Dicembre 2012 *RR*-21.12

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A Game Theory Approach for Regulating Hazmat Transportation

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Abstract

In this paper, we study a novel toll setting policy to regulate hazardous material (hazmat) transportation, where the regulator (e.g., a government authority) aims at minimizing not only the network total risk (see Marcotte et al., 2009), but also spreading the risk in an equitable way over a given road transportation network. The idea is to use a toll setting policy to discourage carriers transporting hazmat from overloading portions of the network with the consequent increase of the risk exposure of the population involved. Specifically, we assume that the toll paid by a carrier on a network link depends on the usage of that link by all carriers. Therefore the route choices of each carrier depend on the other carriers choices, and the tolls deter the carriers from using links with high total risk. The resulting model is a mathematical programming problem with equilibrium constraints (MPEC), where the inner problem is a Nash game having as players the carriers, each one wishing to minimize his/her travel cost (including tolls); the outer problem is addressed by the government authority, whose aim is finding the link tolls that induce the carriers to choose route plans that minimize both the network total risk and the maximum link total risk among the network links (in order to address risk equity). In order to guarantee the stability of the solution, we study conditions for the existence and uniqueness of the Nash equilibrium, and propose a local search heuristic for

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the MPEC problem based on these conditions. Computational results are carried out on a real network, comparing the performance of our toll setting policy with the toll setting approach proposed in Marcotte et al. (2009). The results show the effectiveness of our approach.

Keywords: Hazardous materials transportation, Toll setting, Bilevel optimization, Nash game, Heuristic approach.

1 Introduction

What distinguishes the transportation of dangerous goods or hazardous materials (*hazmats*) from the more general freight transport issues is basically the risk associated with an accidental release of hazmats during their transportation.

Even if statistic data on hazmat transportation (see, e.g., Kara and Verter, 2004) reveal that the number of accidents is very small compared to the number of hazmat shipments, both at world level and at national level, this chance imposes a particular attention to the safety management in order to reduce the occurrence and/or the consequence magnitude of dangerous events whose impact may be very large in terms of fatalities, injuries, large-scale evacuations, and severe environmental damage. More in general, the potential dangers on both the population and the environment make people very sensitive to this kind of transport. For this reason, hazmat transportation has stimulated a relevant research investigation.

Hazmat route planning is one of the main issues in hazmat transportation and deals with selecting where to route hazmat shipment orders among the alternative paths (routes) between origin-destination pairs on a given road network. The choice of a path depends on the different objectives that the distinct actors involved in the decision process want to pursue.

Of course carriers are one of the main actors. From their perspective, shipment contracts can be considered individually and a route decision between a given origin-destination pair needs to be made for each shipment with the objective of minimizing the transportation costs. Thus, for each shipment, this problem focuses on a single-commodity and a single origin-destination route plan, i.e., a "local route planning problem" (for a review on this subject see, e.g., the survey of Erkut et al., 2007).

Indeed, hazmat local route planning suffers from some limitations. Since the plans of the carriers are typically made without taking into account the general context, it may happen that certain links of the transportation network tend to be overloaded with hazmat traffic. This may result in a considerable increase of the accident probability on such road links, leading to inequity on the spatial distribution of the risk over the region where the road transportation network is embedded.

A chance to overcome this difficulty is to consider a government authority charged with the management of all hazmat shipments within and through its jurisdiction area.

Indeed, although the transportation industry is deregulated in many countries, hazmat transportation usually remains part of the governments' mandate mainly due to the associated societal and environmental risks. Hence, it is common to assume that there is a government authority that wants to regulate the hazmat transportation on the network under its jurisdiction. This authority, in contrast with the single carrier, has to consider a transportation problem that involves multi-commodities and multiple origin-destination route decisions, i.e., a "global route planning problem".

The main concern for a government authority is controlling the risk induced by hazmat transportation over the population and the environment. Besides the minimization of the total risk, a government authority should also promote equity in the spatial distribution of risk. This becomes crucial when certain populated zones are exposed to intolerable levels of risk as a result of the carriers' route decisions. Therefore, in hazmat global route planning, the main problem (from the authority's point of view) is that of finding minimum risk routes, while limiting and equitably spreading the risk in any area in which the transportation network is embedded.

Since, typically, the government authority does not have the right to impose specific routes to hazmat carriers, it can only mitigate hazmat transportation risk by means of policies regulating the use of road segments for hazmat shipments. The scenarios are essentially two. In the first one, the authority has the right either to close certain road segments to hazmat vehicles or to limit the amount of hazmat traffic flow on those network links. In the second scenario, the authority uses link tolls to deter the carriers from using certain road segments and consequently induces them to route their shipments on less populated (or risky) links of the network. In the context of global route planning the first policy falls into the field of "hazmat transportation network design" (e.g., see the seminal paper of Kara and Verter, 2004), and the second one belongs to "toll setting policies" (see, e.g., Marcotte et al., 2009). For both cases, typically the problems addressed by the government authority can be modeled as bilevel optimization problems. For a survey on bilevel programming, see, e.g., Colson et al.

(2005, 2007) and Dempe (2003, 2005).

In particular, in the bilevel problem for toll setting studied by Marcotte et al. (2009), the outer problem is addressed by the government authority, or regulator (i.e., the leader in the bilevel problem), that sets tolls on network links that minimize a combination of total population exposure (i.e., the risk) and total carriers' travel cost; this is done taking into account that in the inner problem the carriers (i.e., the followers), each one independently from each other, select routes minimizing the travel cost (including the additional cost due to the tolls), given the link tolls set by the government authority.

In this paper, we propose a new toll setting model that extends the one proposed by Marcotte et al. (2009). A limit of the latter model is that it does not keep into account risk equity. Indeed, it considers unsplittable hazmat shipments: that is, all the trucks used by a carrier are assumed to follow the same route selected by that carrier, with the consequence that it is not possible to distribute and balance (on the whole network) the risk generated by a carrier transporting a large amount of hazmat. Similarly, it may happen that a certain link is used by many carriers, resulting in a very high risk on that link even if the total risk on the network is relatively low.

The aim of the proposed model is to overcome this drawback, assuming a different toll setting policy where the toll paid by a carrier on a link depends also on the total risk induced on that link by all the carriers' route choices. This implies that, differently from the model of Marcotte et al. (2009), in our model the route choices of each carrier depend on the other carriers' choices, and the link tolls are chosen by the government authority also to deter the carriers from using links with high total risk. The resulting model is a mathematical programming problem with equilibrium constraints (MPEC) (see, e.g., Luo et al., 1996, for MPEC), where the inner problem is a Nash game having as players the carriers, each one choosing a hazmat transportation route plan that minimizes his/her travel cost (including tolls); the outer problem is addressed by the government authority, that aims at finding the link tolls that induce the carriers to choose route plans that minimize both the network total risk and the maximum link total risk among the network links (in order to address risk equity).

To guarantee the stability of the solution we find conditions for the existence and uniqueness of the Nash equilibrium, that are valid if all the carriers transport the same hazmat type. These conditions allow us to reformulate the Nash game as a strictly convex optimization problem, reducing therefore our MPEC problem to a bilevel optimization problem. Moreover, we provide toll optimality conditions that allow us to identify the optimal toll setting (with respect to carriers' cost minimization) inducing a given a Nash (unique) equilibrium. To solve real case studies, we propose a local search heuristic for the MPEC problem based on these conditions. Computational results are carried out on a real network, comparing the performance of our toll setting policy with the one proposed in Marcotte et al. (2009).

The remainder of the paper is organized as follows. Section 2 contains an overview of the relevant literature in hazmat global route planning. Section 3 presents the mathematical formulation of the considered hazmat toll setting problem. In Section 4, we show how to solve the Nash game, by identifying existence and uniqueness conditions of the Nash equilibrium. In Section 5, we study the special case with a single hazmat type where the above conditions are valid, and provide additional toll optimality conditions which are exploited in the proposed heuristic approach described in Section 6. Section 7 presents our computational experiments, and Section 8 concludes the paper.

2 Literature overview

Besides minimizing network total risk, one of the main issues addressed in hazmat global route planning is risk equity, and, hence, finding minimum risk routes, assuring, at the same time, an equitable distribution of the risk on the interested area where the given network is embedded. The contributions in this field include the early works of Zografos and Davis (1989), Gopalan et al. (1990), Lindner-Dutton et al. (1991), and Marianov and ReVelle (1998). The works of Akgün et al. (2000), Dell'Olmo et al. (2005), and Carotenuto et al. (2007) on the problem of finding a number of spatially dissimilar paths between an origin and a destination can also be considered in this area, along with more recent works proposing multi-objective approaches for selecting dissimilar paths (see, e.g., Caramia and Giordani, 2009, and Caramia et al., 2010).

The works above reviewed assume that the government authority has the right to impose routes on individual carriers. However, this is not the case in general. Indeed, for example, many governments only have the authority to close certain road segments to hazmat vehicles or to limit the amount of hazmat traffic flow on those links. These kinds of policies are usually categorized as Hazmat Transportation Network Design (HTND) policies, and equity concerns can be incorporated into the design objectives.

Kara and Verter (2004) were the first to study the HTND proposing a bilevel integer programming model by considering the roles of the carriers and of a government authority, where the latter imposes restrictions on the network and the carriers then choose the routes. Since the followers' problem is linear, the bilevel integer programming problem is reformulated as a single-level Mixed Integer Programming (MIP) problem by replacing the followers' problem by its Karush-Kuhn-Tucker (KKT) optimality conditions and by linearizing the complementary slackness constraints. However, the single-level reformulation may fail to find an optimal stable solution for the bilevel model. Indeed, in general, there are multiple minimum-cost route solutions for the followers over the designed network established by the leader, which may induce different total risk values over the network, and, hence, the lack of stability of the solution of the bilevel model provided by the single-level reformulation.

Erkut and Alp (2007) consider the same model of Kara and Verter (2004), restricting the network to a tree, so that there is a single path between each couple of origin-destination pair; clearly, with this restriction, the carriers have no alternative paths on the tree, hence the carriers have no freedom in route selection, with the result that the structure of the proposed model has a single level.

Erkut and Gzara (2008) generalize the model of Kara and Verter (2004) by considering the undirected network case and the design of the same network for all the shipments. They consider the possible lack of stability of the solution of the bilevel model obtained by solving the single-level reformulation, and propose a heuristic solution method that always finds a stable solution. Moreover, they extend the bilevel model to account for the cost/risk trade-off by including cost in the objective function of the leader problem.

Verter and Kara (2008) provide a path-based formulation for the HTND problem studied by Kara and Verter (2004), where the open links chosen by the regulator in the given network determine the set of paths that are available to the carriers. This facilitates the incorporation of carriers' cost concerns in regulator's risk reduction decision, and allows to formulate the problem with a single-level integer programming formulation assuring that the cheapest path among the available ones is used by each carrier.

Bianco et al. (2009) present a bilevel formulation focused on risk equity. Both levels correspond to government agencies: the meta-local authority that aims to minimize the maximum link risk in the whole network, and the regional area authority that aims to minimize the total risk over the network.

Dadkar et al. (2010) and Reilly et al. (2012) focus on a new HTND problem that takes into account security issues by modeling the possible role of a terrorist.

HTND policies can effectively restrict hazmat shipments in order to induce carriers to route shipments on low-risk paths. However, such a restriction could be too much since it does not consider the carriers' priorities, possibly wasting the usability of certain road segments. Moreover, only restricting certain road segments could not rationally direct hazmat transportation along less-populated links. An alternative policy to HTND is proposed by Marcotte et al. (2009) and is based on toll setting. This Hazmat Transportation Toll Setting (HTTS) policy may discourage (but not prevent) hazmat carriers from using certain road segments. The authors show that HTTS policy is a more flexible regulation tool for hazmat transportation than HTND policy. By imposing tolls on certain road segments, the hazmat shipments are expected to be directed on less-populated roads according to the carriers' own selection (due to economic considerations) rather than by governors' restriction. The authors show that the HTTS model dominates the HTND model since the solution set of the former strictly contains the solution set of the latter; therefore, the former model may result in a more attractive policy to regulators since provides more flexible solutions, and at the same time more acceptable to carriers that maintain the freedom of using any link of the network.

Marcotte et al. (2009) propose a bilevel model that minimizes the network total risk and the carriers' total travel cost, including both transportation costs and toll fees on the links. They reduce the bilevel problem to a single-level MIP problem by replacing the followers' problem with its optimality conditions and by linearizing the complementary slackness constraints; they also provide an alternative single-level MIP reformulation where complementary slackness conditions are replaced with the equality between the primal and dual objective functions of the followers' problem. Moreover, they show that the authority can easily find a toll setting inducing a minimum risk solution by inverse optimization, that is, determining the link tolls that induce the carriers to choose the minimum risk route plan; hence, in this case, the problem is not a bilevel problem anymore, but reduces to a single-level problem. However, if the authority wants to keep into account also carriers' cost, the problem is a true bilevel optimization problem.

More recently, Wang et al. (2011) propose a model assuming that both hazmat traffic and regular traffic affect population safety, since congestion increases delay and then accident probabilities. The idea is to control both regular and hazmat traffic via toll setting. The authors make the following assumptions: 1) congestion induced by the traffic flow of hazmat trucks can be ignored; 2) to simplify the model, the users have perfect information of the current status; 3) all the model parameters are deterministic; 4) a single type of hazmat is considered; 5) travel delay is a linear function of traffic congestion; 6) risk is linearly affected by travel delay. The authors provide a bilevel formulation and an equivalent two-stage problem formulation, where the first-stage problem is a non-convex quadratic programming problem and the latter is a linear programming problem.

3 Modeling the HTTS problem

Let $G = (N, A)$ be a direct (road transportation) network, with N being the set of n nodes (intersections in the road network), and A being the set of m directed links or arcs (road segments) between pairs of nodes. Let $A^+(i)$ and $A^-(i)$ be the subset of outgoing links and the subset of ingoing links of node $i \in N$, respectively, that is, $A^+(i) = \{(i,j) \in A : j \in N\}$ and $A^-(i) = \{(j,i) \in A : j \in N\}$.

Given a set H of hazmat type, we consider a set K of p carriers, with carrier $k \in K$ having to satisfy a single shipment order (commodity) of amount b^k of hazmat of type $h(k) \in H$, from origin node s^k to destination node t^k. Accordingly, let e_i^k be equal to $1, -1$ or 0 depending on whether node $i \in N$ is the origin node (i.e., $i \equiv s^k$), the destination node (i.e., $i \equiv t^k$), or a transshipment node (i.e., $i \neq s^k, t^k$) for carrier k.

For the sake of simplicity, we assume that carrier k uses a fleet of homogeneous trucks (vehicles) each one of capacity q^k and traveling at full load (hence, we may assume that the shipment order ratio $\frac{b^k}{q^k}$ represents the number of hazmat trucks used by carrier k for the hazmat shipment).

Let $c_{ij} \geq 0$ be the truck transportation cost (or length) of link $(i,j) \in A$, and let ρ_{ij}^h be the risk (e.g., the number of exposed persons) induced by a truck carrying hazmat of type h through link $(i,j) \in A$; without loss of generality, we assume that $\rho_{ij}^h > 0$, for each $(i,j) \in A$.

According to HTTS policy, we assume that there is a government authority that wishes to mitigate the risk induced by hazmat shipments by regulating hazmat transportation via toll setting. Specifically, in the HTTS problem, the authority wishes to determine the amount of toll fee to be set on all the links of the network, in order to deter the carriers from using certain road segments and encourage them to use, e.g., the less populated ones. The aim is that of limiting as much as possible the risk over the network. On the other side, the carriers make their routing decisions, e.g., minimizing their total transportation and toll cost.

3.1 The model of Marcotte et al. (2009)

Marcotte et al. (2009) model the HTTS problem in terms of the following NP-hard bilevel problem:

$$
\begin{aligned}
\min_{\tau_{ij}^h} \quad & \sum_{k\in K}\sum_{(i,j)\in A} (\rho_{ij}^{h(k)} + \alpha(c_{ij} + \tau_{ij}^{h(k)}))\frac{b^k}{q^k}x_{ij}^k \\
\text{s.t.} \quad & \tau_{ij}^h \geq 0, && \forall\,(i,j)\in A,\ \forall\, h\in H \\
\text{where} \quad & x_{ij}^k \text{ solves:} \\
\min_{x_{ij}^k} \quad & \sum_{k\in K}\sum_{(i,j)\in A} (c_{ij} + \tau_{ij}^{h(k)} + \beta\rho_{ij}^{h(k)})\frac{b^k}{q^k}x_{ij}^k \\
\text{s.t.} \quad & \\
& \sum_{(i,j)\in A^+(i)} x_{ij}^k - \sum_{(j,i)\in A^-(i)} x_{ji}^k = e_i^k, && \forall\, i\in N,\ \forall\, k\in K \\
& x_{ij}^k \in \{0,1\}, && \forall\,(i,j)\in A,\ \forall\, k\in K,
\end{aligned}
\tag{1}
$$

where τ_{ij}^h and x_{ij}^k are the variables controlled by the leader and the followers, respectively. In particular,

- τ_{ij}^h are non-negative variables representing the tolls imposed by the authority on link $(i,j)\in A$, for each truck transporting hazmat of type $h\in H$;
- x_{ij}^k are binary variables modeling the decisions of the carriers, and are equal to 1 if link $(i,j)\in A$ is used by carrier k for the shipment, 0 otherwise.

In this model the leader (i.e., the authority) sets the tolls on the network links in such a way to minimize a weighted combination (through weight parameter α) of the network total risk and carriers' total cost. This is done by keeping into account that, for a given set of values of the link tolls, the followers (i.e., the carriers) choose, independently from each other, the minimum cost routes (assuming, e.g., $\beta = 0$) from their origin node to their destination nodes.

3.2 The proposed model

Differently from the model of Marcotte et al. (2009), we assume that:

1. the authority aims at minimizing not only the network total risk (and the carriers' total cost), but also achieve the risk equity.

