Turbulence in the solar wind, spectra from Voyager 2 data

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Vortical Structures & Wall Turbulence, Prof. Orlandi Anniversary

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Voyager 2 Interstellar Mission

- **Voyager 2** is flying now at 15.6km/s, 104.7 AU from Earth, in the **Heliosheath**, the outermost layer of the heliosphere where the solar wind is slowed by the pressure of interstellar gas
- **Termination Shock** was passed on Sep 5, 2007

source: M. Opher et al.

A turbulence hypothesis for the magnetic field in the **Heliosheath**

"Is the magnetic field in the Heliosheath laminar or a turbulent sea of bubbles?"

source: M. Opher et al.

source: http://voyager.jpl.nasa.gov
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L.L. Orionis colliding with the Orion Nebula. Hubble Space Telescope, February 1995
(Credit: NASA, The Hubble Heritage Team (STScI/AURA))
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Year 1979: V and B data

Velocity and magnetic field data from V2, period 1979 (DOY 1–180). RTN heliographic reference frame.
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Year 1979: V and B data

V2 normalized data (1979)

Velocity and magnetic field data from V2, period 1979 (DOY 1–180). RTN heliographic reference frame.
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Year 1979: V and B data
Year 1979: V and B moments and PDFs

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units: km/s, nT

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normalized PDF of V and B – comparison with a Normal distribution. Evidence of anisotropy.
Year 1979: V and B moments and PDFs

PDF of module – comparison with a $\chi^2$ distribution. High intermittency?

- Evidence of high $Ku(>3)$
- Origin of “intermittency”: advected coherent structures (flux tubes, etc), stochastic Alfvénic fluctuations generated at solar corona and “frozen” in the wind?
- Intermittency interests a broad range of scales
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Autocorrelations

$$R_{ii}(\tau) = \langle x(t)x(t + \tau) \rangle$$
Cross-correlations tensor: off-diagonal terms

\[ R_{ij}(\tau) \equiv \langle x(t)y(t+\tau) \rangle \]
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Cross-correlations tensor: diagonal terms

Summary:

- Averages are computed on 57970 points for V, and 124080 points for B, spanning the whole 180 days period
- Evidence of a 25 days periodicity. Minimum of solar activity in 1979
- High cross-correlation $V_R B_R \rightarrow$ not in-phase
- High cross-correlation $V_R B_T \rightarrow$ not in-phase
- Low Alfvénic one-point correlation (this is often the case in the slow-wind periods)
Data reconstruction techniques

V2 velocity and magnetic field data are discontinuous and irregularly spaced. In the whole year 1979 there is 45% of missing velocity data. These values are about 97% in 2012. To perform an accurate spectral analysis on these kind of data sets, a reconstruction technique may be mandatory. In the following, the effect of two interpolation/recovery methodologies on averaged turbulent spectra will be discussed.

- Linear interpolation
- Maximum likelihood reconstruction and realizations constrained by data

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To test the effects of averaging, interpolating and windowing techniques, two 1D sequences of synthetic turbulence data have been generated from imposed spectral properties:

- **Synt 1** → $E_{3D}(n/n_0) = \frac{(n/n_0)^\beta}{(n/n_0)^{\alpha+\beta}}$

- **Synt 2** → $E_{3D}(n/n_0) = \frac{(n/n_0)^\beta}{(n/n_0)^{\alpha+\beta}} \left[ 1 - \exp\left(\frac{n-n_{tot}}{\gamma} + \epsilon\right) \right]$

$\beta = 2, \alpha = 5/3, n_0 = 11, \gamma = 10^4, \epsilon = 10^{-1}$

**Synt 1** mimics the Kolmogorov inertial range of fluid turbulence, **Synt 2** mimics both the inertial and the dissipative part of the spectrum.

- Synthetic data are scaled on a 180 days time grid ($\Delta t = 100$ s, $n_{tot} = 155520$)

- The same gaps of V2 velocity data are projected on these sequences

- Spectral analysis is carried out.
**Effect of interpolation on Synt 1 data**

\[ L_s = \text{length of reconstructed segments used to compute spectra}; \]
\[ L_g = \text{maximum length of filled gaps} \]

\[ f (\text{Hz}) \]
\[ V^2 (\text{Km}^2/s^2/\text{Hz}) \]

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**Spectral analysis:** synthetic turbulence

**Spectral analysis:** V2 velocity and mag. field data

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Effect of interpolation on **Synt 2** data

- Effect of segmentation: increase in slope of about 5% in the inertial range.
- Effect of linear interpolation: function of $L_g$ (length of “filled” gaps). This interpolation transfers energy to the low frequencies, resulting in an increase (about 6%) in the slope, especially in the high-frequency range ($f > 10^{-3}\text{Hz}$).
Effect of interpolation on Synt 2 data

- Effect of windowing: the Hann window function allows to eliminate spurious energy due to discontinuities ($\approx 1/f$) at the boundary of each segment. The effect is minimal at low wavenumbers. In the high-frequency range, on the one hand a significant increase (up to 23%) of the slope is found to be a function of $L_g$, on the other hand any change in slope of the real spectrum can be followed. Energy correction factor for Hann: $1.63^2$

