SCALING OF THE SHEAR LAYERS
CONSTITUTING THE VORTICITY
COMPONENTS IN A TURBULENT CHANNEL FLOW

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Happy Birthday to someone who is forever young.

- Beginning of 2000’s visiting LEGI-Grenoble (Skiing😊)
- Doing experiments of active control through localized unsteady blowing ➔ Frustrated to not experimentally detect the vortex induced by...
- Paolo set-up his code in our Work Stations ➔ Could detect the structure after a while through DNS
- ➔ DOING DNS since..
DNS

*Large computational domain as in (Hoyas, Jiménez 2006)

** NS with Dispersion Relation Preserving spatial schemes

<table>
<thead>
<tr>
<th>Re,</th>
<th>Re, actual</th>
<th>Resolution ((N_x \times N_y \times N_z))</th>
<th>(\Delta x^*)</th>
<th>(\Delta y^*)</th>
<th>(\Delta z^*)</th>
<th>(L_x/h)</th>
<th>(L_y/h)</th>
<th>CFL</th>
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</thead>
<tbody>
<tr>
<td>180</td>
<td>177.73</td>
<td>771x129x387</td>
<td>8.80</td>
<td>0.49 ((0.31 \eta)) (5.59 ((1.52 \eta))</td>
<td>5.84</td>
<td>12(\pi)</td>
<td>4 (\pi)</td>
<td>0.24</td>
</tr>
<tr>
<td>395</td>
<td>388.77</td>
<td>1691x283x849</td>
<td>8.81</td>
<td>0.48 ((0.33 \eta)) (5.57 ((1.26 \eta))</td>
<td>5.85</td>
<td>12(\pi)</td>
<td>4 (\pi)</td>
<td>0.26</td>
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<tr>
<td>590</td>
<td>580.01</td>
<td>1651x423x1113</td>
<td>8.98</td>
<td>0.48 ((0.34 \eta)) (5.56 ((1.15 \eta))</td>
<td>5.00</td>
<td>8 (\pi)</td>
<td>3 (\pi)</td>
<td>0.33</td>
</tr>
<tr>
<td>1100</td>
<td>1090.82</td>
<td>3079x789x2075</td>
<td>8.98</td>
<td>0.48 ((0.34 \eta)) (5.55 ((0.98 \eta))</td>
<td>5.00</td>
<td>8 (\pi)</td>
<td>3 (\pi)</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Introduction

LSM-VLSM: MAJOR TURBULENT STATISTICS ARE RE DEPENDENT

- **Clustering** of quasi-streamwise vortices ➞ LSM

(takes place at all Reynolds numbers: Tardu (1994, 95, 2002; Adrian’s group, 1999...)

**TRANSPORT** 50 % of shear stress.
Mainly at the median point of log-layer

\[ y^*_R = 0 \equiv y^*_M = 3.9 \text{Re}_\tau^{1/2} \]

**TRANSPORT IS NOT (?) CONTRIBUTION !!**
EXCEPT THE WALL NORMAL VORTICITY INTENSITY
Turbulent intensities of the vorticity components in inner variables

Streamwise vorticity: \( \omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \); Spanwise vorticity: \( \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \); Wall normal vorticity: \( \omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \)

WHY the WALL NORMAL VORTICITY IS INSENSITIVE TO \( Re \)?
Asymptotic behavior of fluctuating velocity field $u$ and $w$ near the wall

- Constant shear stress zone (Townsend, Perry & Marusic...):
  \[ \lim_{y^* \to 0} \frac{u u}{u^2} (y^*) = A_{u u} \ln \frac{y}{h} + B_{u u} (\Pi) \]
  \[ \frac{u u}{u^2} (y^*) = -A_{u u} \ln \frac{y}{h} + B_{u u} (\Pi) \]
  \[ \frac{w w}{u^2} (y^*) = -A_{w w} \ln \frac{y}{h} + B_{w w} (\Pi) \]
  \[ \frac{v v}{u^2} (y^*) = B_{v v} (\Pi) \]

- Near the wall
  \[ \lim_{y^* \to 0} \frac{u u}{U^2} (y^*) = \sigma_{u z}^{+2} \]
  \[ \lim_{y^* \to 0} \frac{w w}{U^2} (y^*) = \sigma_{w x}^{+2} \]

- Modulation of the near wall velocity field in the viscous sublayer by the outer passive eddies (Mathis et al., JFM, 2013)
  \[ \tau_0^+(t^+; \text{Re}_\tau) = \tau_0^+ (t^+) \left[ 1 + \beta' u_{OM}^+(t^+; \text{Re}_\tau) \right] \]
  \[ + \alpha' u_{OM}^+(t^+; \text{Re}_\tau) \]