2. the decision variables controlled by the carriers are continuous;

Therefore, in our model, let x_{ij}^k, with $0 \leq x_{ij}^k \leq 1$, be the decision variables controlled by carrier k, representing the fraction of its shipment routed along link (i, j), for each $(i, j) \in A$. Of course, each carrier k, with $k \in K$, makes routing choice minimizing his own transportation cost, that, without considering additional cost, is equal to $\sum_{(i,j)\in A} c_{ij} \frac{b^k}{q^k} x_{ij}^k$.

The authority, on the basis of the above assumption 1, wishes to minimize the network total risk, that is,

$$\sum_{k\in K} \sum_{(i,j)\in A} \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k,$$

and pursue the risk equity by minimizing the maximum link total risk among the links of the network, that is,

$$\max_{(i,j)\in A} \{ \sum_{k\in K} \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k \}.$$

While Marcotte et al. (2009) (see model formulation (1)) assume that the amount of toll fee that carrier k should pay for using a link is proportional to the number $\frac{b^k}{q^k}$ of its hazmat vehicles routed on that link, and, hence, proportional to the amount of the risk induced by its hazmat shipment on the link, in our model we assume that the amount of the toll paid by a carrier for using a link is a quadratic function of the total amount of the risk that all the carriers' shipments induce on that link.

Therefore, the additional travel cost (due to toll fees) faced by each carrier depends also on the hazmat route plans of the other carriers, since the amount of toll fee that a carrier has to pay for using a link depends on the total risk on that link. The aim of our assumption is to induce the carriers to transport hazmat along routes where the total risk is lower, and at the same time to limit the amount of total risk on each single link.

More in detail, for each link $(i, j) \in A$, let $T_{ij}^{h(k)}$ be the toll fee that carrier k has to pay for each unit of risk induced by its shipment on link (i, j). As the idea is to discourage carrier k from using a link with high total risk, with the latter being induced by carrier k and all the other carriers that use that link, tax $T_{ij}^{h(k)}$ per unit of risk is assumed to be the sum of the following two terms, that is

$$T_{ij}^{h(k)} = t_{ij}^{h(k)} + d_{ij}^{h(k)} \sum_{\ell \in K} \rho_{ij}^{h(\ell)} \frac{b^\ell}{q^\ell} x_{ij}^\ell, \tag{2}$$

where $t_{ij}^{h(k)}$ and $d_{ij}^{h(k)}$ are toll parameters used to weight the risk contribution of carrier k with respect to the risk caused by the whole set of carriers. In particular, toll parameter $t_{ij}^{h(k)}$ is associated to the risk induced by carrier k on link (i, j) (similarly to model (1) of Marcotte et al., 2009), and toll parameter $d_{ij}^{h(k)}$ is related to the total risk induced by all the carriers on the same link.

Therefore the toll fee paid by carrier k for routing a fraction x_{ij}^k of its shipment on link $(i, j) \in A$ is equal to

$$t_{ij}^{h(k)} \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k + d_{ij}^{h(k)} \left(\sum_{\ell \in K} \rho_{ij}^{h(\ell)} \frac{b^\ell}{q^\ell} x_{ij}^\ell \right) \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k. \tag{3}$$

The linear component (weighted by toll parameter $t_{ij}^{h(k)}$) is used to regulate the total risk on the network similarly to the HTTS model (1) of Marcotte et al. (2009), while the quadratic component (weighted by toll parameter $d_{ij}^{h(k)}$) is used to control the maximum link total risk, possibly forcing carrier k to split his order amount b^k along different routes. Thus, given an assignment of values to toll parameters t_{ij}^h and d_{ij}^h by the government authority, the objective function of the subproblem of each carrier k becomes

$$\begin{aligned} \theta_k(\boldsymbol{x}; \mathbf{t}, \mathbf{d}) = \quad & \sum_{(i,j) \in A} c_{ij} \frac{b^k}{q^k} x_{ij}^k + \sum_{(i,j) \in A} t_{ij}^{h(k)} \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k + \\ & \sum_{(i,j) \in A} d_{ij}^{h(k)} \left(\sum_{\ell \in K} \rho_{ij}^{h(\ell)} \frac{b^\ell}{q^\ell} x_{ij}^\ell \right) \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k, \end{aligned} \tag{4}$$

where $\boldsymbol{x}$ denotes the vector containing all the carriers' variables x_{ij}^k, and $\mathbf{t}, \mathbf{d}$ denote the vectors of toll parameters t_{ij}^h, d_{ij}^h, respectively. Therefore, the carriers' problem constitutes a Nash Equilibrium Problem (NEP), where each carrier k is a player and his subproblem is

$$
\begin{aligned}
&\min_{x_{ij}^k} \quad \theta_k(\boldsymbol{x};\mathbf{t},\mathbf{d}) \\
&\text{s.t.} \\
&\qquad \sum_{(i,j)\in A^+(i)} x_{ij}^k - \sum_{(j,i)\in A^-(i)} x_{ji}^k = e_i^k, \quad \forall\, i \in N \\
&\qquad x_{ij}^k \geq 0, \qquad\qquad\qquad\qquad\qquad\quad \forall\,(i,j)\in A.
\end{aligned}
\tag{5}
$$

Due to the compactness of the feasible set of each player's subproblem, the Nash game admits a solution for every value of toll parameters t_{ij}^h and d_{ij}^h. Unfortunately, in general, uniqueness of the equilibrium is not guaranteed, leading to a possible lack of stability of the solution of our HTTS model (6) formulated below.

Taking into account the roles of the authority and of the carriers, the whole model is a bilevel problem, where the leader (i.e, the authority) sets tolls on the network links by choosing the values of toll parameters t_{ij}^h and d_{ij}^h in order to minimize a combination of risk magnitude and carriers travel cost. The followers (i.e., the carriers) are the players of the Nash game, where each player k, with $k \in K$, aims at solving subproblem (5) on the toll network, and hence selects the route plan for his/her shipment by controlling variables x_{ij}^k. In particular the leader wants to minimize the risk magnitude by minimizing the total risk on the network, and the maximum link total risk among the links of the network; in addition the regulator may want to minimize the carriers' total cost. In this situation, we get the following MPEC problem:

$$
\begin{aligned}
&\min_{\mathbf{t},\mathbf{d}} \; z = w_1 \sum_{k\in K} \sum_{(i,j)\in A} \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k + w_2 \cdot \Phi + w_3 \sum_{k\in K} \theta_k(\boldsymbol{x};\boldsymbol{t},\boldsymbol{d}) \\
&\text{s.t.} \\
&\qquad \sum_{k\in K} \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k \leq \Phi, \qquad\qquad \forall\,(i,j)\in A \\
&\qquad t_{ij}^h \geq 0, d_{ij}^h \geq 0, \qquad\qquad\qquad \forall\,(i,j)\in A, \forall\, h \in H \\
&\text{where} \quad \boldsymbol{x} \text{ is a Nash Equilibrium (NE) of} \\
&\qquad \min_{x_{ij}^k} \theta_k(\boldsymbol{x};\mathbf{t},\mathbf{d}) \\
&\qquad \text{s.t.} \\
&\qquad \sum_{(i,j)\in A^+(i)} x_{ij}^k - \sum_{(j,i)\in A^-(i)} x_{ji}^k = e_i^k, \quad \forall\, i \in N \\
&\qquad x_{ij}^k \geq 0, \qquad\qquad\qquad\qquad\qquad \forall\,(i,j)\in A \\
&\qquad \text{for all } k \in K,
\end{aligned}
\tag{6}
$$

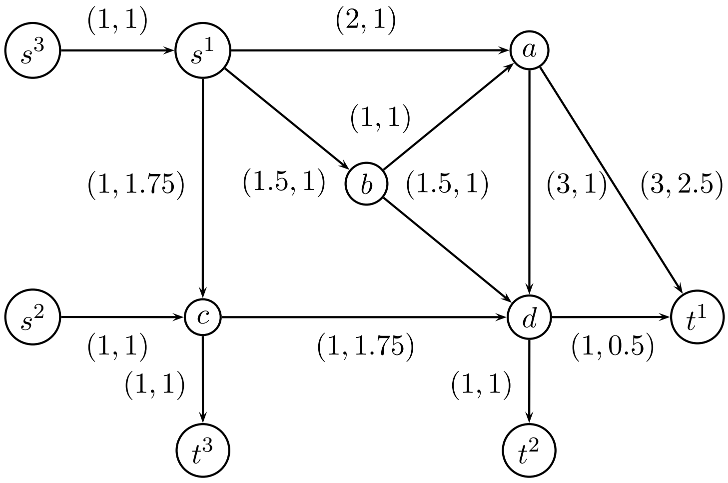

Figure 1: An example: for each link (i, j), the label represents (c_{ij}, ρ^1_{ij}).

where Φ is a continuous variable representing the maximum link total risk, and w_1, w_2, and w_3 are weight factors that allow one to control the preference of the leader with respect to the different aspects he wants to control.

3.3 An example

In order to show the advantages of our HTTS model (6) with respect to the model (1) of Marcotte et al. (2009) (and also w.r.t. HTND policy), let us consider an example on the small network shown in Figure 1, with three carriers (i.e., $K = \{1, 2, 3\}$) all shipping the same type of hazmat (i.e., $h(k) = 1$, for each $k = 1, 2, 3$). Let (s^k, t^k) be the origin-destination pair of the shipment of carrier k. Let us assume that carrier 1 needs two trucks for its shipment and each one of the others only one truck (i.e. $\frac{b^1}{q^1} = 2$, $\frac{b^2}{q^2} = \frac{b^3}{q^3} = 1$). For each link $(i, j) \in A$, the values in brackets shown in the figure represent the cost c_{ij} and the risk ρ^1_{ij} on the link per unit of truck, respectively.

Note that for both carriers 2 and 3 there is a single path connecting the origin to the destination of the shipment: that is, path $(s^2 \to c \to d \to t^2)$ and path $(s^3 \to s^1 \to c \to t^3)$ for carriers 2 and 3, respectively; these two paths pass through link (c, d) and (s^1, c), respectively, where there is the highest population exposure (risk). On the contrary, there are multiple paths available to carrier 1 for traveling from its origin node s^1 to its destination node t^1.

In particular, for carrier 1 the most attractive path is of course the minimum cost path, that is path ($s^1 \to c \to d \to t^1$). If carrier 1 follows this path (i.e., in the *unregulated* scenario) the network total risk will be 15.5, and the maximum link total risk 5.25 (on links (s^1, c) and (c, d)). The total transportation cost will be 12, which of course is the minimum value.

On the other hand, if the authority had the right to impose routes to carriers (i.e., in the *over-regulated* scenario), the authority would choose a minimum risk path for carrier 1, that is, equivalently either path ($s^1 \to a \to d \to t^1$) or path ($s^1 \to b \to d \to t^1$), getting the (minimum) network total risk of value 12.5. In particular to achieve also risk equity, the authority would assign one of these two paths to truck 1 and the other to truck 2 of carrier 1 in order to minimize also the maximum link total risk, which would be equal to the minimum value of 1.75 (on links (s^1, c) and (c, d)). The total transportation cost would be 16.

With a HTND policy (e.g., according to the model of Kara and Verter, 2004), the authority would close link (d, t^1) in order to prevent carrier 1 from using the larger risky path ($s^1 \to c \to d \to t^1$) (note that links (s^1, c) and (c, d) cannot be closed otherwise the other two carriers would not have any available path for their shipments). With this link closure, carrier 1 would be forced to choose path ($s^1 \to a \to t^1$). Therefore with the HTND policy the total risk on the network would be equal to 14.5. The maximum link total risk would be 5 (on link (a, t^1)), and the total transportation cost would be 16.

With the HTTS model (1) of Marcotte et al. (2009), the authority may set the tolls τ_{ij}^1 (e.g., sufficiently large on links (s^1, c) and/or (c, d)) so that carrier 1 chooses the minimum (transportation + toll) cost path ($s^1 \to b \to d \to t^1$) getting a network total risk of minimum possible value, i.e., 12.5. The maximum link total risk would be equal to 2 (on links (s^1, b) and (b, d)), and the total transportation cost equal to $15 + \epsilon$, with ϵ being a small value greater than 0 (including a toll of value $1 + \epsilon$ paid by carrier 3 assuming to have set this value for the toll $\tau_{s^1,c}$ on link (s^1, c)).

The example shows that HTTS policy would induce a lower risk than HTND policy, while the opposite behavior cannot be obtained. This difference is due to the flexibility of HTTS policy that allows to differentiate the behavior among carriers: indeed, it may happen that a carrier chooses a certain link despite the toll, while if a link is closed to the transportation of a given hazmat type, it becomes forbidden for all carriers transporting that type of hazmat.

With our HTTS model (6), we can obtain the same result of model (1) in term of network total risk, but a lower maximum link total risk. Indeed, model (1) provides a network total risk equal to the minimum value 12.5, but it is reached without taking into account the risk equity, and in particular all the amount of the shipment order of a carrier is shipped along the same route. By using our model (6), since the carriers' variables x^k_{ij} are assumed to be continuous, it is possible to set the tolls (e.g., $t^1_{s^1,c} = (3+\epsilon)/\rho^1_{s^1,c}$, $t^1_{a,t^1} > 1/\rho^1_{a,t^1}$, and $d^1_{b,d} = 1$) in order to induce carrier 1 to split its hazmat flow along the two minimum risk paths ($s^1 \to a \to t^1$) and ($s^1 \to b \to d \to t^1$), getting again the (minimum) network total risk of value 12.5, but a lower maximum link total risk of value 1.75 (on links (s^1, c) and (c, d)), which is in particular the minimum possible value. Finally, the total transportation cost is $21 + \epsilon$ (including a toll of value $3 + \epsilon$ paid by carrier 3 on link (s^1, c)). In conclusion, the example shows that model (6) is able to achieve the same network total risk and a better distribution of the risk with respect to model (1) of Marcotte et al. (2009). Therefore, our model can provide solutions dominating those of the latter model (considering the network total risk and the maximum link total risk as performance measures). Note that the opposite cannot occur because the feasible set of model (6) strictly contains the feasible set of model (1): indeed, the latter can be obtained by fixing to zero the value of parameters d^h_{ij} of our model.

4 Properties of the NEP

To simplify the notation, in this section let us denote with $\{1, 2, \dots, m\}$ the set A of the network links, and let $a \in A$ be a generic link. Moreover, let us denote with $x^k \in \Re^m$ the vector of variables controlled by carrier k, with $k = 1, \dots, p$.

We focus on the properties of the Nash game (5) among the carriers. The objective function $\theta_k(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d})$ of player (carrier) k is a continuously differentiable quadratic function of $\boldsymbol{x}$, that is convex with respect to the variables of the player since the hessian with respect to x^k is equal to the diagonal matrix

$$\nabla^2_{x^k}\theta_k(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d}) = \begin{pmatrix} 2d_1^{h(k)}(\rho_1^{h(k)}\frac{b^k}{q^k})^2 & 0 & \dots & 0 \\ 0 & 2d_2^{h(k)}(\rho_2^{h(k)}\frac{b^k}{q^k})^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 2d_m^{h(k)}(\rho_1^{h(k)}\frac{b^k}{q^k})^2 \end{pmatrix}$$

that is positive semidefinite for any $d_a^{h(k)} \geq 0$ of link $a \in A$. Furthermore, denoted with

$$\mathcal{D}_k = \left\{ x^k \in \Re^m \, : \, \sum_{a \in A^+(i)} x_a^k - \sum_{a \in A^-(i)} x_a^k = e_i^k, \quad \forall\, i \in N; \; x_a^k \geq 0, \quad \forall\, a \in A \right\}$$

the feasible set of each player k, we have that $\mathcal{D}_k$ is compact and convex.

