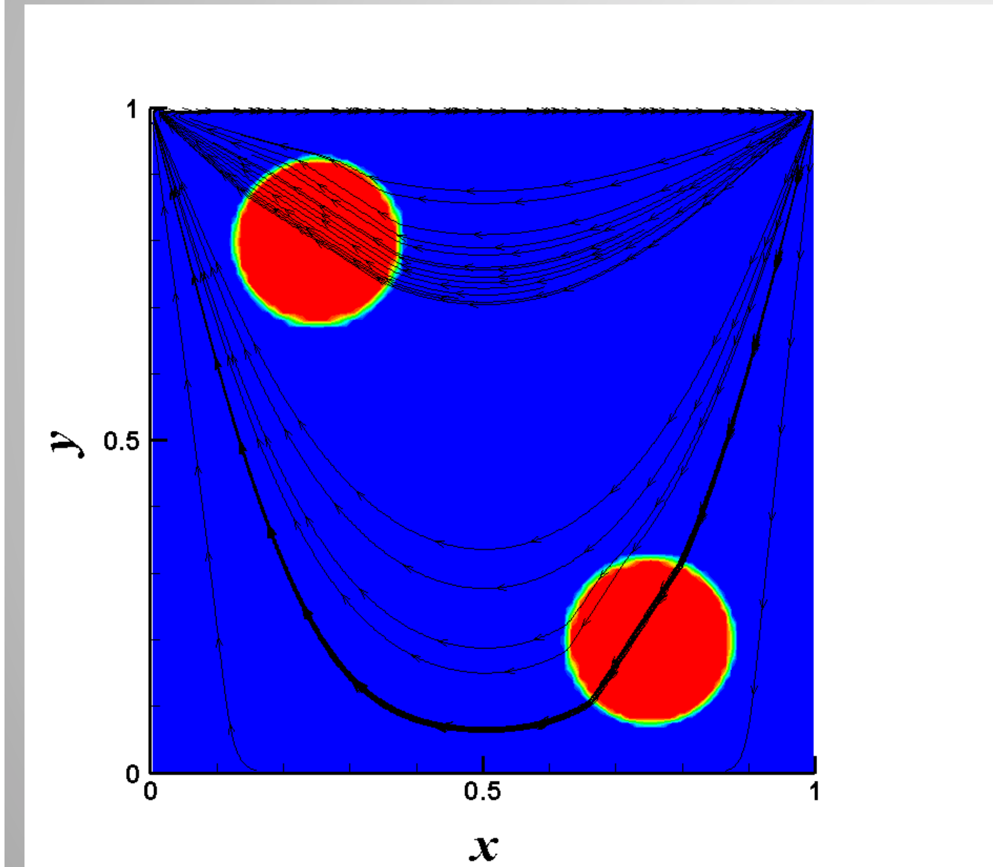
*Pacheco, J. R., Ruiz-Angulo, A., Zenit, R., and Verzicco, R., `Fluid velocity ucttuations in a collision of a sphere with a wall,‘ *Physics of Fluids* **23**(6): 063301, 2011
^Pacheco, J. R. Lopez. J. M. Verzicco, R. `Generation of inertial waves by a sphere settling in a rotating fluid.’ *J. Fluid Mech.* 2012.

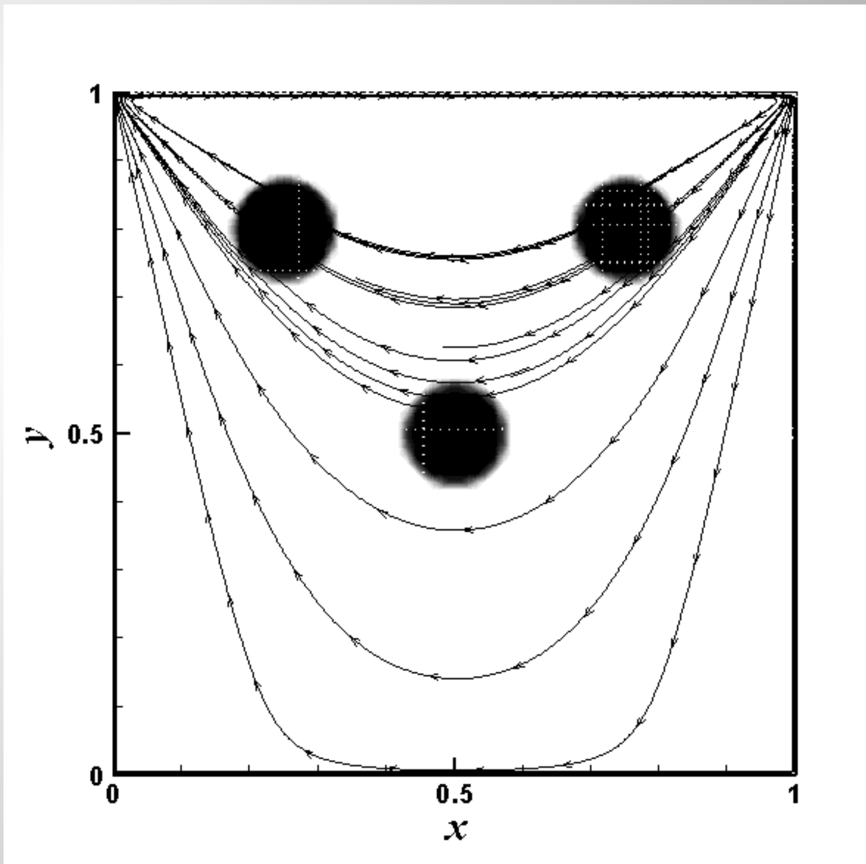
Inertial waves





Chaotic(?) mixing





Pacheco J. R. Mixing enhancement in electro-osmotic flows via modulation of electric fields. *Physics of Fluids*, 20 (9): pp. 093606, 2008.