- $\sigma_{u z}^+ \propto \ln(\text{Re}_\tau)$
- $w$ structurally similar to $u$

  \[ \sigma_{w x}^+ \propto \ln(\text{Re}_\tau) \]
Intensity of shear layers contributing to the wall normal vorticity

\[
\omega_y^{+2} = (\partial u / \partial z)^{+2} + (\partial w / \partial x)^{+2} - 2 (\partial u / \partial z)^{+} (\partial w / \partial x)^{+}
\]

Streamwise \( \partial w / \partial x \) layers play a fundamental role in the generation of the quasi-streamwise vortices through the tilting term

\[
( - d \overline{U} / dy)(\partial w / \partial x)
\]

of the streamwise vorticity transport equation, BUT the contribution of \( (\partial w / \partial x)^{+2} \) to \( \omega_y^{+2} \) is \textit{ONE ORDER} of magnitude smaller compared to the spanwise shear layers

\[
(\partial u / \partial z)^{+2}
\]
Intensity of shear layers contributing to the wall normal vorticity. Minor contributions coming from $w$ streamwise shear layers

*Minor contributions strongly Re dependent*
Major Contribution comes from the spanwise $u$ shear layers (an order of magnitude)

*Remarkably insensitive to $Re$*

*Thin and long low and high speed streaks*

*Among all the shear layer components constituting the vorticity components*

*ONLY the intensity $\left( \frac{\partial u}{\partial z} \right)^2$ IS INSENSITIVE to $Re$*
One dimensional (spanwise) spectral density 
\((y^+=15)\)

\[
\left( \frac{\partial u}{\partial z} \right)^2 = \int_{0}^{\infty} k_z^+ E_{uu}^+(k_z^+) dk_z^+ \quad \Rightarrow \text{Reynolds invariant}
\]

The one dimensional spectral distribution also is only moderately \(Re\) dependent.
Vorticity transport

Transport: \( \frac{\omega_y^2}{2} \)

\( 0 = P - T - \varepsilon + D \)

Production

\( P = \omega_y \omega_i \frac{\partial v}{\partial x_i} = \omega_y \omega_x \frac{\partial v}{\partial x} + \omega_y \omega_y \frac{\partial v}{\partial y} + \omega_y \left( \omega_z + \Omega_z \right) \frac{\partial v}{\partial z} \)

Turbulent transport

\( T = \frac{1}{2} u_i \frac{\partial \omega_y^2}{\partial x_i} = \frac{1}{2} \left( u + U \right) \frac{\partial \omega_y^2}{\partial x} + \frac{1}{2} v \frac{\partial \omega_y^2}{\partial y} + \frac{1}{2} w \frac{\partial \omega_y^2}{\partial z} \)

Dissipation

\( \varepsilon = \nu \left\{ \left( \frac{\partial \omega_y}{\partial x} \right)^2 + \left( \frac{\partial \omega_y}{\partial y} \right)^2 + \left( \frac{\partial \omega_y}{\partial z} \right)^2 \right\} \)

Diffusion

\( D = \nu \frac{1}{2} \frac{\partial^2 \omega_y^2}{\partial x_i \partial x_i} = \nu \frac{1}{2} \left\{ \frac{\partial^2 \omega_y^2}{\partial x^2} + \frac{\partial^2 \omega_y^2}{\partial y^2} + \frac{\partial^2 \omega_y^2}{\partial z^2} \right\} \)
Streamwise vorticity (intensity) transport
Spanwise vorticity transport

![Graph showing spanwise vorticity transport](image-url)
Wall normal vorticity transport

![Graph showing wall normal vorticity transport with various curves representing different processes such as production, transport, dissipation, and diffusion. The graph plots y^* against a log scale on the x-axis and a linear scale on the y-axis, with various markers and line styles to differentiate between the processes.](image)
Vorticity components transport
General remarks

• Maximum production of the streamwise and wall normal vorticity intensities takes place in the median buffer layer at \( y^+ = 10 \) to \( 15 \).

**Despite the fact that \( \omega_y^+ \) is Re independent, the production and dissipation of the wall normal vorticity depends on the Reynolds number. The dissipation is not in equilibrium with the production.

• On the contrary the spanwise vorticity production peaks in the viscous sublayer at \( y^+ = 3 \).