It is well known (see, e.g., Proposition 1.4.2 in Facchinei and Pang, 2003) that given the convexity of $\theta_k(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d})$ and the convexity and compactness of $\mathcal{D}_k$, for each $k = 1, \ldots, p$, Nash game (5) is equivalent to the variational inequality $VI(D, \boldsymbol{F})$, where

$$D = \prod_{k=1}^{p} \mathcal{D}_k, \qquad \boldsymbol{F}(\boldsymbol{x}) = \begin{pmatrix} \nabla_{x^1}\theta_1(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d}) \\ \nabla_{x^2}\theta_2(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d}) \\ \vdots \\ \nabla_{x^p}\theta_p(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d}) \end{pmatrix}. \tag{7}$$

The gradient of the objective function $\theta_k(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d})$ with respect to variables x_a^k of carrier k, with $a \in A$, is a vector of m components whose generic element is given by

$$\frac{\partial \theta_k(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d})}{\partial x_a^k} = c_a \frac{b^k}{q^k} + t_a^{h(k)} \rho_a^{h(k)} \frac{b^k}{q^k} + d_a^{h(k)} \rho_a^{h(k)} \frac{b^k}{q^k} \left(\rho_a^{h(k)} \frac{b^k}{q^k} x_a^k + \sum_{\ell=1}^{p} \rho_a^{h(\ell)} \frac{b^\ell}{q^\ell} x_a^\ell \right), \tag{8}$$

so that

$$\nabla_{x^k}\theta_k(\boldsymbol{x}) = \begin{pmatrix} c_1 \frac{b^k}{q^k} + t_1^{h(k)} \rho_1^{h(k)} \frac{b^k}{q^k} + d_1^{h(k)} \rho_1^{h(k)} \frac{b^k}{q^k} \left(\rho_1^{h(k)} \frac{b^k}{q^k} x_1^k + \sum_{\ell=1}^{p} \rho_1^{h(\ell)} \frac{b^\ell}{q^\ell} x_1^\ell \right) \\ c_2 \frac{b^k}{q^k} + t_2^{h(k)} \rho_2^{h(k)} \frac{b^k}{q^k} + d_2^{h(k)} \rho_2^{h(k)} \frac{b^k}{q^k} \left(\rho_2^{h(k)} \frac{b^k}{q^k} x_2^k + \sum_{\ell=1}^{p} \rho_2^{h(\ell)} \frac{b^\ell}{q^\ell} x_2^\ell \right) \\ \vdots \\ c_m \frac{b^k}{q^k} + t_m^{h(k)} \rho_m^{h(k)} \frac{b^k}{q^k} + d_m^{h(k)} \rho_m^{h(k)} \frac{b^k}{q^k} \left(\rho_m^{h(k)} \frac{b^k}{q^k} x_m^k + \sum_{\ell=1}^{p} \rho_m^{h(\ell)} \frac{b^\ell}{q^\ell} x_m^\ell \right) \end{pmatrix}. \tag{9}$$

Since the sets $\mathcal{D}_k$ are compact, and $\boldsymbol{F}(\boldsymbol{x})$ is a vector of continuous functions, a solution of the $VI(D, \boldsymbol{F})$ (and, hence, a Nash equilibrium of NEP (5)) always exists. A sufficient condition for having uniqueness of the solution is $\boldsymbol{F}(\boldsymbol{x})$ being strictly monotone on D, that is

$$(\boldsymbol{F}(\boldsymbol{x}) - \boldsymbol{F}(\boldsymbol{y}))^{\mathrm{T}}(\boldsymbol{x} - \boldsymbol{y}) > 0, \quad \forall x, y \in D\,, x \neq y.$$

Since our $\boldsymbol{F}(\boldsymbol{x})$ is a vector of affine functions, the Jacobian of $\boldsymbol{F}(\boldsymbol{x})$ (denoted with matrix $JF(\boldsymbol{x})$) is a constant matrix, and a sufficient condition for the strong (and, hence, strict) monotonicity of $\boldsymbol{F}(\boldsymbol{x})$ is $JF(\boldsymbol{x})$ being positive definite. In order to evaluate $JF(\boldsymbol{x})$, we note that

$$\frac{\partial^2\theta_k(\boldsymbol{x};\boldsymbol{t},\boldsymbol{d})}{\partial^2 x_a^k} = 2d_a^{h(k)}\left(\rho_a^{h(k)}\frac{b^k}{q^k}\right)^2; \quad \frac{\partial^2\theta_k(\boldsymbol{x};\boldsymbol{t},\boldsymbol{d})}{\partial x_a^\ell \partial x_a^k} = d_a^{h(k)}\rho_a^{h(k)}\frac{b^k}{q^k}\rho_a^{h(\ell)}\frac{b^\ell}{q^\ell};$$

$$\frac{\partial^2\theta_k(\boldsymbol{x};\boldsymbol{t},\boldsymbol{d})}{\partial x_{a'}^k \partial x_a^k} = 0; \qquad \frac{\partial^2\theta_k(\boldsymbol{x};\boldsymbol{t},\boldsymbol{d})}{\partial x_{a'}^\ell \partial x_a^k} = 0.$$

Therefore, $JF(\boldsymbol{x})$ is the $(p\,m \times p\,m)$ matrix

$$JF(\boldsymbol{x}) = \begin{pmatrix} H_{11} & H_{12} & \dots & H_{1p} \\ H_{21} & H_{22} & \dots & H_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ H_{p1} & H_{p2} & \dots & H_{pp} \end{pmatrix}$$

where

$$H_{k\ell} = \frac{b^k}{q^k}\frac{b^\ell}{q^\ell}\begin{pmatrix} d_1^{h(k)}\rho_1^{h(h)}\rho_1^{h(\ell)} & 0 & \dots & 0 \\ 0 & d_2^{h(k)}\rho_2^{h(k)}\rho_2^{h(\ell)} & \dots & 0 \\ & & & \\ 0 & 0 & \dots & d_m^{h(k)}\rho_m^{h(k)}\rho_m^{h(\ell)} \end{pmatrix}$$

for $k \neq \ell$, and

$$H_{kk} = \left(\frac{b^k}{q^k}\right)^2\begin{pmatrix} 2d_1^{h(k)}\left(\rho_1^{h(k)}\right)^2 & 0 & \dots & 0 \\ 0 & 2d_2^{h(k)}\left(\rho_2^{h(k)}\right)^2 & \dots & 0 \\ & & & \\ 0 & 0 & \dots & 2d_m^{h(k)}\left(\rho_m^{h(k)}\right)^2 \end{pmatrix}.$$

This matrix in general cannot be expected to be positive definite. Note that the matrix $JF(\boldsymbol{x})$ is not symmetric. Furthermore, it is evident from the structure of $JF(\boldsymbol{x})$ that as soon as a single $d_a^{h(k)}$, with $a \in \{1, 2, \dots, m\}$, is equal to zero, then the matrix is not positive definite (an element on the diagonal is zero).

However, if we focus on the particular case where all the carriers transport the same hazardous material h (which is equivalent to remove the superscript $h(k)$), then the following proposition can be stated:

Proposition 4.1 *If all the carriers transport the same hazardous material, i.e.* $h(k) = h$ *for all* $k = 1, \dots, p$*, then for any value of the toll parameters* t_a *and* $d_a > 0$*, for* $a = 1, \dots, m$*:*

(i) *there exists a unique equilibrium of NEP (5).*

(ii) *NEP (5) is equivalent to the following strictly convex optimization problem:*

$$\begin{aligned}
&\min \quad \boldsymbol{\kappa}^T \boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^T JF(\boldsymbol{x})\boldsymbol{x} \\
&s.t. \\
&\qquad \sum_{a\in A^+(i)} x_a^k - \sum_{a\in A^-(i)} x_a^k = e_i^k, \quad \forall\, i \in N, \quad \forall\, k = \{1,\dots,p\} \\
&\qquad x_a^k \geq 0, \quad \forall a \in A, \quad \forall\, k = \{1,\dots,p\},
\end{aligned} \tag{10}$$

where

$$\boldsymbol{\kappa} = \begin{pmatrix} (c_1 + t_1\rho_1)\,\dfrac{b^1}{q^1} \\ \vdots \\ (c_m + t_m\rho_m)\,\dfrac{b^1}{q^1} \\ \vdots \\ (c_1 + t_1\rho_1)\,\dfrac{b^p}{q^p} \\ \vdots \\ (c_m + t_m\rho_m)\,\dfrac{b^p}{q^p} \end{pmatrix}.$$

Proof.

(i) The existence of an equilibrium is implied by the compactness of feasible set $\mathcal{D}_k$ of each player k. As for the uniqueness, under the assumption $h(k) = h$ for all $k = 1,\dots,p$ the jacobian matrix $JF(\boldsymbol{x})$ is symmetric and of the form

$$JF(\boldsymbol{x}) = \begin{pmatrix} 2\left(\frac{b^1}{q^1}\right)^2 H & \frac{b^1 b^2}{q^1 q^2} H & \dots & \frac{b^1 b^p}{q^1 q^p} H \\ \frac{b^2 b^1}{q^2 q^1} H & 2\left(\frac{b^2}{q^2}\right)^2 H & \dots & \frac{b^2 b^p}{q^2 q^p} H \\ \vdots & \vdots & \ddots & \vdots \\ \frac{b^p b^1}{q^p q^1} H & \frac{b^p b^2}{q^p q^2} H & \dots & 2\left(\frac{b^p}{q^p}\right)^2 H \end{pmatrix}$$

where, by denoting with $\mathrm{Diag}(\boldsymbol{v})$ the diagonal matrix having as elements on the diagonal the elements of vector $\boldsymbol{v}$,

$$H = \mathrm{Diag}\left((\rho_a)^2 d_a\right)_{a=1}^m.$$

By setting

$$\boldsymbol{r} = \begin{pmatrix} \frac{b^1}{q^1} \\ \vdots \\ \frac{b^p}{q^p} \end{pmatrix},$$

this matrix can be equivalently rewritten as

$$JF(\boldsymbol{x}) = H \otimes \left(\boldsymbol{r}\boldsymbol{r}^{\mathrm{T}} + \mathrm{Diag}(\boldsymbol{r}) \right),$$

where operator $\otimes$ indicates the Kronecker product of two matrices. Matrix H is a diagonal matrix having eigenvalues equal to the elements of the diagonal, so that it is positive definite if we assume that all the diagonal elements of H are positive (i.e., all the toll parameters d_a and risk coefficient ρ_a are positive). As for the matrix $\boldsymbol{r}\boldsymbol{r}^{\mathrm{T}} + \mathrm{Diag}(\boldsymbol{r})$, it is positive definite since it is a rank one update of the positive definite matrix $\mathrm{Diag}(\boldsymbol{r})$ having as eigenvalues $\frac{b^k}{q^k} > 0$, for $k = 1, \ldots, p$. Therefore, since the Kronecker product of two positive definite matrices is positive definite, then the matrix $JF(\boldsymbol{x})$ is positive definite, implying strict monotonicity of the function $\boldsymbol{F}(\boldsymbol{x})$. Therefore, we have a unique equilibrium.

(ii) As proved in point (i), under the assumption $h(k) = h$, for all $k = 1, \ldots, p$, the jacobian matrix $JF(\boldsymbol{x})$ is symmetric. It is well known (see, e.g., Theorem 1.3.1 in Facchinei and Pang, 2003) that given a vector $\boldsymbol{F}(\boldsymbol{x})$ of continuously differentiable functions, if its jacobian is symmetric, then there exists a real-valued function $\varphi(\boldsymbol{x})$ such that $\nabla\varphi(\boldsymbol{x}) = \boldsymbol{F}(\boldsymbol{x})$, and

$$\varphi(\boldsymbol{x}) = \int_0^1 \boldsymbol{F}^{\mathrm{T}} \left(\boldsymbol{x}_0 + t(\boldsymbol{x} - \boldsymbol{x}_0)\right) (\boldsymbol{x} - \boldsymbol{x}_0)\, dt.$$

By the expression of $\boldsymbol{F}(\boldsymbol{x})$, it follows that

$$\varphi(\boldsymbol{x}) = \boldsymbol{\kappa}^{\mathrm{T}}\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^{\mathrm{T}} JF(\boldsymbol{x})\boldsymbol{x},$$

that is a quadratic function, with hessian equal to $JF(\boldsymbol{x})$ which is proved at point (i) to be positive definite. Therefore, function $\varphi(\boldsymbol{x})$ is strictly convex, so that problem (10) has a unique solution. Finally, the well known minimum principle applied to problem (10) is equivalent to solving the $VI(D, \boldsymbol{F})$, and the thesis follows.

$\square$

The above proposition justifies the attention to the special case where all the carriers transport the same type of hazardous material, situation that occurs in particular in the experiments on the network of Ravenna, Italy, given in Erkut and Gzara (2008) and in the experiments reported in Section 7. In the next section we consider in depth all the implications of Proposition 4.1.

5 The single hazardous material case

From now on, we assume that the hypothesis of proposition 4.1 are verified, i.e., $h(k) = h$, for every carrier $k = 1, \dots, p$.

Point (i) of Proposition 4.1 states that if we restrict toll parameters d_{ij} to strictly positive values, for each $(i, j) \in A$, the equilibrium of Nash game (5) is unique. This result ensures stability of the solutions of the bilevel problem (6), which is an extremely rare result in bilevel programming. Indeed, whenever ties among inner-level solutions (carriers' route plans) occur, in general the bilevel formulation assumes that carriers adopt the one that minimizes the leader's objective. This is true also for the bilevel model of Marcotte et al. (2009), where however in principle a perturbation of the tolls may be used in order to break the ties.

A crucial point for the viability of the game theory approach is how to solve the Nash game (5), possibly by a distributed algorithm requiring a reasonable exchange of information between the players, since it is not realistic to assume the presence of an external agent that, on behalf of the carriers, solves the Nash game and returns the route plans to the carriers.

With this respect, point (ii) of Proposition 4.1 and its proof provide a satisfactory result. Indeed, they imply that, in the special case of a single type of hazmat, Nash game (5) is a *potential game.* Potential game are defined in Monderer and Shapley (1996) as follows: a game where each player $k = 1, \dots, p$ has to solve the problem

$$\begin{array}{ll} \min\limits_{x^k} & \theta_k(\boldsymbol{x}) \\ \text{s.t.} & x^k \in \mathcal{D}_k, \end{array}$$

is an exact potential game if there exist a function $P(\boldsymbol{x})$ such that

$$\nabla_{x^k} P(\boldsymbol{x}) = \nabla_{x^k} \theta_k(\boldsymbol{x}), \text{ for every } k = 1, \dots, p.$$

Proof of point (ii) of Proposition 4.1 implies that the objective function of problem (10) is an exact potential for Nash game (5). Whenever a game is potential, any solution of the problem

$$\begin{array}{ll} \min & P(\boldsymbol{x}) \\ \text{s.t.} & x^k \in \mathcal{D}_k, \quad k = 1, \dots, p \end{array}$$

is a Nash equilibrium of NEP (5).

Another important consequence of Nash game (5) being potential is that a best response algorithm converges to a Nash equilibrium. Therefore, it can be solved by means of distributed algorithms of the following form:

Step 0. Choose a starting point $\boldsymbol{x}_{(0)} = (x^1_{(0)}, \dots, x^p_{(0)}) \in \prod_{k=1}^p \mathcal{D}_k$, and set $i := 0$.

Step 1. If $\boldsymbol{x}_{(i)}$ satisfies a suitable termination criterion: STOP.

Step 2. `FOR` $k = 1, \dots, p$

Compute a solution $x^k_{(i+1)}$ of

$$\begin{array}{ll} \min\limits_{x^k} & \theta_k(x^1_{(i+1)}, \dots, x^{k-1}_{(i+1)}, x^k, x^{k+1}_{(i)}, \dots, x^p_{(i)}) \\ \text{s.t.} & x^k \in \mathcal{D}_k \end{array} \tag{11}$$

`END`

Step 3. Set $\boldsymbol{x}_{(i+1)} := (x^1_{(i+1)}, \dots, x^p_{(i+1)}), i := i + 1$, and go to Step 1.

Looking at the expression (4) of the carrier's objective function $\theta_k(\boldsymbol{x}; \boldsymbol{t}, \boldsymbol{d})$, it results that, for a given set of values of toll parameters t_{ij}, d_{ij}, the only information that the carriers need to exchange is the link total risk $\sum_{\ell=1}^p \rho_{ij}^{h(\ell)} \frac{b^\ell}{q^\ell} x^\ell_{ij}$ on each link $(i, j) \in A$, with no need to know exactly the route plans of the other carriers. Note that this is the minimum requirement on information exchange among the carriers.

This algorithm can also be interpreted equivalently as a Gauss Seidel type algorithm for the optimization problem (10), and it is well known that thanks to the strict convexity of the objective function, it converges to the unique solution.

Given a set of values $(\bar{\boldsymbol{t}}, \bar{\boldsymbol{d}})$ of the toll parameters t_{ij} and d_{ij}, with $\bar{t}_{ij} \geq 0$ and $\bar{d}_{ij} > 0$, for each $(i, j) \in A$, the corresponding Nash equilibrium, denoted with $\bar{\boldsymbol{x}}(\bar{\boldsymbol{t}}, \bar{\boldsymbol{d}})$, is unique. However, in general different toll parameters' values can induce the same equilibrium. An interesting question from the authority point of view, is what is the "best" set of values for the toll parameters with respect to some criterion (e.g., minimizing the total cost paid by the carriers) inducing the Nash equilibrium $\bar{\boldsymbol{x}}(\bar{\boldsymbol{t}}, \bar{\boldsymbol{d}})$.

Assume that corresponding to a given set of non-negative toll parameters' values $\bar{t}_{ij}, \bar{d}_{ij}$ (with $\bar{d}_{ij} > 0$), a certain Nash equilibrium $\bar{\boldsymbol{x}}(\bar{\boldsymbol{t}}, \bar{\boldsymbol{d}})$ has been computed. This implies that $\bar{\boldsymbol{x}} = \bar{\boldsymbol{x}}(\bar{\boldsymbol{t}}, \bar{\boldsymbol{d}})$ is the only point satisfying the following KKT optimality conditions of problem (10) when $t_{ij} = \bar{t}_{ij}$, $d_{ij} = \bar{d}_{ij}$, for every $k = 1, \dots, p$:

$$\begin{gathered}
c_{ij}\frac{b^k}{q^k} + \rho_{ij}\frac{b^k}{q^k}t_{ij} + \left[\rho_{ij}\frac{b^k}{q^k}\left(\rho_{ij}\frac{b^k}{q^k}\bar{x}^k_{ij} + \sum_{\ell=1}^{p}\rho_{ij}\frac{b^\ell}{q^\ell}\bar{x}^\ell_{ij}\right)\right]d_{ij} - \mu^k_i + \mu^k_j - \lambda^k_{ij} = 0, \quad \forall\,(i,j) \in A \\
\lambda^k_{ij}x^k_{ij} = 0, \quad \forall\,(i,j) \in A \\
\sum_{(i,j)\in A^+(i)} x^k_{ij} - \sum_{(j,i)\in A^-(i)} x^k_{ji} = e^k_i, \quad \forall\, i \in N \\
x^k_{ij} \geq 0, \quad \forall\,(i,j) \in A \\
\lambda^k_{ij} \geq 0, \quad \forall\,(i,j) \in A \\
\mu^k_i \text{ free}, \quad \forall\, i \in N,
\end{gathered} \tag{12}$$

where λ^k_{ij} for $(i,j) \in A$, and μ^k_i for $i \in N$ are the dual variables corresponding to the constraints of carrier $k \in K$.

Denoting with $\phi_{ij}(\bar{\boldsymbol{x}}) = \sum_{\ell=1}^{p}\rho_{ij}\frac{b^\ell}{q^\ell}\bar{x}^\ell_{ij}$ the link total risk on link $(i,j) \in A$ corresponding to equilibrium $\bar{\boldsymbol{x}}$, and letting $\gamma^k_{ij}(\bar{\boldsymbol{x}}) = \rho_{ij}\frac{b^k}{q^k}\left(\rho_{ij}\frac{b^k}{q^k}\bar{x}^k_{ij} + \phi_{ij}(\bar{\boldsymbol{x}})\right)$, conditions (12), for each carrier $k = 1, \dots, p$, can be rewritten as the following linear constraints in the variables t_{ij}, d_{ij}, μ^k_i,

$$\begin{aligned}
&c_{ij}\frac{b^k}{q^k} + \rho_{ij}\frac{b^k}{q^k}t_{ij} + \gamma^k_{ij}(\bar{\boldsymbol{x}})d_{ij} - \mu^k_i + \mu^k_j \geq 0, && \forall\,(i,j) \in A\backslash\bar{A}^k \\
&c_{ij}\frac{b^k}{q^k} + \rho_{ij}\frac{b^k}{q^k}t_{ij} + \gamma^k_{ij}(\bar{\boldsymbol{x}})d_{ij} - \mu^k_i + \mu^k_j = 0, && \forall\,(i,j) \in \bar{A}^k \\
&\mu^k_i \text{ free}, && \forall\, i \in N,
\end{aligned} \tag{13}$$

where $\bar{A}^k = \{(i,j) \in A : \bar{x}^k_{ij} > 0\}$, and having omitted the flow conservation constraints on the network nodes and the non-negative constraints on the values of carriers' variables x^k_{ij} because they are obviously satisfied at $\bar{\boldsymbol{x}}$, since $\bar{\boldsymbol{x}} \in \prod_{k=1}^{p}\mathcal{D}_k$.