- Without windowing, the segmentation error doesn’t allow to represent the correct slope, in the general case (see the analysis on Synt 2 data). These cases can be recognized by a flattening in the high-frequency range of the spectrum. Averaging long segments helps.
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V2 velocity spectra at 5 AU (pre-Jupiter)
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**V2 velocity spectra at 5 AU (pre-Jupiter)**

- **V2 velocity data**
  - 1979, DOY 1-180
  - $\alpha$: -1.56
  - $\alpha$: -1.49
  - $\alpha$: -1.60
  - $\alpha$: -1.51

- **V2 kinetic energy**
  - 1979, DOY 1-180
  - $\alpha$: -1.60
  - $\alpha$: -1.54
  - $\alpha$: -1.56
  - $\alpha$: -1.50

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**V2 velocity data**

1. **$V_R$ (Km²/s²/Hz)**
   - $\alpha$: -1.56
   - $\alpha$: -1.49
   - $\alpha$: -1.60
   - $\alpha$: -1.51
   - no recov, $L_s = 3 - 4$ hrs
   - no recov, $L_s = 3 - 4$ hrs, Hann
   - lin. recov, $L_g = 0.5$ hrs, $L_s = 12$ hrs
   - lin. recov, $L_g = 0.5$ hrs, $L_s = 12$ hrs, Hann

2. **$V_N$ (Km²/s²/Hz)**
   - $\alpha$: -1.60
   - $\alpha$: -1.54
   - $\alpha$: -1.52
   - $\alpha$: -1.49
   - $2.5 \times 10^{-3}$ Hz

3. **$E$ (Km²/s²/Hz)**
   - $\alpha$: -1.60
   - $\alpha$: -1.54
   - $\alpha$: -1.56
   - $\alpha$: -1.50

---

**Legend**
- no recov, $L_s = 3 - 4$ hrs
- no recov, $L_s = 3 - 4$ hrs, Hann
- lin. recov, $L_g = 0.5$ hrs, $L_s = 12$ hrs
- lin. recov, $L_g = 0.5$ hrs, $L_s = 12$ hrs, Hann
V2 mag. field spectra at 5 AU (pre-Jupiter)

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V2 spectra at 5 AU (pre-Jupiter)

Velocity:

- The observed frequency range constitute the inertial range
- All computed exponents \((10^{-4} < f < 2 \cdot 10^{-3} \text{ Hz})\) are flatter than the Kolmogorov one:
  \[ \alpha = -1.53 \pm 0.07 \]
- Computed slopes may be slightly overestimated
- A peak is located at \(f = 0.0026 \text{ Hz}\) for T and N components: is it physical or instrumentation-related? (no relation with \(f_{ci}, f_{pi}, f^*\))

Magnetic field:

- Computed exponents \((10^{-4} < f < 2 \cdot 10^{-3})\) higher lower than the velocity ones:
  \[ \alpha = -1.81 \pm 0.09 \]
- Observed steepening for \(f > 3 \cdot 10^{-3} \text{ Hz}\) should not be linked to interpolation issues: the situation recalls that of Synt 2 case, blue (no recovery) and violet (small gaps filled) give the same result.
- Anisotropy is higher with respect to the velocity field
G.B. Rybicki & W.H. Press prediction

• Minimum variance prediction (interpolation):
  \[ y = s + n \] irreg. spaced vector data with errors \( n \)
  \[ s^* = \sum_{i=1}^{M} d_i y_i + x_\ast \] \( s^* \) = true value at a particular point
  \[ \hat{s}^* = S^T [S + N]^{-1} y \] \( \hat{s}^* \) = min. variance estimate for \( s^* \)

Assuming stationary process:
  \[ S_{ij} = \langle s_i s_j \rangle = f(t_i - t_j) \] is the correlation matrix, estimated from data
  \[ N_{ii} = \langle n_i^2 \rangle \] is the errors diagonal matrix \( n_i \to \infty \) in “new” points

The min. variance estimation is not, however, a typical realization of the underlying process.

• Minimum variance prediction + Gaussian process

To obtain a typical realization, a Gaussian process is added to the min. var. estimate:
  \[ s_\ast = u_\ast + \hat{s}_\ast \]

If realizations constrained to data are desired:
  \[ u = V diag(\lambda_1^{1/2}, ..., \lambda_M^{1/2}) r \] where
  \[ \lambda_i = eig(Q), \quad Q = [S^{-1} + N^{-1}]^{-1}, \quad r = rand(\mu = 0, \sigma^2 = 1) \]
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Final considerations and future development

- **V2 data**: it is possible to obtain spectra from incomplete data (at least at 5 AU!)
- **velocity spectra** support the MHD cascade model (Iroshnikov–Kraichnan, -3/2 exponent): $-1.53 \pm 0.07$ exponent
- **magnetic field spectra** much steeper than velocity ones ($-1.81 \pm 0.09$)
- peak at $f = 2.6 \cdot 10^{-3}$ Hz in $V_T$ and $V_N$ spectra only: a feature of solar wind structure or an instrumentation problem? (note: Larmor frequency one order of magnitude higher)
- **Future work**:
  - comparison with V1 data (same exponents and peaks?)
  - analysis of the much challenging *Heliosheath* data (V2: 2007-2013, 97% of voids in data; switch to to *compress sensing* reconstruction method from telecommunication engineering.