• Dissipation is in equilibrium with diffusion next to the wall as usual. Both quantities are Re independent near \( y=0 \) in \( \omega_y^+ \).
Spanwise $u$ shear layers

Transport

\[
\frac{D}{Dt}\left(\frac{\partial u}{\partial z}\right)^2 = -2\frac{\partial u}{\partial z} \frac{\partial v}{\partial z} \frac{\partial (\bar{U} + u)}{\partial y} + 2\left(\frac{\partial u}{\partial z}\right)^2 \frac{\partial v}{\partial y} + 2R \frac{\partial u}{\partial z}
\]

\[R = -\frac{\partial^2 p}{\partial x \partial z} + \nu \nabla^2 \left(\frac{\partial u}{\partial z}\right)\]

- A is a kind of production by tilting of the mean shear
Spanwise $u$ shear layers
Transport

$A^+$ is twice larger than $B^+$
Both of them are remarkably $Re$ independent at $y^+ > 20$
Discussion

WHY $\sigma_{\omega_y}^+$ is invariant with Re?

- One has in Fourier space

$$u = -\frac{ik_z}{k_x^2 + k_z^2} \omega_y + \frac{ik_x}{k_x^2 + k_z^2} \left( \frac{\partial v}{\partial y} \right)$$

(Definition + Continuity)

$$E_{uu} = \frac{k_z^2}{\left(k_x^2 + k_z^2\right)^2} E_{\omega_y \omega_y} + \frac{k_x^2}{\left(k_x^2 + k_z^2\right)^2} E_{\partial v \partial v} - \frac{2k_x k_z}{\left(k_x^2 + k_z^2\right)^2} \left\{ I_{\omega_y} I_{\partial v} + R_{\omega_y} R_{\partial v} \right\}$$

$I$: Imaginary part, $R$: Real part; Premultiplied spectra of each term in wall units $\Rightarrow$

$$A_{\omega_y} = k_x k_z \frac{k_z^2}{\left(k_x^2 + k_z^2\right)^2} E_{\omega_y \omega_y}$$

$$B_{\partial v} = k_x k_z \frac{k_x^2}{\left(k_x^2 + k_z^2\right)^2} E_{\partial v \partial v}$$

$$C_{\omega_y, \partial v} = -\frac{2k_x^2 k_z^2}{\left(k_x^2 + k_z^2\right)^2} \left\{ I_{\omega_y} I_{\partial v} + R_{\omega_y} R_{\partial v} \right\}$$

$$D = B + C$$
Premultiplied spd’s of the ensemble of these terms

\[ k_x k_z E_{uu} \]

\[ A_{\omega_y} = k_x k_z \frac{k_z^2}{(k_x^2 + k_z^2)^2} E_{\omega_x \omega_y} \]

Reto 395(lines) and 1100 (color). Two contours for each spectral density, 0.125 and 0.625 the maximum at the highest Re number (as in Hoyas and Jiménez, PoF, 2006). Note the inactive ridge. The contours in the spectra related to the wall normal vorticity have almost the same magnitude.
Terms coming from the flux of the wall normal velocity and interactions between $v$ and wall normal velocity have different spectral supports and are an order of magnitude smaller.

\[
\frac{k_x k_z}{\left( k_x^2 + k_z^2 \right)^2} \frac{k_x^2}{2} E \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} - \frac{2k_x^2 k_z^2}{\left( k_x^2 + k_z^2 \right)^2} \left( I_{\omega_y} \frac{\partial v}{\partial y} + R_{\omega_y} \frac{\partial v}{\partial y} \right)
\]
Wall normal vorticity is connected to the low pass filtered $u$

$$E_{uu} = \frac{k_z^2}{(k_x^2 + k_z^2)^2} E_{\omega_x\omega_y} \Rightarrow u = -\frac{uk_z}{k_x^2 + k_z^2} \omega_y \Rightarrow \omega_y \equiv \left[-\frac{k_x^2 + k_z^2}{k_z}\right] u$$

LOW PASS FILTER $H(uk_x, uk_z)$

$k_x k_z E_{uu}$ and contours of the filter amplitude $|H|^2$ ⇒ Filters ridges $k_x k_z E_{\omega_x\omega_y}$ INDEPENDENT of $Re_\tau$
CONCLUSION

• Maximum production of the streamwise and wall normal vorticity intensities takes place in the median buffer layer at $y^+=15$.

• On the contrary the spanwise vorticity production peaks in the viscous sublayer at $y^+=3$.

• Wall normal vorticity is dominated by the spanwise $u$ velocity shear layers whose intensity is $Re$ independent.

• It is related to the low pass filtered streamwise velocity field and therefore not influenced by the passive structures. Its intensity distribution in wall units is universal.

• ?