Now, let us assume that, given equilibrium $\bar{\boldsymbol{x}} = \bar{\boldsymbol{x}}(\bar{\boldsymbol{t}}, \bar{\boldsymbol{d}})$, the government authority is interested in finding the optimal vectors $(\boldsymbol{t}^*, \boldsymbol{d}^*)$ of toll parameters that induce the carriers to reach the given equilibrium $\bar{\boldsymbol{x}}$, and such that the total cost $\zeta(\bar{\boldsymbol{x}}; \boldsymbol{t}, \boldsymbol{d})$ paid by the carriers (including toll fees) is minimized, where

$$\begin{aligned}\zeta(\bar{\boldsymbol{x}};\boldsymbol{t},\boldsymbol{d}) &= \sum_{k=1}^{p}\sum_{(i,j)\in A} c_{ij}\frac{b^k}{q^k}\bar{x}_{ij}^k + \sum_{k=1}^{p}\sum_{(i,j)\in A} t_{ij}\rho_{ij}\frac{b^k}{q^k}\bar{x}_{ij}^k \\ &+ \sum_{k=1}^{p}\sum_{(i,j)\in A} d_{ij}\left(\sum_{\ell=1}^{p}\rho_{ij}\frac{b^\ell}{q^\ell}\bar{x}_{ij}^\ell\right)\rho_{ij}\frac{b^k}{q^k}\bar{x}_{ij}^k \\ &= \sum_{k=1}^{p}\sum_{(i,j)\in A} c_{ij}\frac{b^k}{q^k}\bar{x}_{ij}^k + \sum_{(i,j)\in A}\phi_{ij}(\boldsymbol{x})t_{ij} + \sum_{(i,j)\in A}(\phi_{ij}(\boldsymbol{x}))^2 d_{ij}.\end{aligned} \tag{14}$$

Note that $\zeta(\bar{\boldsymbol{x}};\boldsymbol{t},\boldsymbol{d})$ is a linear function of t_{ij} and d_{ij}.

Therefore, finding the optimal toll parameters' values is equivalent to solving a suitable Linear Programming (LP) problem, having the optimality conditions (13) for $k = 1,\ldots,p$ as constraints, and minimizing $\zeta(\bar{\boldsymbol{x}};\boldsymbol{t},\boldsymbol{d})$; that is, the LP problem

$$\begin{aligned}&\min_{t_{ij},d_{ij},\mu_i^k} \zeta(\bar{\boldsymbol{x}};\boldsymbol{t},\boldsymbol{d}) = \sum_{k=1}^{p}\sum_{(i,j)\in A} c_{ij}\frac{b^k}{q^k}\bar{x}_{ij}^k + \sum_{(i,j)\in A}\phi_{ij}(\boldsymbol{x})t_{ij} + \sum_{(i,j)\in A}(\phi_{ij}(\boldsymbol{x}))^2 d_{ij} \\ &\text{s.t.} \\ &\quad c_{ij}\frac{b^k}{q^k} + \rho_{ij}\frac{b^k}{q^k}t_{ij} + \gamma_{ij}^k(\bar{\boldsymbol{x}})d_{ij} - \mu_i^k + \mu_j^k \ge 0, \quad \forall\,(i,j)\in A\backslash\bar{A}^k,\ k=1,\ldots,p \\ &\quad c_{ij}\frac{b^k}{q^k} + \rho_{ij}\frac{b^k}{q^k}t_{ij} + \gamma_{ij}^k(\bar{\boldsymbol{x}})d_{ij} - \mu_i^k + \mu_j^k = 0, \quad \forall\,(i,j)\in \bar{A}^k,\ k=1,\ldots,p \\ &\quad d_{ij}\ge\epsilon, \quad \forall\,(i,j)\in A \\ &\quad t_{ij},d_{ij}\ge 0, \quad \forall\,(i,j)\in A \\ &\quad \mu_i^k \text{ free}, \quad \forall\, i\in N,\ k=1,\ldots,p,\end{aligned} \tag{15}$$

where ϵ is a small value greater than 0.

We note that this problem is well posed. Indeed, it has a nonempty feasible set since toll settings $\bar{t}_{ij},\bar{d}_{ij}$ are feasible and the objective function assumes nonnegative values and hence is bounded from below.

6 A heuristic resolution approach

Even in the favorable case where all the carriers transport the same type of hazardous material, it is hard to solve problem (6) (that, in this case, reduces to a bilevel optimization problem), and few exact algorithms are available for this aim, keeping also in mind that reasonable size networks lead to large size instances of the corresponding bilevel problem.

Therefore, we propose to heuristically solve the problem with a local search algorithm that exploits existence, uniqueness, and efficient computation of the equilibrium $\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})$ of the Nash game (5) among the carriers (i.e., the followers), corresponding to a given set of values $(\boldsymbol{t},\boldsymbol{d})$ of toll parameters t_{ij}, d_{ij} (with $t_{ij} \geq 0$ and $d_{ij} > 0$), for each link $(i,j) \in A$, chosen by the government authority (i.e., the leader). The idea is to explore heuristically the leader's search space using the corresponding Nash equilibrium in order to evaluate the effectiveness of the leader's choice.

Note that, given any toll setting $(\boldsymbol{t}, \boldsymbol{d})$, the corresponding unique equilibrium $\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})$ can be found by solving problem (10) with any valid optimization algorithm for it, including the distributed algorithm given in Section 5. Therefore any given toll setting $(\boldsymbol{t}, \boldsymbol{d})$ (along with the induced unique equilibrium $\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})$) represents a feasible solution of the bilevel problem.

We measure the goodness of a given toll setting (solution) $(\boldsymbol{t}, \boldsymbol{d})$ with the following performance indices, representing the network total risk, the maximum link total risk and the cost paid by the carriers, respectively, given their choices $\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})$:

$$
\begin{aligned}
R_{\text{tot}}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})) &= \sum_{(i,j)\in A} \sum_{k\in K} \rho_{ij} \frac{b^k}{q^k} \bar{x}^k_{ij} = \sum_{(i,j)\in A} \phi_{ij}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d}));\\
\Phi(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})) &= \max_{(i,j)\in A} \left\{ \sum_{k\in K} \rho_{ij} \frac{b^k}{q^k} \bar{x}^k_{ij} \right\} = \max_{(i,j)\in A} \left\{ \phi_{ij}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})) \right\};\\
C_{\text{tot}}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})) &= \sum_{k\in K} \sum_{(i,j)\in A} c_{ij} \frac{b^k}{q^k} \bar{x}^k_{ij} + \sum_{k\in K} \sum_{(i,j)\in A} t_{ij} \rho_{ij} \frac{b^k}{q^k} x^k_{ij}\\
&+ \sum_{k\in K} \sum_{(i,j)\in A} d_{ij} \left(\sum_{\ell\in K} \rho_{ij} \frac{b^\ell}{q^\ell} x^\ell_{ij} \right) \rho_{ij} \frac{b^k}{q^k} x^k_{ij} =\\
&\sum_{k\in K} \sum_{(i,j)\in A} c_{ij} \frac{b^k}{q^k} \bar{x}^k_{ij} + \sum_{(i,j)\in A} t_{ij}\, \phi_{ij}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})) + \sum_{(i,j)\in A} d_{ij} \left[\phi_{ij}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d}))\right]^2.
\end{aligned}
$$

We consider an Old Bachelor Acceptance (OBA) local search approach, that is a Threshold Acceptance (TA) strategy where the threshold changes dynamically, up and down, based on the perceived likelihood of being near a local minimum (see, e.g., Hu et al., 1995). Such classes of local search strategies overcome the typical drawback of standard TA approaches where the threshold is monotonically decreased during the search, with the consequence that after a certain number of iterations if the current solution is not worse than its neighbors the TA algorithm will fail to move to another solution of the neighborhood of the current one. With the OBA strategy the criterion for accepting a neighbor of the current solution is relaxed by increasing the threshold, and after a certain number of fails in moving to another solution, the threshold will become sufficiently large to escape from the current local optimum; therefore in this way OBA applies a sort of diversification phase. Oppositely, when a neighbor is accepted the threshold is decreased to locally intensify the search in order to move toward a local optimum.

Given a (feasible) solution $(\boldsymbol{t}, \boldsymbol{d})$, let $\mathcal{N}(\boldsymbol{t}, \boldsymbol{d})$ be its neighborhood, that is, the set of the solutions (i.e., toll settings) that can be derived from $(\boldsymbol{t}, \boldsymbol{d})$ by changing their values, and in particular where each neighbor can be obtained with the following two steps. First we increase toll parameters d_{ij} (e.g., by doubling their values) of a subset of $m' \leq m$ links of the network, without changing the others toll parameters; let $(\boldsymbol{t}', \boldsymbol{d}')$ be these new toll settings. Then, after having computed the induced unique equilibrium $\bar{\boldsymbol{x}}(\boldsymbol{t}', \boldsymbol{d}')$, we determine the optimal toll setting $(\boldsymbol{t}^*, \boldsymbol{d}^*)$ (with respect to the minimization of the carriers' total cost) inducing that equilibrium, by solving LP problem (15). With this approach we also aim at minimizing (as the last criterion) the carriers' total cost.

In particular, at each iteration of the local search, given the current solution $(\boldsymbol{t}^{curr}, \boldsymbol{d}^{curr})$, first we select at random m' links with a Montecarlo-based sampling method that assumes the selection probability of a link (i, j) being proportional to the ratio between the link total risk and the carriers' total cost (including toll fees) on that link, given the carriers' choices $\bar{\boldsymbol{x}}(t_{ij}^{curr}, d_{ij}^{curr})$ of the carriers. Then, we double the values of toll parameters $d_{i'j'}$, for each selected link $(i'j')$. Therefore, we get the new solution $(\boldsymbol{t}^{new}, \boldsymbol{d}^{new})$, where d_{ij}^{new} are the new values for toll parameters d_{ij} computed as above, while, at this step, toll parameters t_{ij} are assumed unchanged, that is, $t_{ij}^{new} = t_{ij}$, for each $(i, j) \in A$. Clearly, the choice of increasing toll parameters d_{ij} of such m' links goes toward the aim of reducing the link total risk of a subset of links whose values are possibly the largest among the links of the network and that are also the cheapest links for the carriers; moreover, by changing these toll parameters for a sufficiently large number m' of links, we possibly also achieve a reduction of the network total risk.

Secondly, after having computed the unique Nash equilibrium $\bar{\boldsymbol{x}}(\boldsymbol{t}^{new}, \boldsymbol{d}^{new})$, toll parameters d_{ij}^{new} (and also toll parameters t_{ij}^{new}) are possibly updated by solving LP problem (15) in order to get the (optimal) toll parameters $(\boldsymbol{t}^*, \boldsymbol{d}^*)$ inducing the same equilibrium $\bar{\boldsymbol{x}}(\boldsymbol{t}^{new}, \boldsymbol{d}^{new})$ but at the minimum total cost for the carriers. Finally, let t_{ij}^{new} and d_{ij}^{new} be equal to these optimized toll parameters t_{ij}^* and d_{ij}^*, respectively, for each $(i,j) \in A$, that identify the new solution $(\boldsymbol{t}^{new}, \boldsymbol{d}^{new})$ randomly selected from the neighbors of the given current solution $(\boldsymbol{t}^{curr}, \boldsymbol{d}^{curr})$.

We note that, even if in the first step toll parameters d_{ij} can only be increased, their values can be reduced in the second step (along with the values of toll parameters t_{ij}) when we find the optimal toll parameters by solving problem (15), that induce the same equilibrium related to the toll parameters' values determined in the first step.

At the end of each iteration of the local search, the new generated solution $(\boldsymbol{t}^{new}, \boldsymbol{d}^{new})$ becomes the current solution (valid for the next iteration) if

$$z(\boldsymbol{t}^{new}, \boldsymbol{d}^{new}) < z(\boldsymbol{t}^{curr}, \boldsymbol{d}^{curr}) + \delta,$$

where δ is the current threshold value, and

$$z(\boldsymbol{t}, \boldsymbol{d}) = w_1\, R_{\mathrm{tot}}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})) + w_2\, \Phi(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})) + w_3\, C_{\mathrm{tot}}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d}))$$

is the objective function of the leader (i.e., the government authority). Moreover, in this case, we also decrease threshold δ (e.g., by halving its value). Otherwise, we do not update the current solution, while we increase threshold δ (e.g., by doubling its value). In any cases, the value of δ is always assumed to be within a given range $[\delta_{\min}, \delta_{\max}]$. We also test if the new solutions is better than the best solution $(\boldsymbol{t}^{best}, \boldsymbol{d}^{best})$ generated so far, with respect to the objective function $z(\boldsymbol{t}, \boldsymbol{d})$, and in this case we update the best solution and also set the value of threshold δ to its minimum value $\delta_{\min}$.

The heuristic starts assuming an initial toll setting, that is, a given set of values for toll parameters t_{ij}, d_{ij}. Moreover, the initial value of threshold δ is assumed to be equal to $\delta_{\min}$, and the search is repeated for a given *max_iter* number of iterations.

During the search, the heuristic keeps into account the two main objectives of the government authority (i.e., minimizing the network total risk and the maximum link total risk) and also the objective from the carriers' point of view (i.e., minimizing the carriers' total cost, including toll fees), updating the set of non-dominated solutions (i.e., toll settings) with respect to these three objectives, among the solutions generated so far.

In particular, a toll setting $(\boldsymbol{t}, \boldsymbol{d})_1$ is dominated by $(\boldsymbol{t}, \boldsymbol{d})_2$ if

$$\begin{aligned}
&R_{\mathrm{tot}}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})_2) \le R_{\mathrm{tot}}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})_1),\\
&\Phi(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})_2) \le \Phi(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})_1),\\
&C_{\mathrm{tot}}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})_2) \le C_{\mathrm{tot}}(\bar{\boldsymbol{x}}(\boldsymbol{t}, \boldsymbol{d})_1),
\end{aligned}$$

and at least one of the three inequalities is strict.

At the end, the algorithm returns the set of non-dominated solutions generated, along with the best solution $(\boldsymbol{t}^{best}, \boldsymbol{d}^{best})$ with respect to the leader's objective function $z(\boldsymbol{t}, \boldsymbol{d})$ of bilevel problem (6).

7 Computational results

We test our heuristic algorithm on the road transportation network of the city of Ravenna, Italy, used also in Erkut and Alp (2007), and in Erkut and Gzara (2008). The network is composed of 105 nodes and 268 (directed) links.

As in Erkut and Gzara (2008), we consider 35 different OD pairs, that is, $p = |K| = 35$ carriers all shipping the same type of hazmat. Therefore, for each link (i, j), we consider a single value ρ_{ij} for the risk induced by a hazmat truck traveling on link (i, j), which is evaluated as the population in places of assembly (such as schools, churches, hospitals, factories, and office buildings) within 500 m of the link (i.e., the so called *aggregate risk measure*, see Erkut and Gzara, 2008). In order to guarantee that $\rho_{ij} > 0$, for each link $(i, j) \in A$, we assume that $\rho_{ij} = 10^{-1}$ if the original value is equal to 0. Finally, the cost c_{ij} of link (i, j) is evaluated as the length (in meters) of the link.

We consider 5 different instances ($I1$, $I2$, $I3$, $I4$, $I5$) obtained by randomly generating the (order shipment) ratio $\frac{b^k}{q^k}$ related to carrier k in the range $[1, 100]$, for each carrier $k \in K$.

We implemented the heuristic algorithm in C language, using CPLEX 12.4 C callable libraries to solve both the quadratic problem (10) and the linear problem (15). We ran the experiments on a macbook pro with Intel Core 2 Duo and 4 GB of RAM.

In order to evaluate the performance of any solution $\boldsymbol{x}(\boldsymbol{t}, \boldsymbol{d})$ returned by the algorithm, we consider some reference performance values derived from specific hazmat route plans. The first one is obtained by considering the *unregulated* scenario, where the carriers choose their route plans without any risk consideration and without any regulatory action of the government authority. Of course, in this scenario the carriers have the freedom to chose the minimum cost route plans. This corresponds to solving the following minimum cost multi-commodity network flow problem. Let $C_{\min}$ be the minimum carriers' total cost. The resulting linear programming problem is

$$
\begin{aligned}
C_{\min} := \min \quad & \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}\frac{b^k}{q^k}x_{ij}^k \\
& \sum_{(i,j)\in A^+(i)} x_{ij}^k - \sum_{(j,i)\in A^-(i)} x_{ji}^k = e_i^k, \quad \forall\, i \in N, \forall\, k \in K \\
& x_{ij}^k \geq 0, \qquad\qquad\qquad\qquad\qquad \forall\,(i,j) \in A, \forall\, k \in K.
\end{aligned}
\tag{16}
$$

If the authority could directly force the choices of the carriers, that is, in the *over-regulated* scenario, the authority would impose carriers' route plans that optimize the authority's own criteria. In particular, we assume that the authority considers two different (and possibly conflicting) criteria: minimizing the network total risk, and minimizing the maximum link total risk. Depending on which criterion is considered first, two different route plans for the carriers can be found. For example, assuming that the authority aims first at minimizing the network total risk, and secondly at minimizing the maximum link total risk, then the authority aims at finding the solution minimizing the maximum link total risk among the solutions of lowest possible network total risk $R_{\min}$. Let $\Phi_{\min}(R_{\min})$ be the minimum value of the maximum link total risk among route plans of minimum network total risk. This consists in solving the problem

$$
\begin{aligned}
\Phi_{\min}(R_{\min}) := \quad & \min \Phi \\
& \sum_{k\in K} \rho_{ij}^{h(k)}\frac{b^k}{q^k}x_{ij}^k \leq \Phi, \qquad\qquad \forall\,(i,j) \in A \\
& \sum_{k\in K}\sum_{(i,j)\in A} \rho_{ij}^{h(k)}\frac{b^k}{q^k}x_{ij}^k \leq R_{\min} \\
& \sum_{(i,j)\in A^+(i)} x_{ij}^k - \sum_{(ji)\in A^-(i)} x_{ji}^k = e_i^k, \quad \forall\, i \in N, \forall\, k \in K \\
& x_{ij}^k \geq 0, \qquad\qquad\qquad\qquad\qquad \forall\,(i,j) \in A, \forall\, k \in K,
\end{aligned}
\tag{17}
$$

where Φ is a continuous variable representing the maximum link total risk, and $R_{\min}$ is a parameter obtained by solving the following linear programming problem (that represents the routes the authority would choose keeping into account only the network total risk as objective function)

$$
\begin{aligned}
R_{\min} := \min \quad & \sum_{k\in K}\sum_{(i,j)\in A} \rho_{ij}^{h(k)} \frac{b^k}{q^k} x_{ij}^k \\
& \sum_{(i,j)\in A^+(i)} x_{ij}^k - \sum_{(j,i)\in A^-(i)} x_{ji}^k = e_i^k, \quad \forall\, i \in N, \forall\, k \in K \\
& x_{ij}^k \geq 0, \qquad\qquad\qquad\qquad \forall\,(i,j) \in A, \forall\, k \in K.
\end{aligned} \tag{18}
$$

We experiment the proposed algorithm considering the following three different sets of values for the weights w_1, w_2 and w_3 in the leader's objective function of model (6):

S1: $w_1 = 0.4$, $w_2 = 0.3$, $w_3 = 0.3$;

S2: $w_1 = 0.3$, $w_2 = 0.4$, $w_3 = 0.3$;

S3: $w_1 = 0.4$, $w_2 = 0.4$, $w_3 = 0.2$.

We start the algorithm by setting $t_{ij} = 0$ and $d_{ij} = \epsilon$, with $\epsilon = 10^{-3}$, for each $(i,j) \in A$. At each iteration of the local search algorithm, we initially double the values of toll parameters d_{ij} of m' randomly selected links, with m' being 30% of m.

The minimum $\delta_{\min}$ and maximum $\delta_{\max}$ values for threshold δ are fixed to $z'/100$ and z', respectively, with $z' = w_1\, R_{\min} + w_2\, \Phi_{\min}(R_{\min}) + w_3\, C_{\min}$. Threshold δ is initially set at the minimum value $\delta_{\min}$. The update of δ is done at the end of each iteration with the following law: when δ is decided to be decreased, we decrease it by halving its value; otherwise δ is increased by doubling its value. In any cases, the value of δ is always assumed to be within a given range $[\delta_{\min}, \delta_{\max}]$.

For each instance and for each set of weights, we ran $max\_iter = 1000$ iterations of our algorithm, getting for each run a significant number of non-dominated solutions.

In order to evaluate the results of solutions provided by our algorithm, we compare them with the solutions, for different values of parameter α, obtained by solving (using CPLEX 12.4) the single-level MIP formulation of the bilevel model (1) proposed in Marcotte et al. (2009). To make the objective functions comparable, since the equity of the risk cannot be considered in model (1), we choose the values of α equal to the ratio w_3/w_1, and set $\beta = 0$.

For each instance and for each set of weights, we compute:

(i) Performance reference values $R_{\min}$, $\Phi_{\min}(R_{\min})$, and $C_{\min}$.

(ii) Performance values $R_{\rm tot}$, Φ, $C_{\rm tot}$, being the total risk on the network, the maximum link total risk, and the total cost paid by the carriers (including the tolls), respectively, for each non-dominated solutions returned by the algorithm, and for the solutions of model (1).

To simplify the evaluation of the the performance results of a given solution, we normalize their values by considering:

- The ratio $R_{\rm tot}/R_{\rm min}$, representing how far is the network total risk from the minimum possible value $R_{\rm min}$. The closer this value to 1 the better the solution from the point of view of total population exposure.
- The ratio $\Phi/\Phi_{\rm min}(R_{\rm min})$, representing how far is the maximum link total risk from the reference value $\Phi_{\rm min}(R_{\rm min})$, obtained for the *over-regulated* scenario, where we assume the regulator minimizes first the network total risk and then the maximum link total risk. This ratio represents a measure of the equity of the risk (the lower the ratio, the higher the equity), and can be smaller than one, since by relaxing the requirement on the total risk, a lower maximum link risk than the reference value $\Phi_{\rm min}(R_{\rm min})$ can be obtained.
- The ratio $C_{\rm tot}/C_{\rm min}$, representing how far is the carriers' total cost from the minimum possible value $C_{\rm min}$. The closer this value to 1 the better the solution from the carriers' point of view.

For each instance, and for every set of weight parameters, we report on a table the results related to the optimal solution of model (1) of Marcotte et al. (2009) used for the comparison, followed by the results of all the non-dominated solutions produced by our method (ordered by non-decreasing carriers' travel cost values) with carriers' total travel cost less than four times the carriers' total travel cost related to the optimal solution of problem (1). In particular, the table reports the performance ratios $R_{\rm tot}/R_{\rm min}$, $\Phi/\Phi_{\rm min}(R_{\rm min})$, and $C_{\rm tot}/C_{\rm min}$, for each determined solution.

Moreover, for the ease of presentation, we associate three pictures to the table: in the first (see Figure 2) we draw on the 2D plane $(\Phi/\Phi_{\rm min}(R_{\rm min}), R_{tot}/R_{min})$ the three solutions of model (1) (identified with labels "B-alpha=0.5", "B-alpha=0.75", and "B-alpha=1", for $\alpha = 0.50, 0.75, 1.00$, respectively) and all the non-dominated solutions (identified with labels "H-S1", "H-S2", and "H-S3", for weighting set S1, S2, and S3, respectively) produced by our heuristic; in the second picture (see Figure 3) we draw the same solutions as above on the 2D plane ($C_{\text{tot}}/C_{\text{min}}, \Phi/\Phi_{\text{min}}(R_{\text{min}})$), and in the third picture (see Figure 4) we draw the same solutions as above on the 2D plane ($C_{\text{tot}}/C_{\text{min}}, R_{tot}/R_{min}$).

In the following, we provide the results obtained on instance $I1$ listed on Table 1, and showed on Figures 2–4. The other results obtained on the other four instances show similar performance characteristics to those presented here, and are reported for ease of reading in Appendix.

A general behavior that may be observed From Table 1 and Figures 2–4 is that the solutions of our model are not dominated by the solutions of model (1) of Marcotte et al. (2009).

Table 1: The results of test instance $I1$

	$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$		$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$		$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$
$\alpha = 0.75$	2,16	1,12	1,05	$\alpha = 1$	2,3	1,12	1,03	$\alpha = 0.5$	1,94	1,12	1,07
S1	2,53	0,69	1,37	S2	2,53	0,69	1,37	S3	2,53	0,69	1,37
	2,37	0,72	1,44		2,37	0,72	1,44		2,37	0,72	1,44
	2,35	0,73	1,48		2,35	0,73	1,48		2,35	0,73	1,48
	2,26	0,81	1,51		2,26	0,81	1,51		2,26	0,81	1,51
	2,04	0,85	1,52		2,04	0,85	1,52		2,04	0,85	1,52
	2,03	0,85	1,58		2,03	0,85	1,58		2,03	0,85	1,58
	2,17	0,83	1,59		2,17	0,83	1,59		2,17	0,83	1,59
	1,91	0,70	1,62		1,91	0,70	1,62		1,91	0,70	1,62
	1,73	0,70	1,63		1,73	0,70	1,63		1,73	0,70	1,63
	1,90	0,70	1,67		1,90	0,70	1,67		1,90	0,70	1,67
	1,65	0,79	1,69		1,65	0,79	1,69		1,65	0,79	1,69
	1,73	0,70	1,72		1,73	0,70	1,72		1,73	0,70	1,72
	1,56	0,85	1,74		1,56	0,85	1,74		1,56	0,85	1,74
	1,45	0,86	1,75		1,45	0,86	1,75		1,45	0,86	1,75
	1,64	0,80	1,75		1,64	0,80	1,75		1,64	0,80	1,75
	1,43	0,90	1,77		1,43	0,90	1,77		1,43	0,90	1,77
	1,44	0,86	1,78		1,44	0,86	1,78		1,44	0,86	1,78
	1,43	0,90	1,81		1,43	0,90	1,81		1,43	0,90	1,81
	1,26	0,82	1,86		1,26	0,82	1,86		1,26	0,82	1,86
	1,32	0,69	1,86		1,32	0,69	1,86		1,32	0,69	1,86
	1,31	0,69	1,86		1,31	0,69	1,86		1,31	0,69	1,86
	1,24	0,83	1,90		1,24	0,83	1,90		1,24	0,83	1,90
	1,30	0,69	1,90		1,30	0,69	1,90		1,30	0,69	1,90
	1,27	0,69	1,92		1,27	0,69	1,92		1,27	0,69	1,92
	1,23	0,73	1,93		1,23	0,73	1,93		1,23	0,73	1,93
	1,22	0,94	1,99		1,22	0,94	1,99		1,22	0,94	1,99
	1,21	0,94	2,03		1,21	0,94	2,03		1,21	0,94	2,03
	1,19	0,94	2,04		1,19	0,94	2,04		1,19	0,94	2,04
	1,19	0,94	2,04		1,19	0,94	2,04		1,19	0,94	2,04
	1,17	1,02	2,05		1,17	1,02	2,05		1,17	1,02	2,05

	1,17	1,02	2,10		1,20	0,88	2,07		1,17	1,02	2,10
	1,16	1,12	2,23		1,18	0,88	2,08		1,18	1,02	2,23
	1,18	1,02	2,23		1,16	0,88	2,10		1,17	0,92	2,32
	1,18	1,00	2,32		1,15	1,00	2,33		1,15	1,00	2,33
	1,15	1,00	2,33		1,15	0,93	2,33		1,16	0,92	2,33
	1,16	1,00	2,40		1,15	0,96	2,36		1,15	0,92	2,34
	1,15	1,00	2,51		1,22	0,88	2,47		1,15	0,92	2,39
	1,16	0,90	2,55		1,14	0,93	2,53		1,18	0,91	2,40
	1,12	1,12	2,55		1,14	0,93	2,77		1,16	0,92	2,43
	1,14	1,09	2,56		1,16	0,88	2,85		1,16	0,92	2,48
	1,11	1,12	2,57		1,17	0,88	2,92		1,14	0,92	2,48
	1,09	1,12	2,59		1,16	0,88	2,98		1,14	0,93	2,48
	1,13	1,12	2,81		1,13	1,09	3,08		1,13	0,77	2,57
	1,13	1,12	2,81		1,16	0,87	3,12		1,13	0,93	2,61
	1,14	1,12	2,84		1,13	1,09	3,23		1,11	1,10	2,86
	1,12	1,12	2,87		1,11	1,10	3,24		1,10	1,09	2,86
	1,10	1,12	2,89		1,11	1,06	3,63		1,54	0,69	2,89
	1,09	1,12	2,90		1,10	1,11	3,86		1,13	0,73	2,95
	1,13	1,10	2,92		1,10	1,11	4,04		1,10	1,10	2,97
	1,21	0,86	2,92						1,09	1,11	3,08
	1,09	1,12	2,93						1,06	1,12	3,08
	1,18	0,86	2,99						1,09	1,10	3,17
	1,17	0,89	3,05						1,12	0,73	3,32
	1,15	0,91	3,05						1,07	1,11	3,61
	1,16	0,86	3,08						1,09	1,10	3,65
	1,08	1,12	3,08						1,12	0,68	3,95
	1,13	0,93	3,12								
	1,20	0,85	3,20								
	1,18	0,77	3,21								
	1,14	0,88	3,26								
	1,08	1,00	3,31								
	1,07	1,00	3,32								
	1,06	1,00	3,33								
	1,07	1,00	3,33								
	1,13	0,86	3,34								
	1,16	0,86	3,43								
	1,04	1,11	3,43								
	1,06	1,00	3,45								

Referring to instance $I1$, looking at Table 1 and Figure 2, we notice that our solutions almost always have both network total risk and maximum link total risk values lower than those of the solutions of model (1). More than 85% of our solutions have $R_{\text{tot}}/R_{\text{min}}$ values lower than the lowest value of the same ratio achieved by solutions of model (1) (i.e., 1.94). More than half of our solutions have $R_{\text{tot}}/R_{\text{min}} \leq 1.2$. Referring to the ratio $\Phi/\Phi_{\text{min}}(R_{\text{min}})$, model (1) provides always solutions with the same

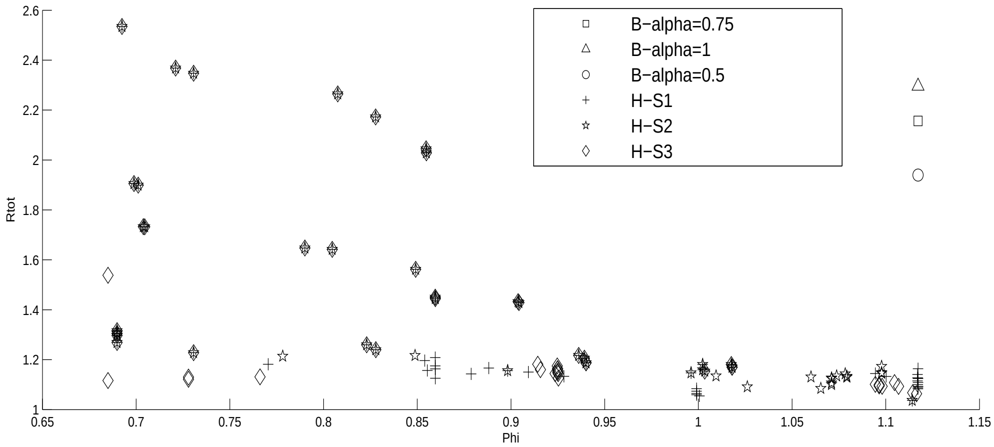

Figure 2: $I1$, 2-D plot with maximum link total risk performance ratio $\Phi/\Phi_{\min}(R_{\min})$ on the X-axis and network total risk performance ratio $R_{\text{tot}}/R_{\min}$ on the Y-axis

value 1.12, while with our model we always obtain non-dominated solutions with values ranging from 0.69 to 1.12. In particular, more than 75% of our solutions have $\Phi/\Phi_{\min}(R_{\min}) \leq 1$.

From Figure 3 (and Table 1), it appears that, in order to have a better risk equity (i.e., lower values of $\Phi/\Phi_{\min}(R_{\min})$), the solutions of our model have a higher total cost. Clearly, this is due to the presence of toll parameters d_{il}, that are constrained to be at least 10^{-3}. The solutions of model (1) have $C_{\text{tot}}/C_{\min}$ values ranging from 1.03 to 1.07. More than 42% of the solutions of our model have values of $C_{\text{tot}}/C_{\min} \leq 2$, while more than 80% of them have $C_{\text{tot}}/C_{\min} \leq 3$.

From Figure 4 (and Table 1), one may notice that more than 35% of our solutions with $R_{\text{tot}}/R_{\min} \leq 1.94$ (i.e., the minimum value achieved by the solutions of Marcotte et al.) have $C_{\text{tot}}/C_{\min} \leq 2$ and more than 80% of our solutions have $C_{\text{tot}}/C_{\min} \leq 3$. More than 90% of our solutions have $R_{\text{tot}}/R_{\min} \leq 2.16$, that is the maximum $R_{\text{tot}}/R_{\min}$ value produced by the Marcotte et al.'s model.

As far as the other instances are considered, one may notice from Figures 5–16 and Tables 2–5 reported in Appendix, that there is a similar performance of our algorithm: it appears that the dispersion of the solutions on the 2D planes considered is almost the same, with slight differences with respect on the percentages computed in the above analysis on instance $I1$.

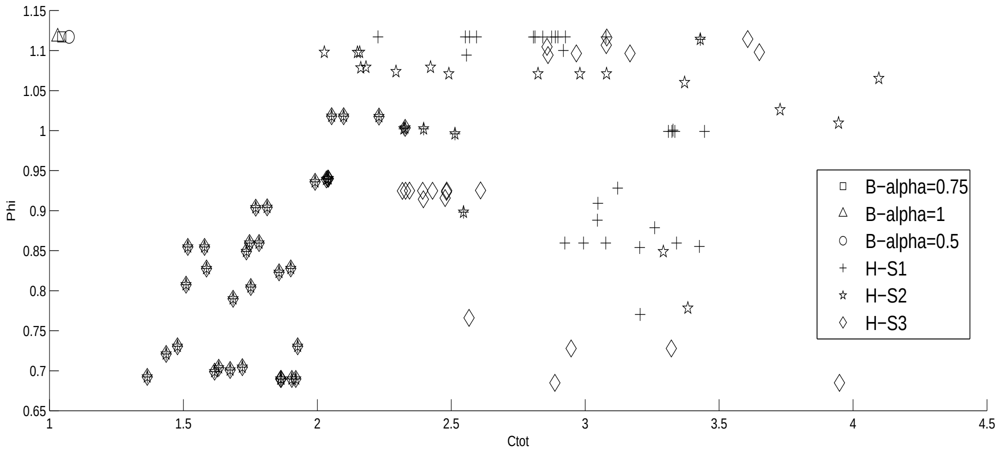

Figure 3: $I1$, 2-D plot with carriers' total cost performance ratio $C_{\text{tot}}/C_{\text{min}}$ on the X-axis and maximum link total risk performance ratio $\Phi/\Phi_{\text{min}}(R_{\text{min}})$ on the Y-axis

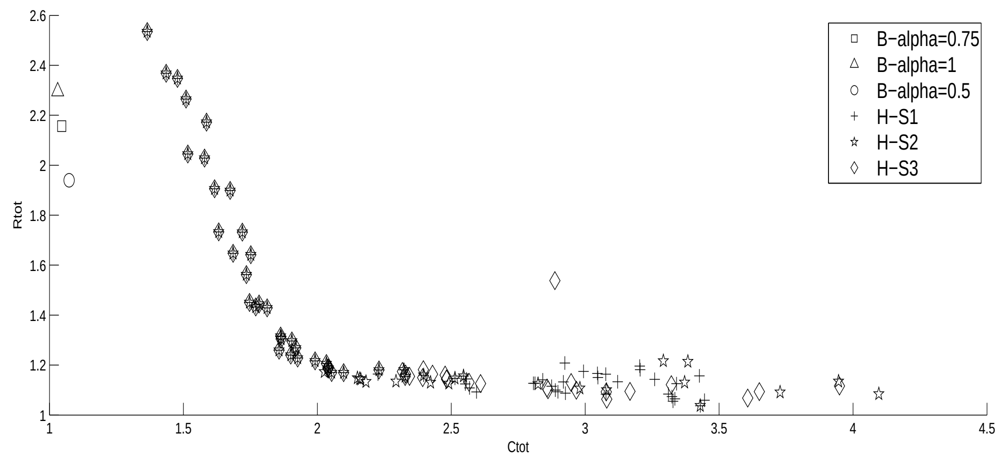

Figure 4: $I1$, 2-D plot with carriers' total cost performance ratio $C_{\text{tot}}/C_{\text{min}}$ on the X-axis and network total risk performance ratio $R_{\text{tot}}/R_{\text{min}}$ on the Y-axis

In conclusions, the overall results show that our algorithm produces very good solutions in terms of population risk exposure and risk equity. Moreover, these solutions are stable and represent a significant improvement with respect to the solutions obtained by the approach proposed in Marcotte et al. (2009).

8 Conclusions

We study a novel toll setting policy to regulate hazardous material transportation, where the regulator (e.g., a government authority) aims both at minimizing the network total risk and at spreading the risk in an equitable way over a given road transportation network. The proposed toll setting policy tends to discourage carriers from overloading portions of the network with the consequent increase of the risk exposure of the population involved. To cope with such a problem we propose a mathematical programming with equilibrium constraint (MPEC) model, where the inner problem is a Nash game having as players the carriers each one wishing to minimize his/her transportation cost (including tolls); the outer problem is addressed by the government authority whose aim is finding the link tolls that induce the carriers to choose hazmat transportation route plans that minimize both the network total risk and the maximum link total risk among the network links (in order to address risk equity). In order to guarantee the stability of the solution of the MPEC problem, we study conditions for the existence and uniqueness of the Nash equilibrium, and propose a local search algorithm for the MPEC problem based on these conditions. Computational results on a real network show that our algorithm is able to produce very good solutions in terms of population risk exposure and risk equity. Moreover, these solutions are stable and represent a significant improvement with respect to the solutions obtained by the toll setting model proposed in Marcotte et al. (2009).

Appendix

Table 2: The results of test instance $I2$

	$\frac{R_{\text{tot}}}{R_{\min}}$	$\frac{\Phi}{\Phi_{\min}(R_{\min})}$	$\frac{C_{\text{tot}}}{C_{\min}}$		$\frac{R_{\text{tot}}}{R_{\min}}$	$\frac{\Phi}{\Phi_{\min}(R_{\min})}$	$\frac{C_{\text{tot}}}{C_{\min}}$		$\frac{R_{\text{tot}}}{R_{\min}}$	$\frac{\Phi}{\Phi_{\min}(R_{\min})}$	$\frac{C_{\text{tot}}}{C_{\min}}$
$\alpha = 0.75$	2,3	1,85	1,03	$\alpha = 1$	2,41	1,85	1,03	$\alpha = 0.5$	2,03	1,85	1,07
S1	2,60	1,14	1,28	S2	2,60	1,14	1,28	S3	2,60	1,14	1,28
	2,34	0,99	1,33		2,34	0,99	1,33		2,34	0,99	1,33

	2,30	0,99	1,36		2,30	0,99	1,36		2,32	1,48	1,36
	2,17	1,18	1,38		2,17	1,18	1,38		2,30	0,99	1,36
	2,13	1,18	1,39		2,13	1,18	1,39		2,17	1,18	1,38
	1,93	0,99	1,42		1,93	0,99	1,42		2,05	1,52	1,39
	1,79	1,18	1,46		1,79	1,18	1,46		2,13	1,18	1,39
	1,93	0,89	1,47		1,93	0,89	1,47		2,04	1,52	1,40
	1,80	1,18	1,48		1,80	1,18	1,48		1,94	1,52	1,42
	1,73	1,00	1,52		1,73	1,00	1,52		2,12	1,52	1,42
	1,72	1,52	1,53		1,66	1,15	1,53		1,93	0,99	1,42
	1,66	1,15	1,53		1,72	0,96	1,56		1,91	1,52	1,46
	1,69	1,05	1,55		1,65	1,15	1,57		1,79	1,18	1,46
	1,59	1,05	1,55		1,52	1,18	1,58		1,93	0,89	1,47
	1,72	0,96	1,56		1,52	1,18	1,59		1,80	1,18	1,48
	1,45	1,05	1,57		1,44	1,33	1,60		1,76	1,52	1,51
	1,44	1,05	1,58		1,44	1,33	1,65		1,73	1,00	1,52
	1,44	1,33	1,60		1,38	1,33	1,68		1,71	1,43	1,53
	1,39	0,84	1,62		1,38	1,33	1,73		1,66	1,15	1,53
	1,28	0,84	1,64		1,38	1,56	1,78		1,44	1,35	1,55
	1,26	1,05	1,70		1,32	1,56	1,78		1,48	1,28	1,56
	1,25	1,06	1,70		1,34	1,56	1,79		1,72	0,96	1,56
	1,24	1,06	1,71		1,23	1,56	1,80		1,65	1,15	1,57
	1,21	1,06	1,72		1,33	1,56	1,80		1,52	1,18	1,58
	1,22	1,05	1,72		1,34	1,56	1,81		1,35	1,35	1,58
	1,14	1,54	1,73		1,25	1,56	1,81		1,52	1,18	1,59
	1,19	1,48	1,74		1,49	1,32	1,81		1,35	1,35	1,60
	1,15	1,52	1,76		1,24	1,56	1,81		1,44	1,33	1,60
	1,18	1,08	1,77		1,23	1,56	1,82		1,50	1,28	1,61
	1,12	1,55	1,77		1,19	1,56	1,83		1,30	1,41	1,62
	1,12	1,54	1,78		1,33	1,56	1,85		1,30	1,56	1,63
	1,11	1,54	1,78		1,18	1,56	1,86		1,25	1,39	1,64
	1,11	1,55	1,79		1,27	1,56	1,86		1,44	1,33	1,65
	1,18	1,27	1,81		1,23	1,56	1,87		1,42	1,31	1,67
	1,18	1,05	1,92		1,25	1,56	1,87		1,22	1,35	1,68
	1,16	0,94	2,00		1,31	1,33	1,87		1,38	1,33	1,68
	1,15	1,11	2,01		1,27	1,43	1,91		1,19	1,56	1,69
	1,13	1,52	2,04		1,29	1,33	1,92		1,38	1,33	1,73
	1,15	1,11	2,04		1,18	1,56	1,93		1,20	1,39	1,76
	1,16	0,94	2,04		1,30	1,12	1,96		1,18	1,56	1,81
	1,18	0,90	2,23		1,28	1,22	1,96		1,16	1,56	1,82
	1,14	1,49	2,32		1,29	1,12	1,96		1,20	1,39	1,83
	1,14	1,26	2,35		1,24	1,22	1,96		1,13	1,56	1,84
	1,15	0,92	2,41		1,14	1,56	1,96		1,19	1,39	1,85
	1,12	1,11	2,43		1,17	1,22	1,99		1,31	1,33	1,87
	1,15	0,92	2,46		1,18	1,13	1,99		1,18	1,42	1,88
	1,11	0,94	2,51		1,15	1,56	2,00		1,19	1,39	1,91
	1,11	1,12	2,56		1,16	0,94	2,00		1,18	1,43	1,92
	1,11	1,12	2,60		1,15	1,11	2,01		1,29	1,33	1,92
	1,10	1,15	2,69		1,11	1,11	2,02		1,16	1,42	1,93
	1,11	0,94	2,95		1,15	1,11	2,02		1,13	1,54	1,94
	1,10	1,12	3,27		1,15	1,11	2,04		1,14	1,52	1,94

	1,10	1,12	3,30		1,16	0,94	2,04		1,13	1,54	1,94
					1,13	1,11	2,05		1,13	1,54	1,95
					1,10	1,11	2,05		1,23	1,35	1,96
					1,14	1,11	2,10		1,14	1,48	1,96
					1,11	0,97	2,11		1,14	1,52	1,96
					1,10	1,56	2,28		1,13	1,52	1,97
					1,10	0,97	2,31		1,34	1,26	1,98
					1,09	1,54	2,38		1,18	1,39	1,99
					1,08	1,55	2,56		1,24	1,26	1,99
					1,09	0,97	2,62		1,20	1,13	2,01
					1,06	1,55	2,62		1,18	1,26	2,01
					1,12	0,95	3,06		1,12	1,51	2,02
					1,06	1,56	3,46		1,17	1,39	2,04
					1,09	1,52	4,11		1,12	1,51	2,05
									1,11	1,56	2,07
									1,29	1,13	2,07
									1,10	1,56	2,07
									1,17	1,26	2,08
									1,20	1,13	2,08
									1,10	1,56	2,09
									1,14	1,39	2,09
									1,11	1,39	2,10
									1,16	1,39	2,13
									1,13	1,30	2,14
									1,15	1,26	2,15
									1,11	1,51	2,17
									1,10	1,51	2,20
									1,13	1,13	2,22
									1,10	1,56	2,31
									1,12	1,30	2,36
									1,10	1,56	2,43
									1,10	1,49	2,45
									1,11	1,44	2,45
									1,09	1,50	2,49
									1,11	1,31	2,51
									1,09	1,56	2,51
									1,11	1,30	2,53
									1,09	1,56	2,55
									1,24	1,11	2,57
									1,18	1,11	2,57
									1,11	1,46	2,59
									1,10	1,48	2,63
									1,10	1,42	2,64
									1,16	1,11	2,65
									1,14	0,99	2,66
									1,11	1,17	2,69
									1,09	1,50	2,72
									1,10	1,28	2,74
									1,10	1,43	2,75
									1,11	1,28	2,75

									1,08	1,50	2,76
									1,10	1,33	2,77
									1,10	1,36	2,99
									1,10	1,18	2,99
									1,07	1,45	3,01
									1,14	0,99	3,01
									1,09	1,36	3,06
									1,07	1,45	3,30
									1,12	1,00	3,38
									1,11	1,00	3,41
									1,09	1,16	3,44
									1,09	1,34	3,48
									1,09	1,36	3,57
									1,08	1,44	3,90
									1,07	1,36	3,95
									1,06	1,56	3,98
									1,06	1,56	4,23

Table 3: The results of test instance $I3$

	$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$		$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$		$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$
$\alpha = 0.75$	2,22	2,08	1,05	$\alpha = 1$	2,26	2,08	1,04	$\alpha = 0.5$	2,02	2,08	1,07
S1	2,57	1,22	1,28	S2	2,57	1,22	1,28	S3	2,57	1,22	1,28
	2,30	1,17	1,34		2,57	1,91	1,33		2,30	1,17	1,34
	2,30	1,17	1,37		2,30	1,17	1,34		2,30	1,17	1,37
	2,23	1,09	1,37		2,30	1,17	1,37		2,23	1,09	1,37
	2,08	1,14	1,38		2,23	1,09	1,37		2,08	1,14	1,38
	1,94	1,13	1,39		2,08	1,14	1,38		1,94	1,13	1,39
	1,94	1,13	1,42		1,94	1,13	1,39		1,94	1,13	1,42
	1,82	1,09	1,42		1,94	1,13	1,42		1,82	1,09	1,42
	1,75	1,42	1,44		1,82	1,09	1,42		1,75	1,42	1,44
	2,10	1,09	1,46		1,75	1,42	1,44		2,10	1,09	1,46
	1,67	1,64	1,46		2,10	1,09	1,46		1,67	1,64	1,46
	1,67	1,42	1,46		1,67	1,64	1,46		1,67	1,42	1,46
	1,82	1,18	1,47		1,67	1,42	1,46		1,82	1,18	1,47
	1,64	1,64	1,49		1,82	1,18	1,47		1,64	1,64	1,49
	1,63	1,84	1,51		1,64	1,64	1,49		1,63	1,84	1,51
	1,66	1,42	1,52		1,63	1,84	1,51		1,66	1,42	1,52
	1,51	1,77	1,52		1,66	1,42	1,52		1,51	1,77	1,52
	1,47	1,77	1,54		1,51	1,77	1,52		1,47	1,77	1,54
	1,44	1,77	1,55		1,47	1,77	1,54		1,44	1,77	1,55
	1,45	1,77	1,56		1,44	1,77	1,55		1,45	1,77	1,56
	1,42	1,77	1,58		1,45	1,77	1,56		1,42	1,77	1,58
	1,31	1,87	1,65		1,42	1,77	1,58		1,64	1,75	1,64
	1,31	1,87	1,68		1,31	1,87	1,65		1,31	1,87	1,65
	1,25	1,89	1,68		1,31	1,87	1,68		1,54	1,62	1,65
	1,23	1,90	1,74		1,25	1,89	1,68		1,50	1,62	1,67

	1,24	1,89	1,75		1,22	1,91	1,73		1,31	1,87	1,68
	1,20	1,89	1,76		1,23	1,90	1,74		1,25	1,89	1,68
	1,20	1,89	1,77		1,18	1,91	1,75		1,54	1,62	1,69
	1,18	1,89	1,77		1,18	1,91	1,75		1,39	1,75	1,70
	1,17	1,89	1,78		1,24	1,89	1,75		1,27	1,62	1,73
	1,17	1,89	1,78		1,20	1,89	1,76		1,24	1,90	1,74
	1,17	1,90	1,83		1,13	1,91	1,77		1,23	1,91	1,75
	1,28	1,83	1,85		1,20	1,89	1,77		1,24	1,75	1,75
	1,23	1,83	1,86		1,18	1,89	1,77		1,47	1,62	1,75
	1,23	1,83	1,86		1,12	1,91	1,77		1,24	1,89	1,75
	1,17	1,83	1,88		1,17	1,89	1,78		1,21	1,90	1,76
	1,29	1,83	1,90		1,15	1,91	1,78		1,18	1,90	1,76
	1,13	1,87	1,92		1,17	1,89	1,78		1,43	1,59	1,77
	1,23	1,83	1,93		1,12	1,91	1,79		1,13	1,91	1,78
	1,14	1,87	1,97		1,11	1,91	1,80		1,17	1,90	1,81
	1,12	1,84	1,99		1,10	1,91	1,80		1,16	1,90	1,82
	1,13	1,83	2,05		1,11	1,91	1,80		1,22	1,71	1,84
	1,13	1,79	2,05		1,09	1,91	1,81		1,12	1,91	1,86
	1,12	1,76	2,09		1,12	1,91	1,82		1,11	1,91	1,87
	1,10	1,76	2,11		1,09	1,91	1,83		1,18	1,90	1,92
	1,14	1,64	2,12		1,17	1,90	1,83		1,21	1,80	1,94
	1,27	1,09	2,31		1,28	1,83	1,85		1,12	1,91	1,98
	1,27	1,09	2,34		1,23	1,83	1,86		1,15	1,91	2,04
	1,22	1,42	2,41		1,23	1,83	1,86		1,15	1,90	2,07
	1,10	1,42	2,43		1,17	1,83	1,88		1,09	1,90	2,13
	1,18	1,42	2,46		1,15	1,87	1,88		1,26	1,60	2,15
	1,09	1,18	2,47		1,11	1,91	1,89		1,19	1,80	2,16
	1,26	1,18	2,56		1,14	1,91	1,90		1,10	1,89	2,22
	1,09	1,88	3,09		1,13	1,87	1,90		1,17	1,75	2,25
	1,07	1,91	3,39		1,29	1,83	1,90		1,22	1,62	2,27
	1,07	1,39	3,41		1,13	1,87	1,92		1,15	1,87	2,28
	1,10	1,07	3,59		1,10	1,91	1,92		1,21	1,62	2,35
	1,07	1,91	3,67		1,23	1,83	1,93		1,14	1,80	2,36
	1,06	1,91	3,72		1,48	1,77	1,95		1,13	1,88	2,36
	1,07	1,39	3,78		1,39	1,77	1,95		1,16	1,60	2,36
	1,09	1,32	3,87		1,25	1,68	1,95		1,09	1,89	2,43
	1,05	1,91	3,90		1,18	1,69	1,96		1,16	1,61	2,48
	1,05	1,91	4,00		1,14	1,87	1,97		1,07	1,91	2,53
	1,07	1,91	4,08		1,12	1,84	1,99		1,09	1,91	2,70
					1,12	1,91	2,01		1,07	1,91	2,71
					1,09	1,91	2,01		1,05	1,91	2,72
					1,27	1,64	2,03		1,08	1,89	2,74
					1,15	1,78	2,09		1,15	1,61	2,76
					1,14	1,77	2,12		1,05	1,91	2,79
					1,16	1,64	2,12		1,14	1,61	2,81
					1,12	1,78	2,13		1,10	1,60	2,83
					1,11	1,91	2,15		1,05	1,91	2,83
					1,12	1,84	2,18		1,09	1,60	2,87
					1,15	1,67	2,26		1,25	1,58	2,91
					1,14	1,78	2,30		1,05	1,91	2,92

					1,36	1,18	2,32		1,26	1,57	2,92
					1,34	1,42	2,37		1,09	1,60	2,99
					1,32	1,42	2,38		1,07	1,89	3,01
					1,06	1,91	2,38		1,09	1,60	3,02
					1,43	1,11	2,43		1,21	1,58	3,02
					1,38	1,15	2,44		1,19	1,58	3,03
					1,30	1,63	2,46		1,14	1,58	3,05
					1,35	1,15	2,46		1,07	1,90	3,06
					1,28	1,43	2,47		1,12	1,56	3,07
					1,36	1,07	2,47		1,47	1,51	3,10
					1,18	1,43	2,48		1,44	1,51	3,12
					1,12	1,85	2,48		1,24	1,55	3,17
					1,22	1,42	2,48		1,54	1,43	3,23
					1,06	1,91	2,48		1,05	1,91	3,25
					1,29	1,11	2,48		1,24	1,55	3,28
					1,33	1,07	2,49		1,15	1,56	3,32
					1,16	1,64	2,54		1,12	1,55	3,33
					1,17	1,48	2,55		1,09	1,58	3,43
					1,28	1,20	2,56		1,32	1,52	3,44
					1,28	1,35	2,59		1,22	1,47	3,44
					1,13	1,69	2,67		1,04	1,91	3,46
					1,18	1,43	2,67		1,27	1,47	3,46
					1,14	1,48	2,67		1,22	1,45	3,51
					1,26	1,01	2,69		1,16	1,50	3,53
					1,10	1,51	2,70		1,08	1,83	3,54
					1,13	1,27	2,71		1,06	1,75	3,55
					1,18	1,01	2,87		1,09	1,60	3,55
					1,40	0,99	3,02		1,06	1,61	3,55
					1,06	1,91	3,05		1,12	1,58	3,56
					1,12	1,51	3,12		1,13	1,52	3,57
					1,14	1,07	3,17		1,04	1,75	3,61
					1,09	1,85	3,18		1,07	1,60	3,70
					1,09	1,85	3,18		1,21	1,48	3,75
					1,08	1,86	3,18		1,45	1,44	3,85
					1,07	1,85	3,19		1,12	1,56	3,90
					1,29	0,99	3,21		1,12	1,55	3,90
					1,09	1,85	3,22		1,07	1,60	3,92
					1,11	1,29	3,27		1,10	1,58	3,92
					1,20	0,96	3,57		1,06	1,60	4,09
					1,05	1,91	3,66				
					1,07	1,86	3,80				
					1,06	1,88	3,88				
					1,05	1,88	4,07				

Table 4: The results of test instance $I4$

	$\frac{R_{\text{tot}}}{R_{\min}}$	$\frac{\Phi}{\Phi_{\min}(R_{\min})}$	$\frac{C_{\text{tot}}}{C_{\min}}$		$\frac{R_{\text{tot}}}{R_{\min}}$	$\frac{\Phi}{\Phi_{\min}(R_{\min})}$	$\frac{C_{\text{tot}}}{C_{\min}}$		$\frac{R_{\text{tot}}}{R_{\min}}$	$\frac{\Phi}{\Phi_{\min}(R_{\min})}$	$\frac{C_{\text{tot}}}{C_{\min}}$
$\alpha = 0.75$	2,32	2,00	1,04	$\alpha = 1$	2,45	2,00	1,03	$\alpha = 0.5$	2,03	2,00	1,08

S1	2,74	1,29	1,29	S2	2,74	1,29	1,29	S3	2,74	1,29	1,29
	2,61	1,29	1,34		2,61	1,29	1,34		2,61	1,29	1,34
	2,57	1,29	1,35		2,57	1,29	1,35		2,57	1,29	1,35
	2,41	1,53	1,38		2,41	1,53	1,38		2,41	1,53	1,38
	2,22	1,72	1,40		2,22	1,72	1,40		2,22	1,72	1,40
	2,36	1,53	1,40		2,36	1,53	1,40		2,36	1,53	1,40
	2,20	1,72	1,42		2,20	1,72	1,42		2,20	1,72	1,42
	2,10	1,53	1,43		2,10	1,53	1,43		2,10	1,53	1,43
	2,02	1,72	1,44		2,02	1,72	1,44		2,02	1,72	1,44
	1,92	1,53	1,45		1,92	1,53	1,45		1,92	1,53	1,45
	1,84	1,53	1,46		1,84	1,53	1,46		1,84	1,53	1,46
	1,82	1,29	1,49		1,82	1,29	1,49		1,82	1,29	1,49
	1,74	1,35	1,50		1,74	1,35	1,50		1,74	1,35	1,50
	1,62	1,35	1,50		1,62	1,35	1,50		1,62	1,35	1,50
	1,58	1,53	1,51		1,58	1,53	1,51		1,58	1,53	1,51
	1,57	1,53	1,52		1,57	1,53	1,52		1,57	1,53	1,52
	1,73	1,21	1,52		1,73	1,21	1,52		1,73	1,21	1,52
	1,49	1,52	1,53		1,49	1,52	1,53		1,49	1,52	1,53
	1,61	1,35	1,53		1,61	1,35	1,53		1,61	1,35	1,53
	1,47	1,50	1,54		1,47	1,50	1,54		1,47	1,50	1,54
	1,40	1,49	1,57		1,40	1,49	1,57		1,40	1,49	1,57
	1,55	1,47	1,58		1,55	1,47	1,58		1,55	1,47	1,58
	1,36	1,49	1,60		1,36	1,49	1,60		1,36	1,49	1,60
	1,38	1,49	1,67		1,38	1,49	1,67		1,72	1,29	1,62
	1,33	1,96	1,73		1,34	1,66	1,70		1,62	1,31	1,62
	1,18	1,91	1,73		1,28	1,51	1,79		1,61	1,31	1,63
	1,17	1,83	1,73		1,31	1,49	1,80		1,37	1,31	1,63
	1,27	1,69	1,74		1,26	1,50	1,87		1,50	1,30	1,64
	1,16	1,69	1,77		1,24	1,52	1,95		1,29	1,31	1,64
	1,31	1,49	1,80		1,21	1,52	1,95		1,25	1,36	1,66
	1,14	1,72	1,86		1,24	1,51	2,04		1,23	1,36	1,66
	1,26	1,50	1,87		1,21	1,51	2,04		1,24	1,36	1,67
	1,24	1,52	1,95		1,20	1,52	2,10		1,23	1,72	1,68
	1,21	1,52	1,95		1,20	1,52	2,11		1,21	1,60	1,68
	1,13	1,69	1,97		1,21	1,51	2,15		1,17	1,36	1,69
	1,12	1,74	2,03		1,19	1,51	2,16		1,71	1,14	1,69
	1,24	1,51	2,04		1,17	1,51	2,17		1,29	1,31	1,70
	1,21	1,51	2,04		1,17	1,66	2,22		1,49	1,16	1,72
	1,10	1,74	2,09		1,20	1,51	2,29		1,56	1,14	1,73
	1,20	1,52	2,10		1,16	1,51	2,33		1,17	1,54	1,73
	1,21	1,51	2,15		1,06	1,51	2,34		1,46	1,07	1,84
	1,17	1,51	2,17		1,17	1,51	2,36		1,15	1,79	1,87
	1,17	1,66	2,22		1,15	1,51	2,56		1,39	1,07	1,96
	1,10	1,74	2,22		1,12	1,51	2,59		1,13	1,89	1,98
	1,16	1,51	2,33		1,19	1,51	2,64		1,15	1,53	2,05
	1,38	1,47	2,45		1,37	1,48	2,70		1,14	1,74	2,11
	1,27	1,42	2,48		1,09	1,51	2,70		1,11	1,89	2,16
	1,09	1,75	2,48		1,19	1,51	2,80		1,31	1,29	2,17
	1,19	1,42	2,50		1,15	1,51	3,00		1,10	1,35	2,23
	1,19	1,42	2,55		1,08	1,51	3,01		1,06	1,96	2,44

	1,15	1,51	2,56		1,16	1,51	3,05		1,05	1,97	2,61
	1,15	1,42	2,57		1,15	1,51	3,11		1,27	1,31	2,74
	1,06	1,64	2,60		1,16	1,51	3,37		1,10	1,82	2,84
	1,12	1,56	2,61		1,22	1,51	3,38		1,06	1,94	2,98
	1,11	1,56	2,66		1,12	1,51	3,47		1,10	1,68	3,42
	1,15	1,49	2,71		1,19	1,51	3,62		1,10	1,46	4,09
	1,12	1,51	2,84		1,34	1,43	3,74		1,10	1,46	4,19
	1,10	1,51	2,88		1,30	1,43	3,75				
	1,07	1,51	2,90		1,35	1,43	3,76				
	1,11	1,47	2,97		1,29	1,43	3,76				
	1,12	1,40	3,45		1,36	1,32	3,80				
	1,10	1,47	3,57		1,38	1,17	3,80				
	1,08	1,47	3,85		1,38	1,06	3,83				
					1,35	1,32	3,84				
					1,29	1,32	3,84				
					1,25	1,17	3,89				
					1,16	1,44	3,94				
					1,16	1,44	3,95				
					1,21	1,14	3,95				
					1,17	1,43	3,97				
					1,16	1,44	3,99				
					1,21	1,25	3,99				
					1,18	1,25	4,01				
					1,15	1,44	4,01				
					1,15	1,43	4,03				
					1,15	1,37	4,07				
					1,10	1,51	4,10				

Table 5: The results of test instance $I5$

	$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$		$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$		$\frac{R_{\text{tot}}}{R_{\text{min}}}$	$\frac{\Phi}{\Phi_{\text{min}}(R_{\text{min}})}$	$\frac{C_{\text{tot}}}{C_{\text{min}}}$
$\alpha = 0.75$	2,12	1,63	1,05	$\alpha = 1$	2,21	1,63	1,04	$\alpha = 0.5$	1,84	1,63	1,08
S1	2,41	1,06	1,23	S2	2,41	1,06	1,23	S3	2,41	1,06	1,23
	2,24	0,93	1,27		2,24	0,93	1,27		2,24	0,93	1,27
	2,23	0,93	1,30		2,23	0,93	1,30		2,23	0,93	1,30
	2,14	0,93	1,30		2,14	0,93	1,30		2,14	0,93	1,30
	1,89	1,06	1,32		1,89	1,06	1,32		1,89	1,06	1,32
	1,86	0,92	1,32		1,86	0,92	1,32		1,86	0,92	1,32
	1,83	1,06	1,32		1,83	1,06	1,32		1,83	1,06	1,32
	1,76	1,06	1,32		1,76	1,06	1,32		1,76	1,06	1,32
	1,73	1,06	1,34		1,73	1,06	1,34		1,73	1,06	1,34
	1,65	1,06	1,35		1,65	1,06	1,35		1,65	1,06	1,35
	1,73	0,93	1,35		1,73	0,93	1,35		1,73	0,93	1,35
	1,71	0,92	1,37		1,71	0,92	1,37		1,71	0,92	1,37
	1,69	0,94	1,38		1,69	0,94	1,38		1,69	0,94	1,38
	1,69	0,94	1,38		1,69	0,94	1,38		1,69	0,94	1,38
	1,61	0,92	1,40		1,61	0,92	1,40		1,61	0,92	1,40

	1,68	0,83	1,42		1,68	0,83	1,42		1,68	0,83	1,42
	1,57	1,06	1,44		1,57	1,06	1,44		1,57	1,06	1,44
	1,67	0,83	1,45		1,67	0,83	1,45		1,67	0,83	1,45
	1,64	0,83	1,48		1,64	0,83	1,48		1,64	0,83	1,48
	1,61	0,90	1,50		1,61	0,90	1,50		1,61	1,06	1,48
	1,58	1,06	1,50		1,58	1,06	1,50		1,61	0,90	1,50
	1,53	1,06	1,55		1,53	1,06	1,55		1,58	1,06	1,50
	1,40	0,93	1,62		1,51	1,47	1,60		1,53	1,06	1,55
	1,59	0,88	1,67		1,40	0,93	1,62		1,41	1,06	1,58
	1,36	1,55	1,69		1,59	0,88	1,67		1,40	0,93	1,62
	1,31	1,25	1,71		1,35	1,55	1,67		1,34	1,25	1,63
	1,25	1,55	1,72		1,31	1,25	1,71		1,59	0,88	1,67
	1,28	1,55	1,72		1,29	1,42	1,72		1,31	1,25	1,71
	1,29	1,42	1,72		1,28	1,42	1,74		1,29	1,42	1,72
	1,36	1,06	1,73		1,36	1,21	1,74		1,28	1,42	1,74
	1,28	1,42	1,74		1,28	1,42	1,74		1,36	1,21	1,74
	1,36	1,21	1,74		1,28	1,42	1,78		1,27	1,40	1,74
	1,28	1,42	1,74		1,30	1,26	1,81		1,23	1,25	1,75
	1,28	1,42	1,78		1,20	1,59	1,83		1,22	1,17	1,83
	1,30	1,26	1,81		1,12	1,63	1,84		1,39	0,97	1,84
	1,21	1,47	1,81		1,23	1,21	1,84		1,19	1,60	1,84
	1,34	1,25	1,84		1,19	1,42	1,86		1,50	0,85	1,88
	1,23	1,21	1,84		1,11	1,63	1,86		1,22	1,26	1,95
	1,23	1,40	1,88		1,23	1,40	1,88		1,28	1,11	1,95
	1,50	0,85	1,88		1,50	0,85	1,88		1,42	0,91	1,97
	1,36	0,94	1,89		1,22	1,24	1,89		1,22	1,45	1,99
	1,35	0,99	1,93		1,13	1,63	1,89		1,16	1,62	2,00
	1,22	1,26	1,95		1,15	1,61	1,90		1,14	1,62	2,00
	1,31	0,99	1,96		1,11	1,63	1,92		1,18	1,61	2,01
	1,18	1,24	1,98		1,13	1,62	1,93		1,13	1,62	2,01
	1,25	1,17	1,99		1,35	0,99	1,93		1,22	1,58	2,04
	1,10	1,51	2,01		1,19	1,44	1,93		1,20	1,60	2,04
	1,31	1,11	2,07		1,31	0,99	1,96		1,37	0,91	2,05
	1,24	1,17	2,13		1,18	1,52	1,98		1,09	1,62	2,06
	1,19	1,12	2,23		1,15	1,61	1,99		1,08	1,63	2,08
	1,27	1,10	2,35		1,10	1,63	2,09		1,19	1,32	2,12
	1,15	1,46	2,74		1,14	1,59	2,10		1,16	1,50	2,19
	1,14	1,51	2,74		1,19	1,32	2,12		1,15	1,58	2,20
	1,14	1,34	2,75		1,14	1,61	2,15		1,39	0,90	2,22
	1,14	1,17	2,77		1,12	1,61	2,17		1,17	1,39	2,25
	1,13	1,46	2,78		1,19	1,12	2,23		1,33	0,98	2,29
	1,22	1,10	2,80		1,10	1,63	2,24		1,16	1,53	2,37
	1,27	0,95	2,86		1,10	1,61	2,28		1,15	1,62	2,37
	1,23	0,95	2,90		1,18	1,58	2,32		1,26	0,90	2,43
	1,32	0,94	2,92		1,08	1,63	2,61		1,18	1,12	2,46
	1,34	0,94	2,94		1,09	1,63	2,91		1,14	1,62	2,46
	1,20	1,09	2,94		1,08	1,63	2,93		1,15	1,39	2,48
	1,15	1,02	2,94		1,07	1,63	2,97		1,15	1,58	2,48
	1,22	0,96	3,00		1,09	1,63	3,04		1,14	1,62	2,54
	1,13	1,06	3,03		1,06	1,63	3,05		1,12	1,62	2,55

	1,13	1,45	3,05		1,05	1,63	3,23		1,12	1,11	2,55
	1,13	1,45	3,06		1,17	1,59	3,31		1,14	1,11	2,57
	1,19	0,96	3,17		1,17	1,58	3,39		1,11	1,62	2,61
	1,11	1,15	3,20		1,05	1,63	3,50		1,09	1,05	2,61
	1,09	1,63	3,25		1,15	1,58	3,67		1,20	1,05	2,64
	1,10	1,15	3,26		1,12	1,58	3,76		1,25	0,99	2,67
	1,18	0,98	3,61		1,11	1,58	3,92		1,07	1,36	2,68
	1,10	1,39	3,70		1,05	1,63	3,94		1,06	1,20	2,74
					1,17	1,57	4,08		1,21	0,99	2,77
									1,26	0,91	3,21
									1,10	1,01	3,31
									1,10	0,99	3,53
									1,09	0,99	4,01

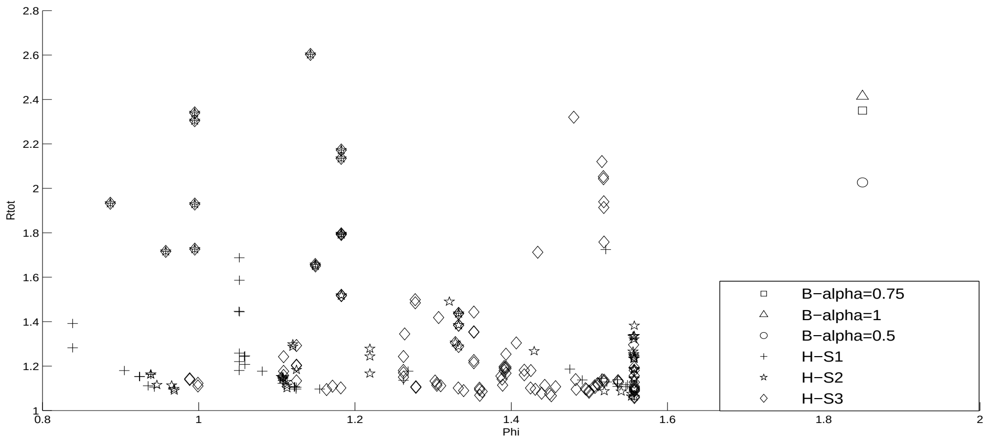

Figure 5: $I2$, 2-D plot with maximum link total risk performance ratio $\Phi/\Phi_{\min}(R_{\min})$ on the X-axis and network total risk performance ratio $R_{\text{tot}}/R_{\min}$ on the Y-axis

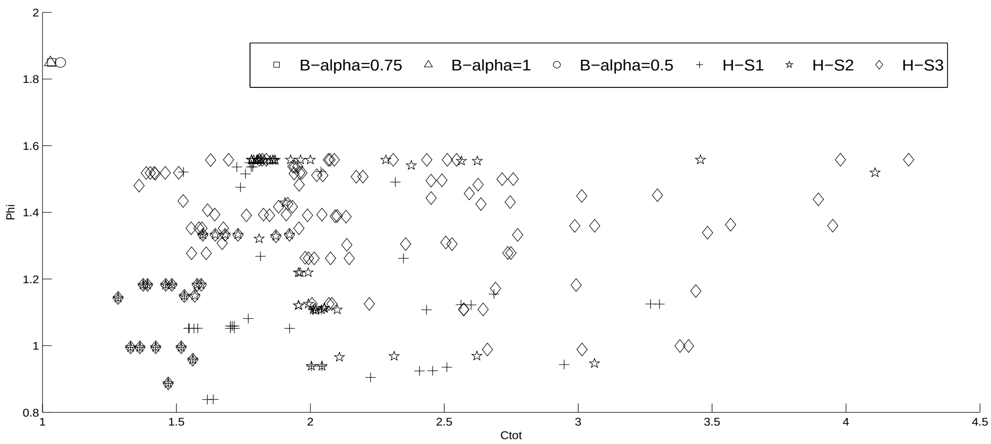

Figure 6: $I2$, 2-D plot with carriers' total cost performance ratio $C_{\text{tot}}/C_{\min}$ on the X-axis and maximum link total risk performance ratio $\Phi/\Phi_{\min}(R_{\min})$ on the Y-axis

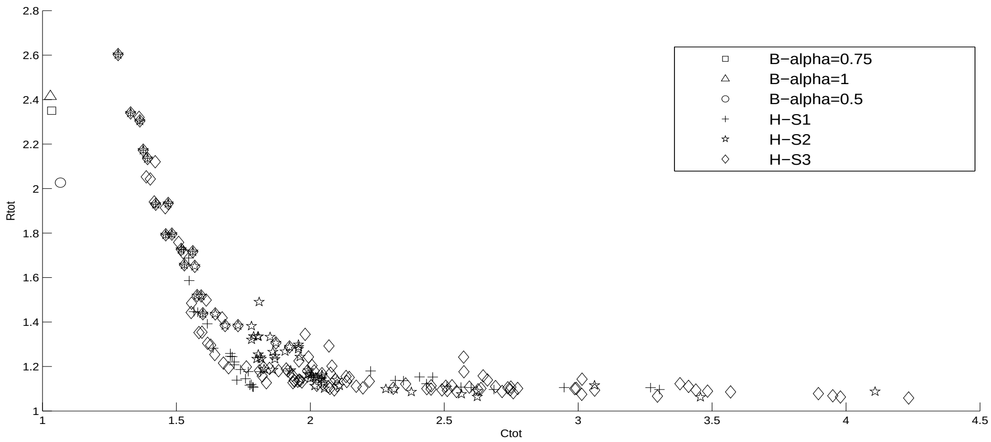

Figure 7: $I2$, 2-D plot with carriers' total cost performance ratio $C_{\mathrm{tot}}/C_{\mathrm{min}}$ on the X-axis and network total risk performance ratio $R_{\mathrm{tot}}/R_{\mathrm{min}}$ on the Y-axis

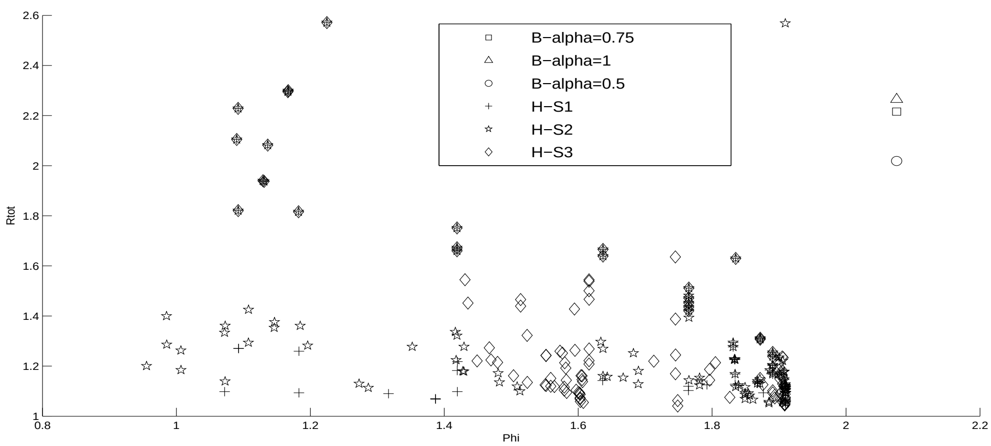

Figure 8: $I3$, 2-D plot with maximum link total risk performance ratio $\Phi/\Phi_{\mathrm{min}}(R_{\mathrm{min}})$ on the X-axis and network total risk performance ratio $R_{\mathrm{tot}}/R_{\mathrm{min}}$ on the Y-axis

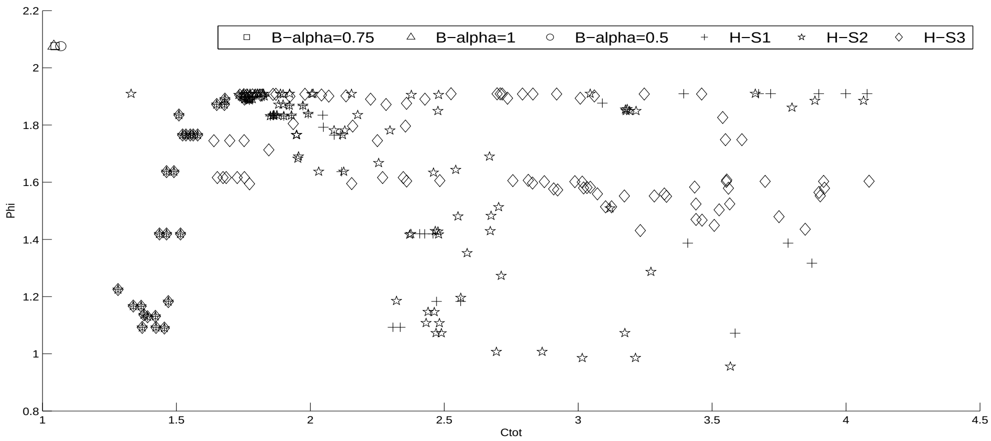

Figure 9: $I3$, 2-D plot with carriers' total cost performance ratio $C_{\text{tot}}/C_{\text{min}}$ on the X-axis and maximum link total risk performance ratio $\Phi/\Phi_{\text{min}}(R_{\text{min}})$ on the Y-axis

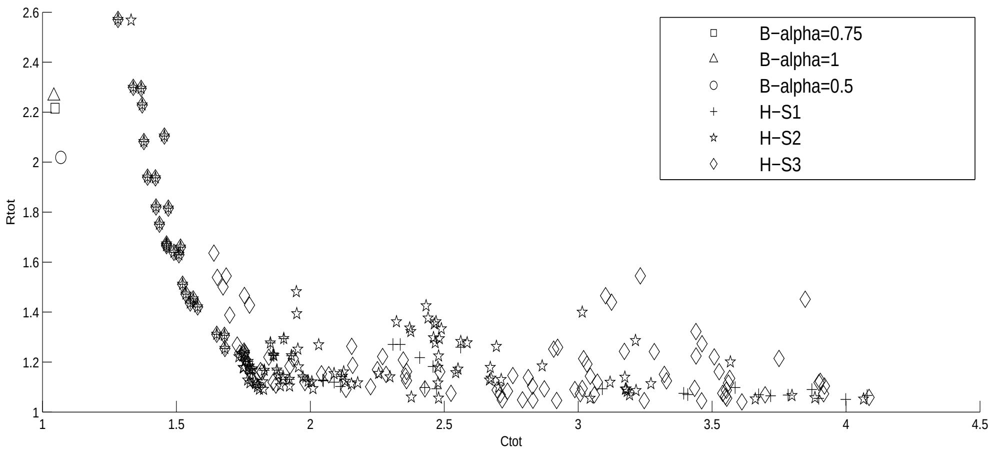

Figure 10: $I3$, 2-D plot with carriers' total cost performance ratio $C_{\text{tot}}/C_{\text{min}}$ on the X-axis and network total risk performance ratio $R_{\text{tot}}/R_{\text{min}}$ on the Y-axis

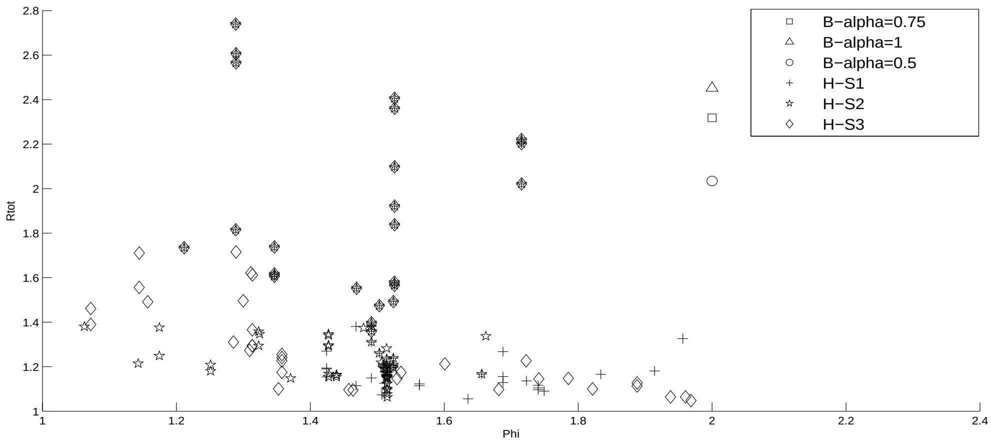

Figure 11: $I4$, 2-D plot with maximum link total risk performance ratio $\Phi/\Phi_{\min}(R_{\min})$ on the X-axis and network total risk performance ratio $R_{\mathrm{tot}}/R_{\min}$ on the Y-axis

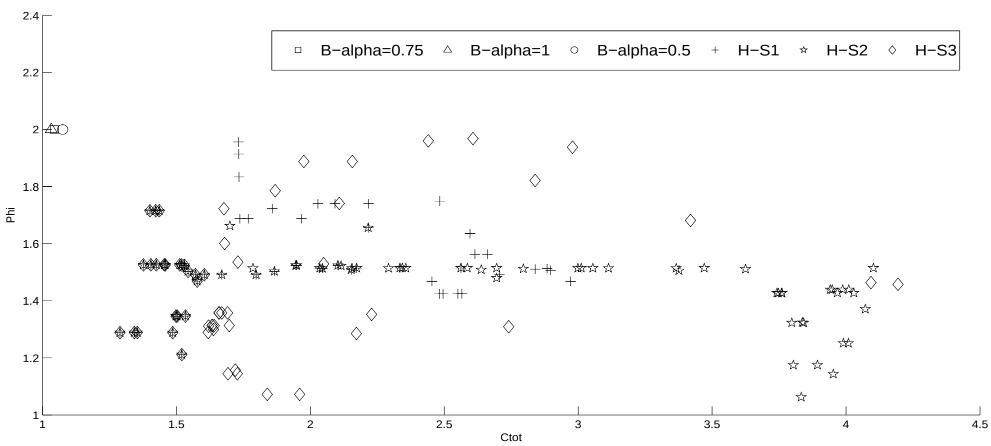

Figure 12: $I4$, 2-D plot with carriers' total cost performance ratio $C_{\mathrm{tot}}/C_{\min}$ on the X-axis and maximum link total risk performance ratio $\Phi/\Phi_{\min}(R_{\min})$ on the Y-axis

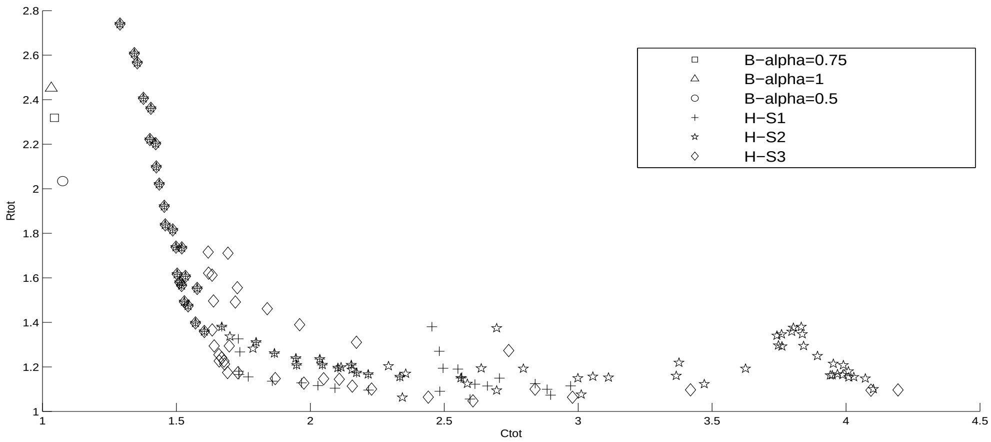

Figure 13: $I4$, 2-D plot with carriers' total cost performance ratio $C_{\text{tot}}/C_{\min}$ on the X-axis and network total risk performance ratio $R_{\text{tot}}/R_{\min}$ on the Y-axis

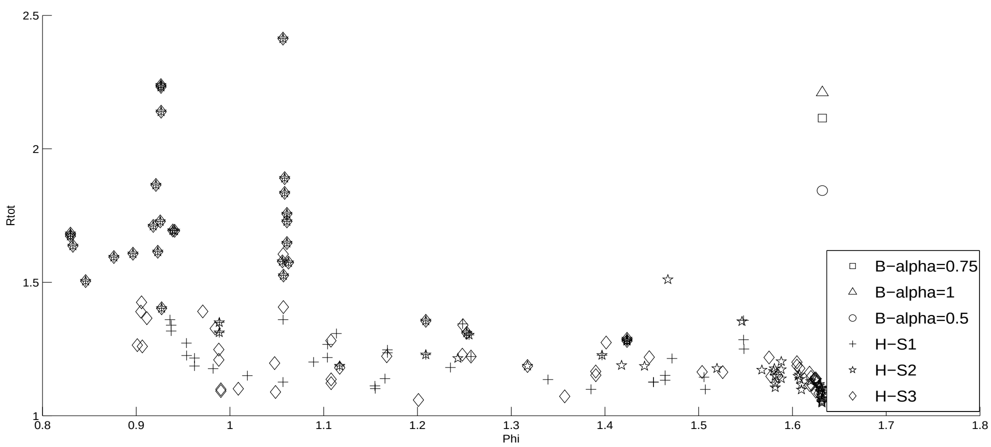

Figure 14: $I5$, 2-D plot with maximum link total risk performance ratio $\Phi/\Phi_{\min}(R_{\min})$ on the X-axis and network total risk performance ratio $R_{\text{tot}}/R_{\min}$ on the Y-axis

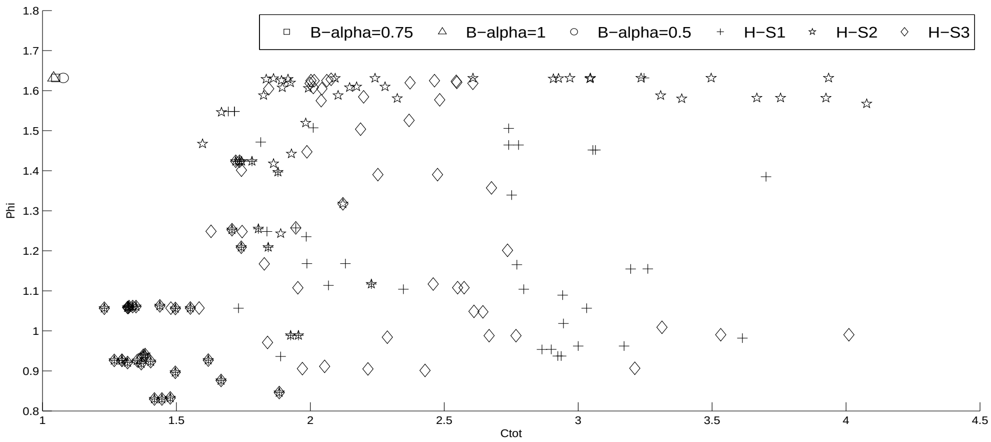

Figure 15: $I5$, 2-D plot with carriers' total cost performance ratio $C_{\text{tot}}/C_{\min}$ on the X-axis and maximum link total risk performance ratio $\Phi/\Phi_{\min}(R_{\min})$ on the Y-axis

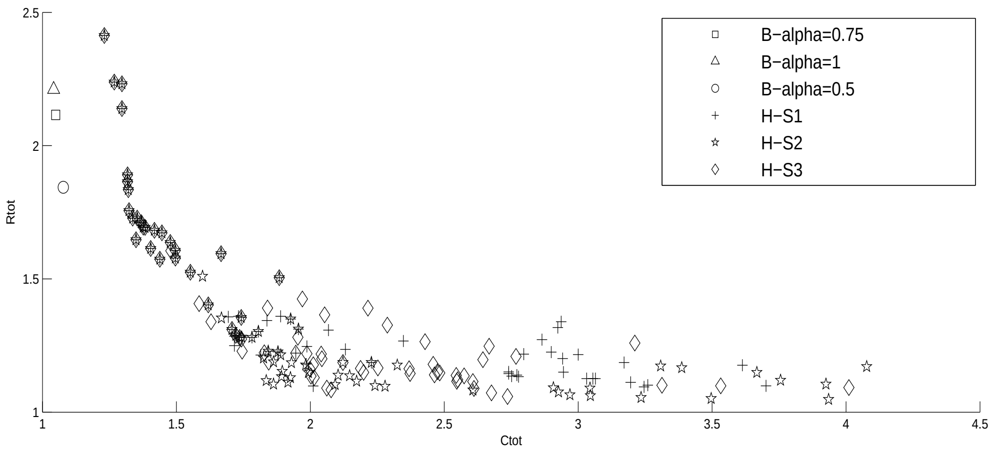

Figure 16: $I5$, 2-D plot with carriers' total cost performance ratio $C_{\text{tot}}/C_{\min}$ on the X-axis and network total risk performance ratio $R_{\text{tot}}/R_{\min}$ on the Y-axis

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